

Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/189-
7.2.2-d-x-^m-a+b-arccosh-c-x-ⁿ

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [166]. This is test number [189].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (166)	0.00 (0)
Mathematica	100.00 (166)	0.00 (0)
Maple	66.87 (111)	33.13 (55)
Maxima	34.34 (57)	65.66 (109)
Sympy	31.93 (53)	68.07 (113)
Fricas	31.33 (52)	68.67 (114)
Giac	23.49 (39)	76.51 (127)
Mupad	19.28 (32)	80.72 (134)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

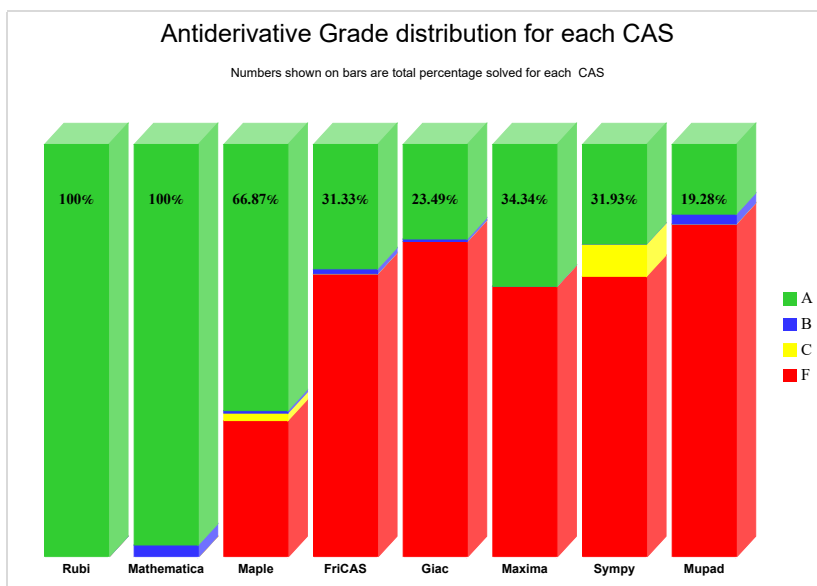
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

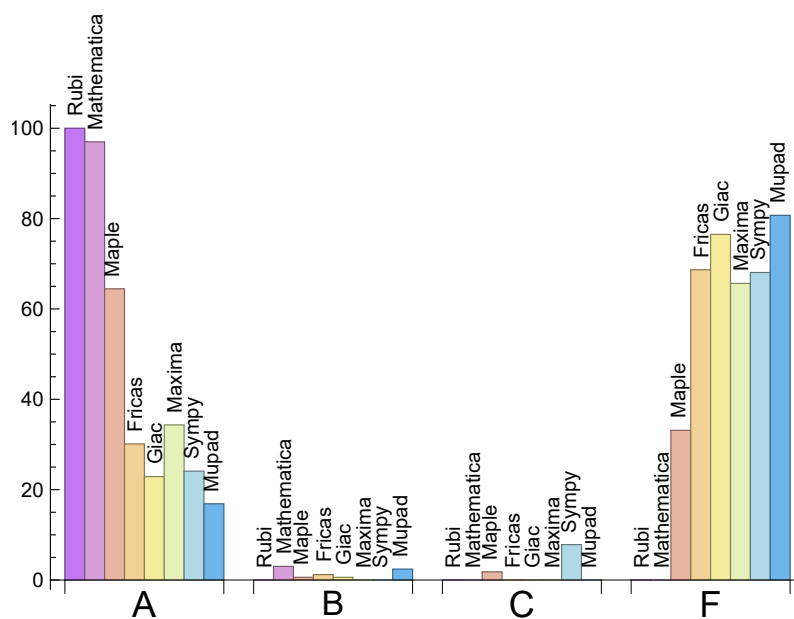
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	96.99	3.01	0.00	0.00
Maple	64.46	0.60	1.81	33.13
Maxima	34.34	0.00	0.00	65.66
Fricas	30.12	1.20	0.00	68.67
Sympy	24.10	0.00	7.83	68.07
Giac	22.89	0.60	0.00	76.51
Mupad	N/A	2.41	0.00	80.72

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**. The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	55	100.00 %	0.00 %	0.00 %
Fricas	114	40.35 %	0.00 %	59.65 %
Giac	127	63.78 %	2.36 %	33.86 %
Maxima	109	100.00 %	0.00 %	0.00 %
Sympy	113	99.12 %	0.00 %	0.88 %
Mupad	134	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

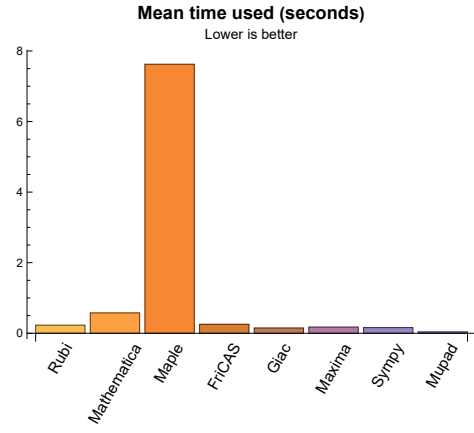
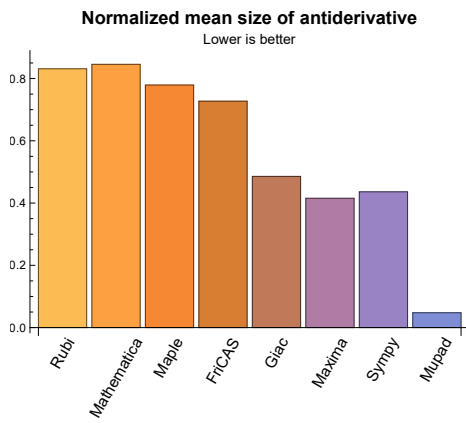
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	106.04	0.83	92.00	1.00
Mathematica	0.58	108.98	0.85	83.50	0.91
Maple	7.62	67.68	0.78	58.00	0.83
Maxima	0.17	34.39	0.42	22.00	0.66
Fricas	0.25	62.96	0.73	60.50	0.81
Sympy	0.16	50.32	0.44	0.00	0.00
Giac	0.15	27.33	0.49	0.00	0.00
Mupad	0.04	3.59	0.05	-1.00	-0.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



1.4 list of integrals that has no closed form antiderivative

{49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 166}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {2, 4, 20, 29, 30, 31, 40, 41, 51, 52, 53, 54, 55, 65, 66, 67, 68, 69, 97, 99, 101, 103, 105, 107, 109, 110, 111, 113, 145, 147, 148, 150, 154, 156, 159, 160, 162}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

The following fields are present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 39, 41, 65, 148, 150 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 25, 26, 27, 29, 31, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 75, 76, 77, 79, 81, 82, 83, 85, 87, 88, 89, 91, 93, 94, 95, 96, 98, 100, 101, 102, 106, 107, 108, 110, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 166 }

B grade: { 104 }

C grade: { 128, 130, 131 }

F grade: { 12, 14, 22, 23, 24, 28, 30, 32, 33, 34, 35, 39, 41, 72, 74, 78, 80, 84, 86, 90, 92, 97, 99, 103, 105, 109, 111, 117, 118, 127, 129, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 138, 139, 140, 141, 166 }

B grade: { }

C grade: { }

F grade: { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 117, 118, 127, 128, 129, 130, 131, 137, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 56, 57, 63, 64, 70, 71, 115, 116, 119, 120, 121, 126, 132, 133, 134, 135, 136, 139, 140, 141, 166 }

B grade: { 7, 138 }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 122, 123, 124, 125, 127, 128, 129, 130, 131, 137, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.6 Sympy

A grade: { 12, 13, 14, 15, 16, 32, 33, 34, 35, 36, 37, 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 136, 166 }

B grade: { }

C grade: { 1, 2, 3, 4, 5, 22, 23, 24, 25, 26, 133, 134, 135 }

F grade: { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 117, 118, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.7 Giac

A grade: { 4, 5, 7, 8, 9, 10, 11, 16, 21, 26, 37, 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 124, 125, 132, 135, 136, 166 }

B grade: { 19 }

C grade: { }

F grade: { 1, 2, 3, 6, 12, 13, 14, 15, 17, 18, 20, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 117, 118, 122, 123, 126, 127, 128, 129, 130, 131, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.1.8 Mupad

A grade: { 49, 50, 56, 57, 63, 64, 70, 71, 77, 83, 89, 95, 96, 102, 108, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 132, 166 }

B grade: { 4, 5, 135, 136 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 117, 118, 127, 128, 129, 130, 131, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA.

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA.	grade	A	A	A	A	A	A	C	F(-2)	F
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	93	93	55	52	68	61	76	0	-1
	N.S.	1	1.00	0.59	0.56	0.73	0.66	0.82	0.00	-0.01
	time (sec)	N/A	0.025	0.029	1.464	0.263	0.344	0.318	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	98	77	59	68	0	-1
N.S.	1	1.00	0.92	1.27	1.00	0.77	0.88	0.00	-0.01
time (sec)	N/A	0.022	0.045	1.484	0.260	0.343	0.202	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	43	48	52	54	0	-1
N.S.	1	1.00	0.71	0.66	0.74	0.80	0.83	0.00	-0.02
time (sec)	N/A	0.017	0.019	1.473	0.265	0.364	0.127	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	61	76	56	48	44	70	39
N.S.	1	1.00	1.24	1.55	1.14	0.98	0.90	1.43	0.80
time (sec)	N/A	0.011	0.019	1.464	0.286	0.349	0.085	0.397	0.040

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	25	37	26	35	26
N.S.	1	1.00	1.00	0.97	0.83	1.23	0.87	1.17	0.87
time (sec)	N/A	0.005	0.048	0.273	0.288	0.333	0.057	0.384	0.318

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	66	0	0	0	0	-1
N.S.	1	1.00	0.98	1.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.027	2.015	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	57	55	22	65	0	36	-1
N.S.	1	1.00	1.78	1.72	0.69	2.03	0.00	1.12	-0.03
time (sec)	N/A	0.013	0.020	1.473	0.464	0.365	0.000	0.392	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	40	27	38	0	50	-1
N.S.	1	1.00	0.92	1.05	0.71	1.00	0.00	1.32	-0.03
time (sec)	N/A	0.009	0.007	1.460	0.476	0.361	0.000	0.412	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	84	43	90	0	62	-1
N.S.	1	1.00	1.20	1.29	0.66	1.38	0.00	0.95	-0.02
time (sec)	N/A	0.020	0.056	2.033	0.450	0.446	0.000	0.395	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	50	48	48	0	77	-1
N.S.	1	1.00	0.68	0.76	0.73	0.73	0.00	1.17	-0.02
time (sec)	N/A	0.016	0.017	1.668	0.466	0.463	0.000	0.397	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	104	104	63	101	0	85	-1
N.S.	1	1.00	1.12	1.12	0.68	1.09	0.00	0.91	-0.01
time (sec)	N/A	0.028	0.032	1.492	0.460	0.363	0.000	0.397	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	80	0	99	99	122	0	-1
N.S.	1	1.00	0.61	0.00	0.75	0.75	0.92	0.00	-0.01
time (sec)	N/A	0.314	0.081	180.000	0.261	0.345	0.475	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	81	0	92	99	0	-1
N.S.	1	1.00	0.73	0.76	0.00	0.87	0.93	0.00	-0.01
time (sec)	N/A	0.282	0.054	1.682	0.000	0.376	0.331	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	64	0	70	82	85	0	-1
N.S.	1	1.00	0.71	0.00	0.78	0.91	0.94	0.00	-0.01
time (sec)	N/A	0.199	0.072	180.000	0.259	0.372	0.194	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	58	0	73	60	0	-1
N.S.	1	1.00	0.91	0.91	0.00	1.14	0.94	0.00	-0.02
time (sec)	N/A	0.162	0.037	1.636	0.000	0.334	0.126	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	39	32	59	39	62	-1
N.S.	1	1.00	1.00	1.00	0.82	1.51	1.00	1.59	-0.03
time (sec)	N/A	0.082	0.018	1.770	0.256	0.341	0.079	0.399	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	98	0	0	0	0	-1
N.S.	1	1.00	1.02	1.58	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.023	1.808	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	92	138	0	0	0	0	-1
N.S.	1	1.00	1.53	2.30	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.140	0.196	4.605	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	81	39	65	0	98	-1
N.S.	1	1.00	1.00	1.69	0.81	1.35	0.00	2.04	-0.02
time (sec)	N/A	0.128	0.015	3.673	0.465	0.337	0.000	0.426	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	144	171	0	0	0	0	-1
N.S.	1	1.00	1.26	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.185	4.670	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	69	123	72	85	0	147	-1
N.S.	1	1.00	0.73	1.29	0.76	0.89	0.00	1.55	-0.01
time (sec)	N/A	0.232	0.061	4.153	0.473	0.381	0.000	0.447	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	130	0	165	151	206	0	-1
N.S.	1	1.00	0.56	0.00	0.71	0.65	0.89	0.00	-0.00
time (sec)	N/A	0.488	0.082	180.000	0.277	0.385	0.704	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	143	0	0	142	170	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.78	0.93	0.00	-0.01
time (sec)	N/A	0.425	0.100	180.000	0.000	0.334	0.474	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	103	0	116	124	138	0	-1
N.S.	1	1.00	0.66	0.00	0.75	0.80	0.89	0.00	-0.01
time (sec)	N/A	0.305	0.071	180.000	0.265	0.348	0.319	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	113	88	0	112	102	0	-1
N.S.	1	1.00	1.06	0.82	0.00	1.05	0.95	0.00	-0.01
time (sec)	N/A	0.240	0.060	1.656	0.000	0.346	0.192	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	61	57	90	63	98	-1
N.S.	1	1.00	1.00	0.90	0.84	1.32	0.93	1.44	-0.01
time (sec)	N/A	0.115	0.019	1.782	0.250	0.407	0.122	0.401	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	132	0	0	0	0	-1
N.S.	1	1.00	0.94	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.037	2.566	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	128	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.127	1.899	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	92	116	0	0	0	0	-1
N.S.	1	1.00	0.94	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.574	3.599	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	201	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.401	3.050	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	220	191	0	0	0	0	-1
N.S.	1	1.00	1.26	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.384	0.450	4.013	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	175	0	0	208	275	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.68	0.90	0.00	-0.00
time (sec)	N/A	1.394	0.117	180.000	0.000	0.367	1.512	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	158	0	201	189	248	0	-1
N.S.	1	1.00	0.58	0.00	0.73	0.69	0.91	0.00	-0.00
time (sec)	N/A	1.030	0.092	180.000	0.282	0.371	1.024	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	143	0	0	176	197	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.82	0.92	0.00	-0.00
time (sec)	N/A	0.845	0.083	180.000	0.000	0.349	0.697	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	122	0	143	154	165	0	-1
N.S.	1	1.00	0.67	0.00	0.79	0.85	0.91	0.00	-0.01
time (sec)	N/A	0.573	0.081	180.000	0.271	0.367	0.469	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	104	0	138	110	0	-1
N.S.	1	1.00	0.87	0.87	0.00	1.15	0.92	0.00	-0.01
time (sec)	N/A	0.396	0.055	1.753	0.000	0.361	0.318	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	73	112	70	125	-1
N.S.	1	1.00	1.00	0.92	0.95	1.45	0.91	1.62	-0.01
time (sec)	N/A	0.195	0.022	1.805	0.274	0.448	0.173	0.461	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	165	0	0	0	0	-1
N.S.	1	1.00	1.00	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.024	2.007	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	478	0	0	0	0	0	-1
N.S.	1	1.00	3.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.462	1.849	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	112	149	0	0	0	0	-1
N.S.	1	1.00	0.97	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.772	3.642	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	595	0	0	0	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	2.148	3.149	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	0	-1
N.S.	1	1.00	0.73	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.088	2.398	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	0	-1
N.S.	1	1.00	0.77	0.77	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.073	2.280	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	0	-1
N.S.	1	1.00	0.76	0.76	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.068	2.476	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	0	-1
N.S.	1	1.00	0.83	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.052	0.055	2.119	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	0	0	0	0	-1
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.045	1.697	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	0	-1
N.S.	1	1.00	1.00	0.93	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	0.020	2.261	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.012	0.021	1.665	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	0.155	0.941	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	0.336	1.929	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	101	83	0	0	0	0	-1
N.S.	1	1.00	1.38	1.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.159	2.392	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	54	0	0	0	0	-1
N.S.	1	1.00	0.95	0.89	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.183	2.084	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	59	0	0	0	0	-1
N.S.	1	1.00	0.98	1.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.176	2.486	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	28	0	0	0	0	-1
N.S.	1	1.00	1.05	0.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.175	2.454	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	60	33	0	0	0	0	-1
N.S.	1	1.00	1.54	0.85	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.123	0.090	1.818	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	3.215	180.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	5.882	1.816	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	107	123	0	0	0	0	-1
N.S.	1	1.00	1.05	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.434	0.119	2.322	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	82	0	0	0	0	-1
N.S.	1	1.00	0.86	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.393	0.128	2.078	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	84	0	0	0	0	-1
N.S.	1	1.00	0.81	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.332	0.119	1.853	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	43	0	0	0	0	-1
N.S.	1	1.00	0.99	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.036	2.293	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	45	0	0	0	0	-1
N.S.	1	1.00	1.00	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.125	0.033	2.504	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.009	0.520	180.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	1.648	1.832	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	356	175	0	0	0	0	-1
N.S.	1	1.00	2.09	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.408	0.283	2.326	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	188	114	0	0	0	0	-1
N.S.	1	1.00	1.21	0.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.277	2.053	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	183	121	0	0	0	0	-1
N.S.	1	1.00	1.20	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.440	0.276	1.823	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	131	60	0	0	0	0	-1
N.S.	1	1.00	1.25	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.218	2.282	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	114	67	0	0	0	0	-1
N.S.	1	1.00	1.33	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.150	1.833	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	7.637	0.971	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	9.916	1.992	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	162	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.324	0.081	7.428	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	101	150	0	0	0	0	-1
N.S.	1	1.00	0.73	1.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.068	9.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	100	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.065	4.416	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	65	73	0	0	0	0	-1
N.S.	1	1.00	0.70	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.058	3.489	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	45	43	0	0	0	0	-1
N.S.	1	1.00	0.85	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.147	0.031	3.029	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	0.233	1.211	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	152	0	0	0	0	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	0.088	6.906	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	101	242	0	0	0	0	-1
N.S.	1	1.00	0.48	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	0.078	10.083	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	100	0	0	0	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.395	0.071	4.306	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	84	105	0	0	0	0	-1
N.S.	1	1.00	0.66	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.296	0.090	3.903	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	45	68	0	0	0	0	-1
N.S.	1	1.00	0.52	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.025	3.506	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	0.229	1.283	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	162	0	0	0	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.134	0.074	7.266	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	101	321	0	0	0	0	-1
N.S.	1	1.00	0.39	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.887	0.063	10.067	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	100	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.686	0.063	4.429	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	92	139	0	0	0	0	-1
N.S.	1	1.00	0.59	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.455	0.119	3.783	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	45	81	0	0	0	0	-1
N.S.	1	1.00	0.45	0.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.032	3.529	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	0.252	1.235	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	150	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.086	8.224	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	101	67	0	0	0	0	-1
N.S.	1	1.00	0.93	0.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.068	9.333	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	100	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.074	5.433	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	37	0	0	0	0	-1
N.S.	1	1.00	0.78	0.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.029	3.980	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	26	0	0	0	0	-1
N.S.	1	1.00	1.05	0.60	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.024	2.710	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	0.188	1.247	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	0.508	3.447	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	201	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.231	5.975	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	124	191	0	0	0	0	-1
N.S.	1	1.00	0.87	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.131	10.016	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	139	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.136	3.994	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	85	0	0	0	0	-1
N.S.	1	1.00	0.71	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.080	5.233	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	66	0	0	0	0	-1
N.S.	1	1.00	1.12	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.052	3.669	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	0.295	1.298	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	278	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	1.142	6.432	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	175	277	0	0	0	0	-1
N.S.	1	1.00	1.02	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.513	0.554	9.899	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	194	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.426	0.496	4.017	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	122	0	0	0	0	-1
N.S.	1	1.00	0.67	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.198	3.844	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	121	84	0	0	0	0	-1
N.S.	1	1.00	1.36	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.120	3.489	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	0.275	1.646	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	374	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	1.487	6.220	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	291	366	0	0	0	0	-1
N.S.	1	1.00	1.19	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	0.406	9.391	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	286	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.580	4.204	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	91	153	0	0	0	0	-1
N.S.	1	1.00	0.58	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.217	4.134	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	147	111	0	0	0	0	-1
N.S.	1	1.00	1.20	0.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	0.144	3.674	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	0.282	1.332	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.171	1.772	5.296	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.161	1.676	4.394	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	143	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.252	4.729	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.069	4.651	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	1.323	3.773	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	1.345	3.878	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	1.357	3.948	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	2.959	1.158	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	2.887	1.067	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	2.745	1.010	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	2.849	1.122	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.012	1.433	0.663	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	144	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.147	3.307	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	80	0	0	0	0	-1
N.S.	1	1.00	0.83	0.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.079	4.073	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.085	1.492	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	38	0	0	0	0	-1
N.S.	1	1.00	0.98	0.64	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.037	2.378	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	40	0	0	0	0	-1
N.S.	1	1.00	0.88	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.021	2.332	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	0.265	0.771	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	105	114	87	73	87	0	-1
N.S.	1	1.00	1.25	1.36	1.04	0.87	1.04	0.00	-0.01
time (sec)	N/A	0.025	0.029	1.972	0.274	0.353	0.218	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	55	58	65	71	0	-1
N.S.	1	1.00	0.76	0.77	0.82	0.92	1.00	0.00	-0.01
time (sec)	N/A	0.021	0.040	2.697	0.266	0.367	0.144	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	76	91	66	61	61	80	47
N.S.	1	1.00	1.38	1.65	1.20	1.11	1.11	1.45	0.85
time (sec)	N/A	0.013	0.027	2.122	0.265	0.384	0.095	0.477	0.411

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	34	30	43	31	41	31
N.S.	1	1.00	1.00	0.97	0.86	1.23	0.89	1.17	0.89
time (sec)	N/A	0.009	0.059	0.697	0.262	0.346	0.062	0.412	0.452

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	75	0	0	0	0	-1
N.S.	1	1.00	0.87	1.36	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.083	0.031	2.529	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	65	66	30	74	0	0	-1
N.S.	1	1.00	1.76	1.78	0.81	2.00	0.00	0.00	-0.03
time (sec)	N/A	0.018	0.053	1.945	0.463	0.373	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	52	36	48	0	0	-1
N.S.	1	1.00	1.12	1.21	0.84	1.12	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.015	1.908	0.469	0.342	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	101	91	52	100	0	0	-1
N.S.	1	1.00	1.42	1.28	0.73	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.079	1.954	0.480	0.388	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	50	62	57	60	0	0	-1
N.S.	1	1.00	0.69	0.86	0.79	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.024	1.956	0.470	0.355	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	214	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	0.348	180.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.405	0.286	180.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	100	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	0.161	180.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	540	0	0	0	0	0	-1
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.784	1.617	180.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	165	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.503	0.758	180.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	269	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.468	180.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	924	0	0	0	0	0	-1
N.S.	1	1.00	2.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.234	9.234	180.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	207	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.819	1.340	180.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	452	0	0	0	0	0	-1
N.S.	1	1.00	2.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.467	1.635	180.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	195	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.257	180.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	104	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.169	180.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	100	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.073	180.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	247	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.485	180.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.870	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	132	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.169	180.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	340	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.856	1.432	180.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	157	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.562	1.082	180.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	192	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.721	180.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	394	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.080	1.736	180.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	175	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.567	1.149	180.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	214	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.484	0.492	180.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.285	180.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	164	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.168	180.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	87	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.098	9.589	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.571	180.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [163] had the largest ratio of [18]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	8	0.500
2	A	5	5	1.00	8	0.625
3	A	4	4	1.00	8	0.500
4	A	3	3	1.00	6	0.500
5	A	2	2	1.00	4	0.500
6	A	5	5	1.00	8	0.625
7	A	3	3	1.00	8	0.375
8	A	2	2	1.00	8	0.250
9	A	5	5	1.00	8	0.625
10	A	4	4	1.00	8	0.500
11	A	7	5	1.00	8	0.625
12	A	7	5	1.00	10	0.500
13	A	6	4	1.00	10	0.400
14	A	5	5	1.00	10	0.500
15	A	4	4	1.00	8	0.500
16	A	3	3	1.00	6	0.500
17	A	6	6	1.00	10	0.600
18	A	7	5	1.00	10	0.500
19	A	3	3	1.00	10	0.300
20	A	9	7	1.00	10	0.700
21	A	5	5	1.00	10	0.500
22	A	16	7	1.00	10	0.700
23	A	12	7	1.00	10	0.700
24	A	9	7	1.00	10	0.700
25	A	6	5	1.00	8	0.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	6	0.500
27	A	7	7	1.00	10	0.700
28	A	9	6	1.00	10	0.600
29	A	7	7	1.00	10	0.700
30	A	13	9	1.00	10	0.900
31	A	10	9	1.00	10	0.900
32	A	23	4	1.00	10	0.400
33	A	19	6	1.00	10	0.600
34	A	14	4	1.00	10	0.400
35	A	11	6	1.00	10	0.600
36	A	7	4	1.00	8	0.500
37	A	5	3	1.00	6	0.500
38	A	8	7	1.00	10	0.700
39	A	11	7	1.00	10	0.700
40	A	8	8	1.00	10	0.800
41	A	19	10	1.00	10	1.000
42	A	7	3	1.00	10	0.300
43	A	6	3	1.00	10	0.300
44	A	6	3	1.00	10	0.300
45	A	5	3	1.00	10	0.300
46	A	5	3	1.00	10	0.300
47	A	4	4	1.00	8	0.500
48	A	2	2	1.00	6	0.333
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	5	2	1.00	10	0.200
52	A	4	2	1.00	10	0.200
53	A	4	2	1.00	10	0.200
54	A	2	2	1.00	8	0.250
55	A	3	3	1.00	6	0.500
56	A	0	0	0.00	0	0.000
57	A	0	0	0.00	0	0.000
58	A	14	5	1.00	10	0.500
59	A	12	6	1.00	10	0.600
60	A	10	6	1.00	10	0.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	7	1.00	8	0.875
62	A	4	4	1.00	6	0.667
63	A	0	0	0.00	0	0.000
64	A	0	0	0.00	0	0.000
65	A	12	4	1.00	10	0.400
66	A	9	4	1.00	10	0.400
67	A	10	6	1.00	10	0.600
68	A	5	5	1.00	8	0.625
69	A	5	4	1.00	6	0.667
70	A	0	0	0.00	0	0.000
71	A	0	0	0.00	0	0.000
72	A	19	7	1.00	12	0.583
73	A	14	7	1.00	12	0.583
74	A	14	7	1.00	12	0.583
75	A	9	7	1.00	10	0.700
76	A	7	6	1.00	8	0.750
77	A	0	0	0.00	0	0.000
78	A	41	10	1.00	12	0.833
79	A	25	10	1.00	12	0.833
80	A	22	10	1.00	12	0.833
81	A	11	10	1.00	10	1.000
82	A	8	7	1.00	8	0.875
83	A	0	0	0.00	0	0.000
84	A	44	10	1.00	12	0.833
85	A	27	9	1.00	12	0.750
86	A	24	10	1.00	12	0.833
87	A	12	9	1.00	10	0.900
88	A	9	7	1.00	8	0.875
89	A	0	0	0.00	0	0.000
90	A	18	6	1.00	12	0.500
91	A	13	6	1.00	12	0.500
92	A	13	6	1.00	12	0.500
93	A	8	7	1.00	10	0.700
94	A	6	5	1.00	8	0.625
95	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	0	0	0.00	0	0.000
97	A	17	5	1.00	12	0.417
98	A	12	5	1.00	12	0.417
99	A	12	5	1.00	12	0.417
100	A	6	5	1.00	10	0.500
101	A	7	6	1.00	8	0.750
102	A	0	0	0.00	0	0.000
103	A	34	8	1.00	12	0.667
104	A	24	9	1.00	12	0.750
105	A	22	9	1.00	12	0.750
106	A	11	10	1.00	10	1.000
107	A	8	7	1.00	8	0.875
108	A	0	0	0.00	0	0.000
109	A	32	7	1.00	12	0.583
110	A	21	7	1.00	12	0.583
111	A	22	9	1.00	12	0.750
112	A	9	8	1.00	10	0.800
113	A	9	7	1.00	8	0.875
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	0	0	0.00	0	0.000
117	A	2	2	1.00	10	0.200
118	A	4	4	1.00	8	0.500
119	A	0	0	0.00	0	0.000
120	A	0	0	0.00	0	0.000
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	0	0	0.00	0	0.000
127	A	12	4	1.00	10	0.400
128	A	9	4	1.00	10	0.400
129	A	9	4	1.00	10	0.400
130	A	6	5	1.00	8	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	6	0.500
132	A	0	0	0.00	0	0.000
133	A	5	5	1.00	12	0.417
134	A	4	4	1.00	12	0.333
135	A	3	3	1.00	10	0.300
136	A	3	2	1.00	8	0.250
137	A	5	5	1.00	12	0.417
138	A	3	3	1.00	12	0.250
139	A	2	2	1.00	12	0.167
140	A	5	5	1.00	12	0.417
141	A	4	4	1.00	12	0.333
142	A	14	7	1.00	16	0.438
143	A	9	7	1.00	14	0.500
144	A	7	6	1.00	12	0.500
145	A	22	10	1.00	16	0.625
146	A	11	10	1.00	14	0.714
147	A	8	7	1.00	12	0.583
148	A	24	10	1.00	16	0.625
149	A	12	9	1.00	14	0.643
150	A	9	7	1.00	12	0.583
151	A	13	6	1.00	16	0.375
152	A	8	7	1.00	14	0.500
153	A	6	5	1.00	12	0.417
154	A	12	5	1.00	16	0.312
155	A	6	5	1.00	14	0.357
156	A	7	6	1.00	12	0.500
157	A	22	9	1.00	16	0.562
158	A	11	10	1.00	14	0.714
159	A	8	7	1.00	12	0.583
160	A	22	9	1.00	16	0.562
161	A	9	8	1.00	14	0.571
162	A	9	7	1.00	12	0.583
163	A	2	2	1.00	18	0.111
164	A	2	2	1.00	16	0.125
165	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1	$\int x^4 \cosh^{-1}(ax) dx$	66
3.2	$\int x^3 \cosh^{-1}(ax) dx$	70
3.3	$\int x^2 \cosh^{-1}(ax) dx$	74
3.4	$\int x \cosh^{-1}(ax) dx$	78
3.5	$\int \cosh^{-1}(ax) dx$	82
3.6	$\int \frac{\cosh^{-1}(ax)}{x} dx$	85
3.7	$\int \frac{\cosh^{-1}(ax)}{x^2} dx$	89
3.8	$\int \frac{\cosh^{-1}(ax)}{x^3} dx$	92
3.9	$\int \frac{\cosh^{-1}(ax)}{x^4} dx$	95
3.10	$\int \frac{\cosh^{-1}(ax)}{x^5} dx$	99
3.11	$\int \frac{\cosh^{-1}(ax)}{x^6} dx$	103
3.12	$\int x^4 \cosh^{-1}(ax)^2 dx$	107
3.13	$\int x^3 \cosh^{-1}(ax)^2 dx$	111
3.14	$\int x^2 \cosh^{-1}(ax)^2 dx$	115
3.15	$\int x \cosh^{-1}(ax)^2 dx$	119
3.16	$\int \cosh^{-1}(ax)^2 dx$	123
3.17	$\int \frac{\cosh^{-1}(ax)^2}{x} dx$	126
3.18	$\int \frac{\cosh^{-1}(ax)^2}{x^2} dx$	130
3.19	$\int \frac{\cosh^{-1}(ax)^2}{x^3} dx$	134
3.20	$\int \frac{\cosh^{-1}(ax)^2}{x^4} dx$	138
3.21	$\int \frac{\cosh^{-1}(ax)^2}{x^5} dx$	143
3.22	$\int x^4 \cosh^{-1}(ax)^3 dx$	147
3.23	$\int x^3 \cosh^{-1}(ax)^3 dx$	152
3.24	$\int x^2 \cosh^{-1}(ax)^3 dx$	157
3.25	$\int x \cosh^{-1}(ax)^3 dx$	162

3.26	$\int \cosh^{-1}(ax)^3 dx$	166
3.27	$\int \frac{\cosh^{-1}(ax)^3}{x} dx$	170
3.28	$\int \frac{\cosh^{-1}(ax)^3}{x^2} dx$	175
3.29	$\int \frac{\cosh^{-1}(ax)^3}{x^3} dx$	179
3.30	$\int \frac{\cosh^{-1}(ax)^3}{x^4} dx$	184
3.31	$\int \frac{\cosh^{-1}(ax)^3}{x^5} dx$	189
3.32	$\int x^5 \cosh^{-1}(ax)^4 dx$	194
3.33	$\int x^4 \cosh^{-1}(ax)^4 dx$	198
3.34	$\int x^3 \cosh^{-1}(ax)^4 dx$	203
3.35	$\int x^2 \cosh^{-1}(ax)^4 dx$	207
3.36	$\int x \cosh^{-1}(ax)^4 dx$	211
3.37	$\int \cosh^{-1}(ax)^4 dx$	215
3.38	$\int \frac{\cosh^{-1}(ax)^4}{x} dx$	219
3.39	$\int \frac{\cosh^{-1}(ax)^4}{x^2} dx$	224
3.40	$\int \frac{\cosh^{-1}(ax)^4}{x^3} dx$	229
3.41	$\int \frac{\cosh^{-1}(ax)^4}{x^4} dx$	234
3.42	$\int \frac{x^6}{\cosh^{-1}(ax)} dx$	239
3.43	$\int \frac{x^5}{\cosh^{-1}(ax)} dx$	242
3.44	$\int \frac{x^4}{\cosh^{-1}(ax)} dx$	245
3.45	$\int \frac{x^3}{\cosh^{-1}(ax)} dx$	248
3.46	$\int \frac{x^2}{\cosh^{-1}(ax)} dx$	251
3.47	$\int \frac{x}{\cosh^{-1}(ax)} dx$	254
3.48	$\int \frac{1}{\cosh^{-1}(ax)} dx$	257
3.49	$\int \frac{1}{x \cosh^{-1}(ax)} dx$	260
3.50	$\int \frac{1}{x^2 \cosh^{-1}(ax)} dx$	263
3.51	$\int \frac{x^4}{\cosh^{-1}(ax)^2} dx$	266
3.52	$\int \frac{x^3}{\cosh^{-1}(ax)^2} dx$	269
3.53	$\int \frac{x^2}{\cosh^{-1}(ax)^2} dx$	273
3.54	$\int \frac{x}{\cosh^{-1}(ax)^2} dx$	276
3.55	$\int \frac{1}{\cosh^{-1}(ax)^2} dx$	279
3.56	$\int \frac{1}{x \cosh^{-1}(ax)^2} dx$	282
3.57	$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$	285
3.58	$\int \frac{x^4}{\cosh^{-1}(ax)^3} dx$	288
3.59	$\int \frac{x^3}{\cosh^{-1}(ax)^3} dx$	293
3.60	$\int \frac{x^2}{\cosh^{-1}(ax)^3} dx$	298
3.61	$\int \frac{x}{\cosh^{-1}(ax)^3} dx$	303
3.62	$\int \frac{1}{\cosh^{-1}(ax)^3} dx$	308

3.63	$\int \frac{1}{x \cosh^{-1}(ax)^3} dx$	312
3.64	$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} dx$	315
3.65	$\int \frac{x^4}{\cosh^{-1}(ax)^4} dx$	318
3.66	$\int \frac{x^3}{\cosh^{-1}(ax)^4} dx$	323
3.67	$\int \frac{x^2}{\cosh^{-1}(ax)^4} dx$	328
3.68	$\int \frac{x}{\cosh^{-1}(ax)^4} dx$	334
3.69	$\int \frac{1}{\cosh^{-1}(ax)^4} dx$	339
3.70	$\int \frac{1}{x \cosh^{-1}(ax)^4} dx$	344
3.71	$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} dx$	347
3.72	$\int x^4 \sqrt{\cosh^{-1}(ax)} dx$	351
3.73	$\int x^3 \sqrt{\cosh^{-1}(ax)} dx$	356
3.74	$\int x^2 \sqrt{\cosh^{-1}(ax)} dx$	361
3.75	$\int x \sqrt{\cosh^{-1}(ax)} dx$	366
3.76	$\int \sqrt{\cosh^{-1}(ax)} dx$	371
3.77	$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} dx$	375
3.78	$\int x^4 \cosh^{-1}(ax)^{3/2} dx$	378
3.79	$\int x^3 \cosh^{-1}(ax)^{3/2} dx$	384
3.80	$\int x^2 \cosh^{-1}(ax)^{3/2} dx$	389
3.81	$\int x \cosh^{-1}(ax)^{3/2} dx$	394
3.82	$\int \cosh^{-1}(ax)^{3/2} dx$	399
3.83	$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} dx$	404
3.84	$\int x^4 \cosh^{-1}(ax)^{5/2} dx$	407
3.85	$\int x^3 \cosh^{-1}(ax)^{5/2} dx$	413
3.86	$\int x^2 \cosh^{-1}(ax)^{5/2} dx$	418
3.87	$\int x \cosh^{-1}(ax)^{5/2} dx$	424
3.88	$\int \cosh^{-1}(ax)^{5/2} dx$	429
3.89	$\int \frac{\cosh^{-1}(ax)^{5/2}}{x} dx$	434
3.90	$\int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}} dx$	437
3.91	$\int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}} dx$	441
3.92	$\int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}} dx$	445
3.93	$\int \frac{x}{\sqrt{\cosh^{-1}(ax)}} dx$	449
3.94	$\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx$	453

3.95	$\int \frac{1}{x \sqrt{\cosh^{-1}(ax)}} dx$	457
3.96	$\int \frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}} dx$	460
3.97	$\int \frac{x^4}{\cosh^{-1}(ax)^{3/2}} dx$	463
3.98	$\int \frac{x^3}{\cosh^{-1}(ax)^{3/2}} dx$	467
3.99	$\int \frac{x^2}{\cosh^{-1}(ax)^{3/2}} dx$	471
3.100	$\int \frac{x}{\cosh^{-1}(ax)^{3/2}} dx$	475
3.101	$\int \frac{1}{\cosh^{-1}(ax)^{3/2}} dx$	479
3.102	$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} dx$	483
3.103	$\int \frac{x^4}{\cosh^{-1}(ax)^{5/2}} dx$	486
3.104	$\int \frac{x^3}{\cosh^{-1}(ax)^{5/2}} dx$	491
3.105	$\int \frac{x^2}{\cosh^{-1}(ax)^{5/2}} dx$	496
3.106	$\int \frac{x}{\cosh^{-1}(ax)^{5/2}} dx$	501
3.107	$\int \frac{1}{\cosh^{-1}(ax)^{5/2}} dx$	506
3.108	$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} dx$	511
3.109	$\int \frac{x^4}{\cosh^{-1}(ax)^{7/2}} dx$	514
3.110	$\int \frac{x^3}{\cosh^{-1}(ax)^{7/2}} dx$	519
3.111	$\int \frac{x^2}{\cosh^{-1}(ax)^{7/2}} dx$	524
3.112	$\int \frac{x}{\cosh^{-1}(ax)^{7/2}} dx$	529
3.113	$\int \frac{1}{\cosh^{-1}(ax)^{7/2}} dx$	534
3.114	$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$	539
3.115	$\int x^m \cosh^{-1}(ax)^4 dx$	542
3.116	$\int x^m \cosh^{-1}(ax)^3 dx$	545
3.117	$\int x^m \cosh^{-1}(ax)^2 dx$	548
3.118	$\int x^m \cosh^{-1}(ax) dx$	551
3.119	$\int \frac{x^m}{\cosh^{-1}(ax)} dx$	555
3.120	$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx$	558
3.121	$\int \frac{x^m}{\cosh^{-1}(ax)^3} dx$	561
3.122	$\int x^m \cosh^{-1}(ax)^{3/2} dx$	564
3.123	$\int x^m \sqrt{\cosh^{-1}(ax)} dx$	566
3.124	$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}} dx$	569
3.125	$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}} dx$	572
3.126	$\int (dx)^m \cosh^{-1}(ax)^n dx$	575
3.127	$\int x^4 \cosh^{-1}(ax)^n dx$	577
3.128	$\int x^3 \cosh^{-1}(ax)^n dx$	581
3.129	$\int x^2 \cosh^{-1}(ax)^n dx$	585

3.130	$\int x \cosh^{-1}(ax)^n dx$	588
3.131	$\int \cosh^{-1}(ax)^n dx$	592
3.132	$\int \frac{\cosh^{-1}(ax)^n}{x} dx$	595
3.133	$\int x^3 (a + b \cosh^{-1}(cx)) dx$	598
3.134	$\int x^2 (a + b \cosh^{-1}(cx)) dx$	602
3.135	$\int x (a + b \cosh^{-1}(cx)) dx$	606
3.136	$\int (a + b \cosh^{-1}(cx)) dx$	610
3.137	$\int \frac{a+b \cosh^{-1}(cx)}{x} dx$	613
3.138	$\int \frac{a+b \cosh^{-1}(cx)}{x^2} dx$	617
3.139	$\int \frac{a+b \cosh^{-1}(cx)}{x^3} dx$	620
3.140	$\int \frac{a+b \cosh^{-1}(cx)}{x^4} dx$	623
3.141	$\int \frac{a+b \cosh^{-1}(cx)}{x^5} dx$	627
3.142	$\int x^2 \sqrt{a + b \cosh^{-1}(cx)} dx$	631
3.143	$\int x \sqrt{a + b \cosh^{-1}(cx)} dx$	636
3.144	$\int \sqrt{a + b \cosh^{-1}(cx)} dx$	641
3.145	$\int x^2 (a + b \cosh^{-1}(cx))^{3/2} dx$	645
3.146	$\int x (a + b \cosh^{-1}(cx))^{3/2} dx$	651
3.147	$\int (a + b \cosh^{-1}(cx))^{3/2} dx$	656
3.148	$\int x^2 (a + b \cosh^{-1}(cx))^{5/2} dx$	661
3.149	$\int x (a + b \cosh^{-1}(cx))^{5/2} dx$	667
3.150	$\int (a + b \cosh^{-1}(cx))^{5/2} dx$	672
3.151	$\int \frac{x^2}{\sqrt{a + b \cosh^{-1}(cx)}} dx$	677
3.152	$\int \frac{x}{\sqrt{a + b \cosh^{-1}(cx)}} dx$	682
3.153	$\int \frac{1}{\sqrt{a + b \cosh^{-1}(cx)}} dx$	687
3.154	$\int \frac{x^2}{(a+b \cosh^{-1}(cx))^{3/2}} dx$	691
3.155	$\int \frac{x}{(a+b \cosh^{-1}(cx))^{3/2}} dx$	696
3.156	$\int \frac{1}{(a+b \cosh^{-1}(cx))^{3/2}} dx$	700
3.157	$\int \frac{x^2}{(a+b \cosh^{-1}(cx))^{5/2}} dx$	705
3.158	$\int \frac{x}{(a+b \cosh^{-1}(cx))^{5/2}} dx$	710
3.159	$\int \frac{1}{(a+b \cosh^{-1}(cx))^{5/2}} dx$	716
3.160	$\int \frac{x^2}{(a+b \cosh^{-1}(cx))^{7/2}} dx$	721
3.161	$\int \frac{x}{(a+b \cosh^{-1}(cx))^{7/2}} dx$	727

3.162	$\int \frac{1}{(a+b \cosh^{-1}(cx))^{7/2}} dx$	732
3.163	$\int \sqrt{fx} (a+b \cosh^{-1}(cx))^2 dx$	737
3.164	$\int (dx)^m (a+b \cosh^{-1}(cx))^2 dx$	740
3.165	$\int (dx)^m (a+b \cosh^{-1}(cx)) dx$	743
3.166	$\int \frac{(dx)^m}{a+b \cosh^{-1}(cx)} dx$	747

3.1 $\int x^4 \cosh^{-1}(ax) dx$

Optimal. Leaf size=93

$$-\frac{8\sqrt{-1+ax}\sqrt{1+ax}}{75a^5} - \frac{4x^2\sqrt{-1+ax}\sqrt{1+ax}}{75a^3} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax)$$

[Out] 1/5*x^5*arccosh(a*x)-8/75*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-4/75*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-1/25*x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 102, 12, 75}

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}}{75a^5} - \frac{4x^2\sqrt{ax-1}\sqrt{ax+1}}{75a^3} + \frac{1}{5}x^5 \cosh^{-1}(ax) - \frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{25a}$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcCosh[a*x],x]

[Out] (-8*sqrt[-1 + a*x]*sqrt[1 + a*x])/(75*a^5) - (4*x^2*sqrt[-1 + a*x]*sqrt[1 + a*x])/(75*a^3) - (x^4*sqrt[-1 + a*x]*sqrt[1 + a*x])/(25*a) + (x^5*ArcCosh[a*x])/5

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 75

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 102

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4 \cosh^{-1}(ax) dx &= \frac{1}{5}x^5 \cosh^{-1}(ax) - \frac{1}{5}a \int \frac{x^5}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax) - \frac{\int \frac{4x^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{25a} \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax) - \frac{4 \int \frac{x^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{25a} \\
&= -\frac{4x^2 \sqrt{-1+ax} \sqrt{1+ax}}{75a^3} - \frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax) - \frac{4 \int \frac{\sqrt{-1+ax}}{\sqrt{-1+ax}} dx}{25a} \\
&= -\frac{4x^2 \sqrt{-1+ax} \sqrt{1+ax}}{75a^3} - \frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax) - \frac{8 \int \frac{\sqrt{-1+ax}}{\sqrt{-1+ax}} dx}{25a} \\
&= -\frac{8\sqrt{-1+ax} \sqrt{1+ax}}{75a^5} - \frac{4x^2 \sqrt{-1+ax} \sqrt{1+ax}}{75a^3} - \frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 55, normalized size = 0.59

$$-\frac{\sqrt{-1+ax} \sqrt{1+ax} (8 + 4a^2x^2 + 3a^4x^4)}{75a^5} + \frac{1}{5}x^5 \cosh^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^4*ArcCosh[a*x], x]

[Out] -1/75*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(8 + 4*a^2*x^2 + 3*a^4*x^4))/a^5 + (x^5 *ArcCosh[a*x])/5

Maple [A]

time = 1.46, size = 52, normalized size = 0.56

method	result	size
--------	--------	------

derivativedivides	$\frac{\frac{a^5 x^5 \operatorname{arccosh}(ax) - \sqrt{ax-1} \sqrt{ax+1} (3a^4 x^4 + 4a^2 x^2 + 8)}{5}}{a^5}$	52
default	$\frac{\frac{a^5 x^5 \operatorname{arccosh}(ax) - \sqrt{ax-1} \sqrt{ax+1} (3a^4 x^4 + 4a^2 x^2 + 8)}{5}}{a^5}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccosh(a*x),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{a^5} * (1/5 * a^5 * x^5 * \operatorname{arccosh}(a*x) - 1/75 * (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)} * (3*a^4*x^4 + 4*a^2*x^2 + 8))$

Maxima [A]

time = 0.26, size = 68, normalized size = 0.73

$$\frac{1}{5} x^5 \operatorname{arccosh}(ax) - \frac{1}{75} \left(\frac{3 \sqrt{a^2 x^2 - 1} x^4}{a^2} + \frac{4 \sqrt{a^2 x^2 - 1} x^2}{a^4} + \frac{8 \sqrt{a^2 x^2 - 1}}{a^6} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x),x, algorithm="maxima")`

[Out] $\frac{1}{5} * x^5 * \operatorname{arccosh}(a*x) - \frac{1}{75} * (3 * \sqrt{a^2 * x^2 - 1} * x^4 / a^2 + 4 * \sqrt{a^2 * x^2 - 1} * x^2 / a^4 + 8 * \sqrt{a^2 * x^2 - 1} / a^6) * a$

Fricas [A]

time = 0.34, size = 61, normalized size = 0.66

$$\frac{15 a^5 x^5 \log(ax + \sqrt{a^2 x^2 - 1}) - (3 a^4 x^4 + 4 a^2 x^2 + 8) \sqrt{a^2 x^2 - 1}}{75 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x),x, algorithm="fricas")`

[Out] $\frac{1}{75} * (15 * a^5 * x^5 * \log(ax + \sqrt{a^2 * x^2 - 1}) - (3 * a^4 * x^4 + 4 * a^2 * x^2 + 8) * \sqrt{a^2 * x^2 - 1}) / a^5$

Sympy [C] Result contains complex when optimal does not.

time = 0.32, size = 76, normalized size = 0.82

$$\begin{cases} \frac{x^5 \operatorname{acosh}(ax)}{5} - \frac{x^4 \sqrt{a^2 x^2 - 1}}{25a} - \frac{4x^2 \sqrt{a^2 x^2 - 1}}{75a^3} - \frac{8\sqrt{a^2 x^2 - 1}}{75a^5} & \text{for } a \neq 0 \\ \frac{i\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x),x)
```

```
[Out] Piecewise((x**5*acosh(a*x)/5 - x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*x**2*sqrt(a**2*x**2 - 1)/(75*a**3) - 8*sqrt(a**2*x**2 - 1)/(75*a**5), Ne(a, 0)), (I*pi*x**5/10, True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acosh}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acosh(a*x),x)
```

```
[Out] int(x^4*acosh(a*x), x)
```

3.2 $\int x^3 \cosh^{-1}(ax) dx$

Optimal. Leaf size=77

$$-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}}{32a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{16a} - \frac{3\cosh^{-1}(ax)}{32a^4} + \frac{1}{4}x^4\cosh^{-1}(ax)$$

[Out] $-3/32*\operatorname{arccosh}(a*x)/a^4+1/4*x^4*\operatorname{arccosh}(a*x)-3/32*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-1/16*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5883, 102, 12, 92, 54}

$$-\frac{3\cosh^{-1}(ax)}{32a^4} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}}{32a^3} + \frac{1}{4}x^4\cosh^{-1}(ax) - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{16a}$$

Antiderivative was successfully verified.

[In] `Int[x^3*ArcCosh[a*x], x]`

[Out] $(-3*x*\sqrt{-1+a*x}*\sqrt{1+a*x})/(32*a^3) - (x^3*\sqrt{-1+a*x}*\sqrt{1+a*x})/(16*a) - (3*ArcCosh[a*x])/(32*a^4) + (x^4*ArcCosh[a*x])/4$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 54

`Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]`

Rule 92

`Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]`

Rule 102

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x`

```
)^(p + 1)/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
+ b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))] + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^n_.)*((d_.)*(x_.))^m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(b
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cosh^{-1}(ax) dx &= \frac{1}{4}x^4 \cosh^{-1}(ax) - \frac{1}{4}a \int \frac{x^4}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{16a} + \frac{1}{4}x^4 \cosh^{-1}(ax) - \frac{\int \frac{3x^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{16a} \\
&= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{16a} + \frac{1}{4}x^4 \cosh^{-1}(ax) - \frac{3 \int \frac{x^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{16a} \\
&= -\frac{3x \sqrt{-1+ax} \sqrt{1+ax}}{32a^3} - \frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{16a} + \frac{1}{4}x^4 \cosh^{-1}(ax) - \frac{3 \int \frac{1}{\sqrt{-1+ax}} dx}{16a} \\
&= -\frac{3x \sqrt{-1+ax} \sqrt{1+ax}}{32a^3} - \frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{16a} - \frac{3 \cosh^{-1}(ax)}{32a^4} + \frac{1}{4}x^4 \cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 71, normalized size = 0.92

$$\frac{ax \sqrt{-1+ax} \sqrt{1+ax} (3 + 2a^2x^2) - 8a^4x^4 \cosh^{-1}(ax) + 6 \tanh^{-1} \left(\sqrt{\frac{-1+ax}{1+ax}} \right)}{32a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCosh[a*x], x]
```

```
[Out] -1/32*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(3 + 2*a^2*x^2) - 8*a^4*x^4*ArcCosh
[a*x] + 6*ArcTanh[Sqrt[(-1 + a*x)/(1 + a*x)]])/a^4
```

Maple [A]

time = 1.48, size = 98, normalized size = 1.27

method	result
derivativedivides	$\frac{a^4 x^4 \operatorname{arccosh}(ax)}{4} - \frac{\sqrt{ax-1} \sqrt{ax+1} \left(\sqrt{a^2 x^2 - 1} a^3 x^3 + 3ax \sqrt{a^2 x^2 - 1} + 3 \ln(ax + \sqrt{a^2 x^2 - 1}) \right)}{32 a^4 \sqrt{a^2 x^2 - 1}}$
default	$\frac{a^4 x^4 \operatorname{arccosh}(ax)}{4} - \frac{\sqrt{ax-1} \sqrt{ax+1} \left(\sqrt{a^2 x^2 - 1} a^3 x^3 + 3ax \sqrt{a^2 x^2 - 1} + 3 \ln(ax + \sqrt{a^2 x^2 - 1}) \right)}{32 a^4 \sqrt{a^2 x^2 - 1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccosh(a*x)-1/32*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(2*(a^2*x^2-1)^(1/2)*a^3*x^3+3*a*x*(a^2*x^2-1)^(1/2)+3*ln(a*x+(a^2*x^2-1)^(1/2)))/(a^2*x^2-1)^(1/2))
```

Maxima [A]

time = 0.26, size = 77, normalized size = 1.00

$$\frac{1}{4} x^4 \operatorname{arccosh}(ax) - \frac{1}{32} \left(\frac{2 \sqrt{a^2 x^2 - 1} x^3}{a^2} + \frac{3 \sqrt{a^2 x^2 - 1} x}{a^4} + \frac{3 \log(2 a^2 x + 2 \sqrt{a^2 x^2 - 1} a)}{a^5} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccosh(a*x) - 1/32*(2*sqrt(a^2*x^2 - 1)*x^3/a^2 + 3*sqrt(a^2*x^2 - 1)*x/a^4 + 3*log(2*a^2*x + 2*sqrt(a^2*x^2 - 1)*a)/a^5)*a
```

Fricas [A]

time = 0.34, size = 59, normalized size = 0.77

$$\frac{(8 a^4 x^4 - 3) \log(ax + \sqrt{a^2 x^2 - 1}) - (2 a^3 x^3 + 3 a x) \sqrt{a^2 x^2 - 1}}{32 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 - 1)) - (2*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 - 1))/a^4
```

Sympy [C] Result contains complex when optimal does not.

time = 0.20, size = 68, normalized size = 0.88

$$\begin{cases} \frac{x^4 \operatorname{acosh}(ax)}{4} - \frac{x^3 \sqrt{a^2 x^2 - 1}}{16a} - \frac{3x \sqrt{a^2 x^2 - 1}}{32a^3} - \frac{3 \operatorname{acosh}(ax)}{32a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*acosh(a*x),x)

[Out] Piecewise((x**4*acosh(a*x)/4 - x**3*sqrt(a**2*x**2 - 1)/(16*a) - 3*x*sqrt(a**2*x**2 - 1)/(32*a**3) - 3*acosh(a*x)/(32*a**4), Ne(a, 0)), (I*pi*x**4/8, True))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccosh(a*x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*acosh(a*x),x)

[Out] int(x^3*acosh(a*x), x)

3.3 $\int x^2 \cosh^{-1}(ax) dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{9a^3} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{9a} + \frac{1}{3}x^3 \cosh^{-1}(ax)$$

[Out] $1/3*x^3*\operatorname{arccosh}(a*x)-2/9*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-1/9*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 102, 12, 75}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{9a^3} + \frac{1}{3}x^3 \cosh^{-1}(ax) - \frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{9a}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCosh[a*x], x]`

[Out] $(-2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(9*a^3) - (x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(9*a) + (x^3*\operatorname{ArcCosh}[a*x])/3$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 75

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]`

Rule 102

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]`

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x^2 \cosh^{-1}(ax) dx &= \frac{1}{3}x^3 \cosh^{-1}(ax) - \frac{1}{3}a \int \frac{x^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\ &= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{9a} + \frac{1}{3}x^3 \cosh^{-1}(ax) - \frac{\int \frac{2x}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{9a} \\ &= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{9a} + \frac{1}{3}x^3 \cosh^{-1}(ax) - \frac{2 \int \frac{x}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{9a} \\ &= -\frac{2\sqrt{-1+ax} \sqrt{1+ax}}{9a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{9a} + \frac{1}{3}x^3 \cosh^{-1}(ax) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 46, normalized size = 0.71

$$-\frac{\sqrt{-1+ax} \sqrt{1+ax} (2+a^2x^2)}{9a^3} + \frac{1}{3}x^3 \cosh^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcCosh[a*x], x]

[Out] -1/9*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2))/a^3 + (x^3*ArcCosh[a*x])/3

Maple [A]

time = 1.47, size = 43, normalized size = 0.66

method	result	size
derivativedivides	$\frac{a^3 x^3 \operatorname{arccosh}(ax)}{3} - \frac{\sqrt{ax-1} \sqrt{ax+1} (a^2 x^2 + 2)}{a^3}$	43
default	$\frac{a^3 x^3 \operatorname{arccosh}(ax)}{3} - \frac{\sqrt{ax-1} \sqrt{ax+1} (a^2 x^2 + 2)}{a^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arccosh(a*x), x, method=_RETURNVERBOSE)

[Out] $1/a^3*(1/3*a^3*x^3*\operatorname{arccosh}(a*x)-1/9*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}*(a^2*x^2+2)$
)

Maxima [A]

time = 0.26, size = 48, normalized size = 0.74

$$\frac{1}{3} x^3 \operatorname{arccosh}(ax) - \frac{1}{9} a \left(\frac{\sqrt{a^2 x^2 - 1} x^2}{a^2} + \frac{2 \sqrt{a^2 x^2 - 1}}{a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x),x, algorithm="maxima")`

[Out] $1/3*x^3*\operatorname{arccosh}(a*x) - 1/9*a*(\operatorname{sqrt}(a^2*x^2 - 1)*x^2/a^2 + 2*\operatorname{sqrt}(a^2*x^2 - 1)/a^4)$

Fricas [A]

time = 0.36, size = 52, normalized size = 0.80

$$\frac{3 a^3 x^3 \log \left(a x + \sqrt{a^2 x^2 - 1} \right) - (a^2 x^2 + 2) \sqrt{a^2 x^2 - 1}}{9 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x),x, algorithm="fricas")`

[Out] $1/9*(3*a^3*x^3*\log(a*x + \operatorname{sqrt}(a^2*x^2 - 1)) - (a^2*x^2 + 2)*\operatorname{sqrt}(a^2*x^2 - 1))/a^3$

Sympy [C] Result contains complex when optimal does not.

time = 0.13, size = 54, normalized size = 0.83

$$\begin{cases} \frac{x^3 \operatorname{acosh}(ax)}{3} - \frac{x^2 \sqrt{a^2 x^2 - 1}}{9a} - \frac{2 \sqrt{a^2 x^2 - 1}}{9a^3} & \text{for } a \neq 0 \\ \frac{i\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acosh(a*x),x)`

[Out] `Piecewise((x**3*acosh(a*x)/3 - x**2*sqrt(a**2*x**2 - 1)/(9*a) - 2*sqrt(a**2*x**2 - 1)/(9*a**3), Ne(a, 0)), (I*pi*x**3/6, True))`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{acosh}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(a*x),x)
```

```
[Out] int(x^2*acosh(a*x), x)
```

3.4 $\int x \cosh^{-1}(ax) dx$

Optimal. Leaf size=49

$$-\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{4a} - \frac{\cosh^{-1}(ax)}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)$$

[Out] $-1/4*\operatorname{arccosh}(a*x)/a^2+1/2*x^2*\operatorname{arccosh}(a*x)-1/4*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 92, 54}

$$-\frac{\cosh^{-1}(ax)}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax) - \frac{x\sqrt{ax-1}\sqrt{ax+1}}{4a}$$

Antiderivative was successfully verified.

[In] Int[x*ArcCosh[a*x],x]

[Out] $-1/4*(x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/a - \operatorname{ArcCosh}[a*x]/(4*a^2) + (x^2*\operatorname{ArcCosh}[a*x])/2$

Rule 54

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] :> Simp[ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 92

Int[((a_) + (b_)*(x_))²*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)ⁿ*(e + f*x)^p*Simp[a²*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

Rule 5883

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])ⁿ/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int x \cosh^{-1}(ax) dx &= \frac{1}{2}x^2 \cosh^{-1}(ax) - \frac{1}{2}a \int \frac{x^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x\sqrt{-1+ax} \sqrt{1+ax}}{4a} + \frac{1}{2}x^2 \cosh^{-1}(ax) - \frac{\int \frac{1}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{4a} \\
&= -\frac{x\sqrt{-1+ax} \sqrt{1+ax}}{4a} - \frac{\cosh^{-1}(ax)}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 61, normalized size = 1.24

$$\frac{ax\sqrt{-1+ax} \sqrt{1+ax} - 2a^2x^2 \cosh^{-1}(ax) + 2 \tanh^{-1}\left(\sqrt{\frac{-1+ax}{1+ax}}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*ArcCosh[a*x],x]

[Out] -1/4*(a*x*sqrt[-1 + a*x]*sqrt[1 + a*x] - 2*a^2*x^2*ArcCosh[a*x] + 2*ArcTanh[sqrt[(-1 + a*x)/(1 + a*x)]])/a^2

Maple [A]

time = 1.46, size = 76, normalized size = 1.55

method	result	size
derivativedivides	$\frac{\frac{a^2x^2 \operatorname{arccosh}(ax)}{2} - \frac{\sqrt{ax-1} \sqrt{ax+1} \left(ax \sqrt{a^2x^2-1} + \ln\left(ax + \sqrt{a^2x^2-1}\right) \right)}{4\sqrt{a^2x^2-1}}}{a^2}$	76
default	$\frac{\frac{a^2x^2 \operatorname{arccosh}(ax)}{2} - \frac{\sqrt{ax-1} \sqrt{ax+1} \left(ax \sqrt{a^2x^2-1} + \ln\left(ax + \sqrt{a^2x^2-1}\right) \right)}{4\sqrt{a^2x^2-1}}}{a^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arccosh(a*x),x,method=_RETURNVERBOSE)

[Out] 1/a^2*(1/2*a^2*x^2*arccosh(a*x)-1/4*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(a*x*(a^2*x^2-1)^(1/2)+ln(a*x+(a^2*x^2-1)^(1/2)))/(a^2*x^2-1)^(1/2))

Maxima [A]

time = 0.29, size = 56, normalized size = 1.14

$$\frac{1}{2}x^2 \operatorname{arccosh}(ax) - \frac{1}{4}a \left(\frac{\sqrt{a^2x^2-1} x}{a^2} + \frac{\log\left(2a^2x + 2\sqrt{a^2x^2-1} a\right)}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccosh(a*x),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 \operatorname{arccosh}(ax) - \frac{1}{4}a(\sqrt{a^2x^2 - 1})x/a^2 + \log(2a^2x + 2\sqrt{a^2x^2 - 1})/a^3$

Fricas [A]

time = 0.35, size = 48, normalized size = 0.98

$$-\frac{\sqrt{a^2x^2 - 1} ax - (2a^2x^2 - 1) \log(ax + \sqrt{a^2x^2 - 1})}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccosh(a*x),x, algorithm="fricas")

[Out] $-1/4(\sqrt{a^2x^2 - 1})ax - (2a^2x^2 - 1)\log(ax + \sqrt{a^2x^2 - 1})/a^2$

Sympy [C] Result contains complex when optimal does not.

time = 0.09, size = 44, normalized size = 0.90

$$\begin{cases} \frac{x^2 \operatorname{acosh}(ax)}{2} - \frac{x\sqrt{a^2x^2 - 1}}{4a} - \frac{\operatorname{acosh}(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{i\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*acosh(a*x),x)

[Out] Piecewise((x**2*acosh(a*x)/2 - x*sqrt(a**2*x**2 - 1)/(4*a) - acosh(a*x)/(4*a**2), Ne(a, 0)), (I*pi*x**2/4, True))

Giac [A]

time = 0.40, size = 70, normalized size = 1.43

$$\frac{1}{2}x^2 \log(ax + \sqrt{a^2x^2 - 1}) - \frac{1}{4}a \left(\frac{\sqrt{a^2x^2 - 1} x}{a^2} - \frac{\log(|-x|a| + \sqrt{a^2x^2 - 1}|)}{a^2|a|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccosh(a*x),x, algorithm="giac")

[Out] $\frac{1}{2}x^2 \log(ax + \sqrt{a^2x^2 - 1}) - \frac{1}{4}a(\sqrt{a^2x^2 - 1})x/a^2 - \log(\operatorname{abs}(-x\operatorname{abs}(a) + \sqrt{a^2x^2 - 1}))/a^2\operatorname{abs}(a)$

Mupad [B]

time = 0.04, size = 39, normalized size = 0.80

$$x \operatorname{acosh}(ax) \left(\frac{x}{2} - \frac{1}{4a^2x} \right) - \frac{x \sqrt{ax-1} \sqrt{ax+1}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acosh(a*x),x)`

[Out] `x*acosh(a*x)*(x/2 - 1/(4*a^2*x)) - (x*(a*x - 1)^(1/2)*(a*x + 1)^(1/2))/(4*a)`

3.5 $\int \cosh^{-1}(ax) dx$

Optimal. Leaf size=30

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a} + x \cosh^{-1}(ax)$$

[Out] $x \operatorname{arccosh}(ax) - (ax-1)^{1/2} (ax+1)^{1/2} / a$

Rubi [A]

time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5879, 75}

$$x \cosh^{-1}(ax) - \frac{\sqrt{ax-1}\sqrt{ax+1}}{a}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x], x]`

[Out] `-((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a) + x*ArcCosh[a*x]`

Rule 75

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]`

Rule 5879

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c^n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

Rubi steps

$$\begin{aligned} \int \cosh^{-1}(ax) dx &= x \cosh^{-1}(ax) - a \int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}} dx \\ &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a} + x \cosh^{-1}(ax) \end{aligned}$$

Mathematica [A]

time = 0.05, size = 30, normalized size = 1.00

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a} + x \cosh^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x],x]

[Out] $-\left(\sqrt{-1 + ax} \sqrt{1 + ax}\right)/a + x \operatorname{ArcCosh}[ax]$

Maple [A]

time = 0.27, size = 29, normalized size = 0.97

method	result	size
derivativeldivides	$\frac{ax \operatorname{arccosh}(ax) - \sqrt{ax - 1} \sqrt{ax + 1}}{a}$	29
default	$\frac{ax \operatorname{arccosh}(ax) - \sqrt{ax - 1} \sqrt{ax + 1}}{a}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x),x,method=_RETURNVERBOSE)

[Out] $1/a*(ax*\operatorname{arccosh}(ax)-(ax-1)^{(1/2)}*(ax+1)^{(1/2)})$

Maxima [A]

time = 0.29, size = 25, normalized size = 0.83

$$\frac{ax \operatorname{arccosh}(ax) - \sqrt{a^2x^2 - 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x),x, algorithm="maxima")

[Out] $(ax*\operatorname{arccosh}(ax) - \operatorname{sqrt}(a^2*x^2 - 1))/a$

Fricas [A]

time = 0.33, size = 37, normalized size = 1.23

$$\frac{ax \log\left(ax + \sqrt{a^2x^2 - 1}\right) - \sqrt{a^2x^2 - 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x),x, algorithm="fricas")

[Out] $(ax*\log(ax + \operatorname{sqrt}(a^2*x^2 - 1)) - \operatorname{sqrt}(a^2*x^2 - 1))/a$

Sympy [C] Result contains complex when optimal does not.

time = 0.06, size = 26, normalized size = 0.87

$$\begin{cases} x \operatorname{acosh}(ax) - \frac{\sqrt{a^2x^2 - 1}}{a} & \text{for } a \neq 0 \\ \frac{i\pi x}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x),x)

[Out] Piecewise((x*acosh(a*x) - sqrt(a**2*x**2 - 1)/a, Ne(a, 0)), (I*pi*x/2, True))

Giac [A]

time = 0.38, size = 35, normalized size = 1.17

$$x \log \left(ax + \sqrt{a^2 x^2 - 1} \right) - \frac{\sqrt{a^2 x^2 - 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x),x, algorithm="giac")

[Out] x*log(a*x + sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)/a

Mupad [B]

time = 0.32, size = 26, normalized size = 0.87

$$x \operatorname{acosh}(ax) - \frac{\sqrt{ax-1} \sqrt{ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x),x)

[Out] x*acosh(a*x) - ((a*x - 1)^(1/2)*(a*x + 1)^(1/2))/a

3.6 $\int \frac{\cosh^{-1}(ax)}{x} dx$

Optimal. Leaf size=43

$$-\frac{1}{2} \cosh^{-1}(ax)^2 + \cosh^{-1}(ax) \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{1}{2} \text{PolyLog}\left(2, -e^{2 \cosh^{-1}(ax)}\right)$$

[Out] $-1/2*\text{arccosh}(a*x)^2 + \text{arccosh}(a*x)*\ln(1+(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2) + 1/2*\text{polylog}(2, -(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)$

Rubi [A]

time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5882, 3799, 2221, 2317, 2438}

$$\frac{1}{2} \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right) - \frac{1}{2} \cosh^{-1}(ax)^2 + \cosh^{-1}(ax) \log\left(e^{2 \cosh^{-1}(ax)} + 1\right)$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]/x,x]`

[Out] $-1/2*\text{ArcCosh}[a*x]^2 + \text{ArcCosh}[a*x]*\text{Log}[1 + E^{(2*\text{ArcCosh}[a*x])}] + \text{PolyLog}[2, -E^{(2*\text{ArcCosh}[a*x])}]/2$

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x
_Symbol] := Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x))))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x} dx &= \text{Subst}\left(\int x \tanh(x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{1}{2} \cosh^{-1}(ax)^2 + 2\text{Subst}\left(\int \frac{e^{2x} x}{1 + e^{2x}} dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{1}{2} \cosh^{-1}(ax)^2 + \cosh^{-1}(ax) \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) - \text{Subst}\left(\int \log(1 + e^{2x}) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{1}{2} \cosh^{-1}(ax)^2 + \cosh^{-1}(ax) \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) - \frac{1}{2} \text{Subst}\left(\int \frac{\log(1 + x)}{x} dx, x, e^{2 \cosh^{-1}(ax)}\right) \\
&= -\frac{1}{2} \cosh^{-1}(ax)^2 + \cosh^{-1}(ax) \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{1}{2} \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 42, normalized size = 0.98

$$\frac{1}{2} \left(\cosh^{-1}(ax) \left(\cosh^{-1}(ax) + 2 \log\left(1 + e^{-2 \cosh^{-1}(ax)}\right) \right) - \text{PolyLog}\left(2, -e^{-2 \cosh^{-1}(ax)}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/x,x]
```

```
[Out] (ArcCosh[a*x]*(ArcCosh[a*x] + 2*Log[1 + E^(-2*ArcCosh[a*x])]) - PolyLog[2,
-E^(-2*ArcCosh[a*x])])/2
```

Maple [A]

time = 2.02, size = 66, normalized size = 1.53

method	result
derivativedivides	$-\frac{\text{arccosh}(ax)^2}{2} + \text{arccosh}(ax) \ln\left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2\right) + \frac{\text{polylog}\left(2, -\left(ax + \sqrt{ax-1} \sqrt{ax+1}\right)\right)}{2}$

default	$-\frac{\operatorname{arccosh}(ax)^2}{2} + \operatorname{arccosh}(ax) \ln \left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2 \right) + \frac{\operatorname{polylog} \left(2, - \left(ax + \sqrt{ax-1} \sqrt{ax+1} \right)^2 \right)}{2}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)/x,x,method=_RETURNVERBOSE)`

[Out] $-1/2*\operatorname{arccosh}(a*x)^2 + \operatorname{arccosh}(a*x)*\ln(1+(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2) + 1/2*\operatorname{polylog}(2,-(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x,x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)/x, x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)/x, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)/x,x)`

[Out] `Integral(acosh(a*x)/x, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a x)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/x,x)
```

```
[Out] int(acosh(a*x)/x, x)
```

3.7 $\int \frac{\cosh^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=32

$$-\frac{\cosh^{-1}(ax)}{x} + a \operatorname{ArcTan}\left(\sqrt{-1+ax} \sqrt{1+ax}\right)$$

[Out] $-\operatorname{arccosh}(a*x)/x+a*\arctan((a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})$

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5883, 94, 211}

$$a \operatorname{ArcTan}\left(\sqrt{ax-1} \sqrt{ax+1}\right) - \frac{\cosh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcCosh}[a*x]/x^2, x]$

[Out] $-(\operatorname{ArcCosh}[a*x]/x) + a*\operatorname{ArcTan}[\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]]$

Rule 94

$\operatorname{Int}[1/(\operatorname{Sqrt}[(a_.) + (b_.)*(x_.)]*\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] \rightarrow \operatorname{Dist}[b*f, \operatorname{Subst}[\operatorname{Int}[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, \operatorname{Sqrt}[a + b*x]*\operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \operatorname{EqQ}[2*b*d*e - f*(b*c + a*d), 0]$

Rule 211

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{PosQ}[a/b]$

Rule 5883

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^n/(d*(m+1))), x] - \operatorname{Dist}[b*c*(n/(d*(m+1))), \operatorname{Int}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^{(n-1)})/(\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[-1 + c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x\} \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^2} dx &= -\frac{\cosh^{-1}(ax)}{x} + a \int \frac{1}{x\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= -\frac{\cosh^{-1}(ax)}{x} + a^2 \text{Subst} \left(\int \frac{1}{a+ax^2} dx, x, \sqrt{-1+ax}\sqrt{1+ax} \right) \\
&= -\frac{\cosh^{-1}(ax)}{x} + a \tan^{-1} \left(\sqrt{-1+ax}\sqrt{1+ax} \right)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 57, normalized size = 1.78

$$-\frac{\cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+a^2x^2} \text{ArcTan}\left(\sqrt{-1+a^2x^2}\right)}{\sqrt{-1+ax}\sqrt{1+ax}}$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]/x^2,x]`

```
[Out] -(ArcCosh[a*x]/x) + (a*Sqrt[-1 + a^2*x^2]*ArcTan[Sqrt[-1 + a^2*x^2]])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Maple [A]

time = 1.47, size = 55, normalized size = 1.72

method	result	size
derivativedivides	$a \left(-\frac{\text{arccosh}(ax)}{ax} - \frac{\sqrt{ax-1}\sqrt{ax+1} \arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{\sqrt{a^2x^2-1}} \right)$	55
default	$a \left(-\frac{\text{arccosh}(ax)}{ax} - \frac{\sqrt{ax-1}\sqrt{ax+1} \arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{\sqrt{a^2x^2-1}} \right)$	55

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)/x^2,x,method=_RETURNVERBOSE)`

```
[Out] a*(-1/a/x*arccosh(a*x)-(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)))
```

Maxima [A]

time = 0.46, size = 22, normalized size = 0.69

$$-a \arcsin\left(\frac{1}{a|x|}\right) - \frac{\text{arccosh}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x^2,x, algorithm="maxima")

[Out] -a*arcsin(1/(a*abs(x))) - arccosh(a*x)/x

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(28) = 56.

time = 0.36, size = 65, normalized size = 2.03

$$\frac{2ax \arctan\left(-ax + \sqrt{a^2x^2 - 1}\right) + (x - 1) \log\left(ax + \sqrt{a^2x^2 - 1}\right) + x \log\left(-ax + \sqrt{a^2x^2 - 1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x^2,x, algorithm="fricas")

[Out] (2*a*x*arctan(-a*x + sqrt(a^2*x^2 - 1)) + (x - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + x*log(-a*x + sqrt(a^2*x^2 - 1)))/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)/x**2,x)

[Out] Integral(acosh(a*x)/x**2, x)

Giac [A]

time = 0.39, size = 36, normalized size = 1.12

$$a \arctan\left(\sqrt{a^2x^2 - 1}\right) - \frac{\log\left(ax + \sqrt{a^2x^2 - 1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x^2,x, algorithm="giac")

[Out] a*arctan(sqrt(a^2*x^2 - 1)) - log(a*x + sqrt(a^2*x^2 - 1))/x

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)/x^2,x)

[Out] int(acosh(a*x)/x^2, x)

3.8 $\int \frac{\cosh^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=38

$$\frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x} - \frac{\cosh^{-1}(ax)}{2x^2}$$

[Out] $-1/2*\operatorname{arccosh}(a*x)/x^2+1/2*a*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x$

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5883, 97}

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}}{2x} - \frac{\cosh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcCosh}[a*x]/x^3, x]$

[Out] $(a*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(2*x) - \operatorname{ArcCosh}[a*x]/(2*x^2)$

Rule 97

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[b*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}*((e + f*x)^{(p+1)})/((m+1)*(b*c - a*d)*(b*e - a*f)), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& \operatorname{EqQ}[\operatorname{Simplify}[m + n + p + 3], 0] \&\& \operatorname{EqQ}[a*d*f*(m+1) + b*c*f*(n+1) + b*d*e*(p+1), 0] \&\& \operatorname{NeQ}[m, -1]$

Rule 5883

$\operatorname{Int}[(a_. + \operatorname{ArcCosh}[(c_.)*(x_.)]*(b_.))^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^n/(d*(m+1))), x] - \operatorname{Dist}[b*c*(n/(d*(m+1))), \operatorname{Int}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^{(n-1)})/(\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[-1+c*x])], x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^{-1}(ax)}{x^3} dx &= -\frac{\cosh^{-1}(ax)}{2x^2} + \frac{1}{2}a \int \frac{1}{x^2\sqrt{-1+ax}\sqrt{1+ax}} dx \\ &= \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{2x} - \frac{\cosh^{-1}(ax)}{2x^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 35, normalized size = 0.92

$$\frac{ax\sqrt{-1+ax}\sqrt{1+ax} - \cosh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]/x^3,x]``[Out] (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] - ArcCosh[a*x])/(2*x^2)`**Maple [A]**

time = 1.46, size = 40, normalized size = 1.05

method	result	size
derivativedivides	$a^2 \left(-\frac{\operatorname{arccosh}(ax)}{2a^2x^2} + \frac{\sqrt{ax-1}\sqrt{ax+1}}{2ax} \right)$	40
default	$a^2 \left(-\frac{\operatorname{arccosh}(ax)}{2a^2x^2} + \frac{\sqrt{ax-1}\sqrt{ax+1}}{2ax} \right)$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)/x^3,x,method=_RETURNVERBOSE)``[Out] a^2*(-1/2/a^2/x^2*arccosh(a*x)+1/2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/x)`**Maxima [A]**

time = 0.48, size = 27, normalized size = 0.71

$$\frac{\sqrt{a^2x^2-1}a}{2x} - \frac{\operatorname{arccosh}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)/x^3,x, algorithm="maxima")``[Out] 1/2*sqrt(a^2*x^2 - 1)*a/x - 1/2*arccosh(a*x)/x^2`**Fricas [A]**

time = 0.36, size = 38, normalized size = 1.00

$$\frac{\sqrt{a^2x^2-1}ax - \log\left(ax + \sqrt{a^2x^2-1}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)/x^3,x, algorithm="fricas")`

[Out] $1/2*(\sqrt{a^2*x^2 - 1}*a*x - \log(a*x + \sqrt{a^2*x^2 - 1}))/x^2$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)/x**3,x)`

[Out] `Integral(acosh(a*x)/x**3, x)`

Giac [A]

time = 0.41, size = 50, normalized size = 1.32

$$\frac{a|a|}{\left(x|a| - \sqrt{a^2x^2 - 1}\right)^2 + 1} - \frac{\log\left(ax + \sqrt{a^2x^2 - 1}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x^3,x, algorithm="giac")`

[Out] $a*\operatorname{abs}(a)/((x*\operatorname{abs}(a) - \sqrt{a^2*x^2 - 1})^2 + 1) - 1/2*\log(a*x + \sqrt{a^2*x^2 - 1})/x^2$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)/x^3,x)`

[Out] `int(acosh(a*x)/x^3, x)`

3.9 $\int \frac{\cosh^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=65

$$\frac{a\sqrt{-1+ax}\sqrt{1+ax}}{6x^2} - \frac{\cosh^{-1}(ax)}{3x^3} + \frac{1}{6}a^3\text{ArcTan}\left(\sqrt{-1+ax}\sqrt{1+ax}\right)$$

[Out] $-1/3*\text{arccosh}(a*x)/x^3+1/6*a^3*\text{arctan}((a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))+1/6*a*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x^2$

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5883, 105, 12, 94, 211}

$$\frac{1}{6}a^3\text{ArcTan}\left(\sqrt{ax-1}\sqrt{ax+1}\right) - \frac{\cosh^{-1}(ax)}{3x^3} + \frac{a\sqrt{ax-1}\sqrt{ax+1}}{6x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]/x^4,x]

[Out] $(a*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(6*x^2) - \text{ArcCosh}[a*x]/(3*x^3) + (a^3*\text{ArcTan}[\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x]])/6$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 94

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 105

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])

Rule 211

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^{-1}(ax)}{x^4} dx &= -\frac{\cosh^{-1}(ax)}{3x^3} + \frac{1}{3}a \int \frac{1}{x^3 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{6x^2} - \frac{\cosh^{-1}(ax)}{3x^3} + \frac{1}{6}a \int \frac{a^2}{x\sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{6x^2} - \frac{\cosh^{-1}(ax)}{3x^3} + \frac{1}{6}a^3 \int \frac{1}{x\sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{6x^2} - \frac{\cosh^{-1}(ax)}{3x^3} + \frac{1}{6}a^4 \text{Subst}\left(\int \frac{1}{a+ax^2} dx, x, \sqrt{-1+ax} \sqrt{1+ax}\right) \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{6x^2} - \frac{\cosh^{-1}(ax)}{3x^3} + \frac{1}{6}a^3 \tan^{-1}\left(\sqrt{-1+ax} \sqrt{1+ax}\right)
 \end{aligned}$$

Mathematica [A]

time = 0.06, size = 78, normalized size = 1.20

$$\frac{-2 \cosh^{-1}(ax) + \frac{ax \left(-1 + a^2 x^2 + a^2 x^2 \sqrt{-1 + a^2 x^2} \operatorname{ArcTan}\left(\sqrt{-1 + a^2 x^2}\right) \right)}{\sqrt{-1 + ax} \sqrt{1 + ax}}}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]/x^4,x]

[Out] (-2*ArcCosh[a*x] + (a*x*(-1 + a^2*x^2 + a^2*x^2*Sqrt[-1 + a^2*x^2]*ArcTan[Sqrt[-1 + a^2*x^2]]))/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(6*x^3)

Maple [A]

time = 2.03, size = 84, normalized size = 1.29

method	result	size
derivativedivides	$a^3 \left(-\frac{\operatorname{arccosh}(ax)}{3a^3x^3} - \frac{\sqrt{ax-1} \sqrt{ax+1} \left(\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) a^2x^2 - \sqrt{a^2x^2-1} \right)}{6\sqrt{a^2x^2-1} a^2x^2} \right)$	84
default	$a^3 \left(-\frac{\operatorname{arccosh}(ax)}{3a^3x^3} - \frac{\sqrt{ax-1} \sqrt{ax+1} \left(\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) a^2x^2 - \sqrt{a^2x^2-1} \right)}{6\sqrt{a^2x^2-1} a^2x^2} \right)$	84

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)/x^4,x,method=_RETURNVERBOSE)`

[Out] $a^3 \left(-\frac{1}{3} \frac{\operatorname{arccosh}(ax)}{a^3x^3} - \frac{1}{6} \frac{(ax-1)^{1/2} (ax+1)^{1/2} \left(\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) a^2x^2 - \sqrt{a^2x^2-1} \right)}{a^2x^2 (a^2x^2-1)^{1/2}} \right) / \frac{1}{a^2x^2}$

Maxima [A]

time = 0.45, size = 43, normalized size = 0.66

$$-\frac{1}{6} \left(a^2 \arcsin\left(\frac{1}{a|x|}\right) - \frac{\sqrt{a^2x^2-1}}{x^2} \right) a - \frac{\operatorname{arcosh}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x^4,x, algorithm="maxima")`

[Out] $-\frac{1}{6} \frac{(a^2 \arcsin(1/(a \operatorname{abs}(x))) - \sqrt{a^2x^2-1}/x^2) a - 1/3 \operatorname{arccosh}(ax)}{x^3}$

Fricas [A]

time = 0.45, size = 90, normalized size = 1.38

$$\frac{2a^3x^3 \arctan(-ax + \sqrt{a^2x^2-1}) + 2x^3 \log(-ax + \sqrt{a^2x^2-1}) + \sqrt{a^2x^2-1} ax + 2(x^3-1) \log(ax + \sqrt{a^2x^2-1})}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x^4,x, algorithm="fricas")`

[Out] $\frac{1}{6} \frac{(2a^3x^3 \arctan(-ax + \sqrt{a^2x^2-1}) + 2x^3 \log(-ax + \sqrt{a^2x^2-1}) + \sqrt{a^2x^2-1} ax + 2(x^3-1) \log(ax + \sqrt{a^2x^2-1}))}{x^3}$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)/x**4,x)

[Out] Integral(acosh(a*x)/x**4, x)

Giac [A]

time = 0.40, size = 62, normalized size = 0.95

$$\frac{a^4 \arctan\left(\sqrt{a^2 x^2 - 1}\right) + \frac{\sqrt{a^2 x^2 - 1} a^2}{x^2}}{6 a} - \frac{\log\left(ax + \sqrt{a^2 x^2 - 1}\right)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x^4,x, algorithm="giac")

[Out] 1/6*(a^4*arctan(sqrt(a^2*x^2 - 1)) + sqrt(a^2*x^2 - 1)*a^2/x^2)/a - 1/3*log(a*x + sqrt(a^2*x^2 - 1))/x^3

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a x)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)/x^4,x)

[Out] int(acosh(a*x)/x^4, x)

3.10 $\int \frac{\cosh^{-1}(ax)}{x^5} dx$

Optimal. Leaf size=66

$$\frac{a\sqrt{-1+ax}\sqrt{1+ax}}{12x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{6x} - \frac{\cosh^{-1}(ax)}{4x^4}$$

[Out] $-1/4*\operatorname{arccosh}(a*x)/x^4+1/12*a*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x^3+1/6*a^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x$

Rubi [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 105, 12, 97}

$$\frac{a^3\sqrt{ax-1}\sqrt{ax+1}}{6x} - \frac{\cosh^{-1}(ax)}{4x^4} + \frac{a\sqrt{ax-1}\sqrt{ax+1}}{12x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]/x^5,x]

[Out] $(a*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(12*x^3) + (a^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(6*x) - \operatorname{ArcCosh}[a*x]/(4*x^4)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 97

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]

Rule 105

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^5} dx &= -\frac{\cosh^{-1}(ax)}{4x^4} + \frac{1}{4}a \int \frac{1}{x^4 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{12x^3} - \frac{\cosh^{-1}(ax)}{4x^4} + \frac{1}{12}a \int \frac{2a^2}{x^2 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{12x^3} - \frac{\cosh^{-1}(ax)}{4x^4} + \frac{1}{6}a^3 \int \frac{1}{x^2 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{12x^3} + \frac{a^3 \sqrt{-1+ax} \sqrt{1+ax}}{6x} - \frac{\cosh^{-1}(ax)}{4x^4}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 45, normalized size = 0.68

$$\frac{ax\sqrt{-1+ax} \sqrt{1+ax} (1+2a^2x^2) - 3\cosh^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]/x^5, x]

[Out] (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(1 + 2*a^2*x^2) - 3*ArcCosh[a*x])/(12*x^4)

Maple [A]

time = 1.67, size = 50, normalized size = 0.76

method	result	size
derivativedivides	$a^4 \left(-\frac{\operatorname{arccosh}(ax)}{4a^4x^4} + \frac{\sqrt{ax-1} \sqrt{ax+1} (2a^2x^2+1)}{12a^3x^3} \right)$	50
default	$a^4 \left(-\frac{\operatorname{arccosh}(ax)}{4a^4x^4} + \frac{\sqrt{ax-1} \sqrt{ax+1} (2a^2x^2+1)}{12a^3x^3} \right)$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)/x^5,x,method=_RETURNVERBOSE)`

[Out] $a^4 \cdot (-1/4/a^4/x^4 \cdot \operatorname{arccosh}(a \cdot x) + 1/12 \cdot (a \cdot x - 1)^{(1/2)} \cdot (a \cdot x + 1)^{(1/2)} \cdot (2 \cdot a^2 \cdot x^2 + 1)/a^3/x^3)$

Maxima [A]

time = 0.47, size = 48, normalized size = 0.73

$$\frac{1}{12} \left(\frac{2 \sqrt{a^2 x^2 - 1} a^2}{x} + \frac{\sqrt{a^2 x^2 - 1}}{x^3} \right) a - \frac{\operatorname{arccosh}(ax)}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x^5,x, algorithm="maxima")`

[Out] $1/12 \cdot (2 \cdot \sqrt{a^2 \cdot x^2 - 1} \cdot a^2/x + \sqrt{a^2 \cdot x^2 - 1}/x^3) \cdot a - 1/4 \cdot \operatorname{arccosh}(a \cdot x)/x^4$

Fricas [A]

time = 0.46, size = 48, normalized size = 0.73

$$\frac{(2 a^3 x^3 + a x) \sqrt{a^2 x^2 - 1} - 3 \log(ax + \sqrt{a^2 x^2 - 1})}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)/x^5,x, algorithm="fricas")`

[Out] $1/12 \cdot ((2 \cdot a^3 \cdot x^3 + a \cdot x) \cdot \sqrt{a^2 \cdot x^2 - 1} - 3 \cdot \log(a \cdot x + \sqrt{a^2 \cdot x^2 - 1})) / x^4$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)/x**5,x)`

[Out] `Integral(acosh(a*x)/x**5, x)`

Giac [A]

time = 0.40, size = 77, normalized size = 1.17

$$\frac{\left(3 \left(x|a| - \sqrt{a^2 x^2 - 1} \right)^2 + 1 \right) a^3 |a|}{3 \left(\left(x|a| - \sqrt{a^2 x^2 - 1} \right)^2 + 1 \right)^3} - \frac{\log(ax + \sqrt{a^2 x^2 - 1})}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x^5,x, algorithm="giac")

[Out] $\frac{1}{3}*(3*(x*\text{abs}(a) - \sqrt{a^2*x^2 - 1})^2 + 1)*a^3*\text{abs}(a)/((x*\text{abs}(a) - \sqrt{a^2*x^2 - 1})^2 + 1)^3 - 1/4*\log(a*x + \sqrt{a^2*x^2 - 1})/x^4$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\text{acosh}(a x)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)/x^5,x)

[Out] int(acosh(a*x)/x^5, x)

3.11 $\int \frac{\cosh^{-1}(ax)}{x^6} dx$

Optimal. Leaf size=93

$$\frac{a\sqrt{-1+ax}\sqrt{1+ax}}{20x^4} + \frac{3a^3\sqrt{-1+ax}\sqrt{1+ax}}{40x^2} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{3}{40}a^5 \operatorname{ArcTan}\left(\sqrt{-1+ax}\sqrt{1+ax}\right)$$

[Out] $-1/5*\operatorname{arccosh}(a*x)/x^5+3/40*a^5*\arctan((a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})+1/20*a*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x^4+3/40*a^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x^2$

Rubi [A]

time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5883, 105, 12, 94, 211}

$$\frac{3}{40}a^5 \operatorname{ArcTan}\left(\sqrt{ax-1}\sqrt{ax+1}\right) + \frac{3a^3\sqrt{ax-1}\sqrt{ax+1}}{40x^2} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{a\sqrt{ax-1}\sqrt{ax+1}}{20x^4}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]/x^6,x]`

[Out] $(a*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(20*x^4) + (3*a^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(40*x^2) - \operatorname{ArcCosh}[a*x]/(5*x^5) + (3*a^5*\operatorname{ArcTan}[\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]])/40$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 94

`Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]`

Rule 105

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])`

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 5883

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*((d_)*(x_))^(m_), x_Symbol] := Simp[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^{-1}(ax)}{x^6} dx &= -\frac{\cosh^{-1}(ax)}{5x^5} + \frac{1}{5}a \int \frac{1}{x^5 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{20x^4} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{1}{20}a \int \frac{3a^2}{x^3 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{20x^4} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{1}{20}(3a^3) \int \frac{1}{x^3 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{20x^4} + \frac{3a^3 \sqrt{-1+ax} \sqrt{1+ax}}{40x^2} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{1}{40}(3a^3) \int \frac{1}{x \sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{20x^4} + \frac{3a^3 \sqrt{-1+ax} \sqrt{1+ax}}{40x^2} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{1}{40}(3a^5) \int \frac{1}{x \sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{20x^4} + \frac{3a^3 \sqrt{-1+ax} \sqrt{1+ax}}{40x^2} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{1}{40}(3a^6) \text{Subst}\left(\int \frac{1}{x \sqrt{-1+ax} \sqrt{1+ax}} dx, x, \sqrt{-1+ax}\right) \\
 &= \frac{a\sqrt{-1+ax} \sqrt{1+ax}}{20x^4} + \frac{3a^3 \sqrt{-1+ax} \sqrt{1+ax}}{40x^2} - \frac{\cosh^{-1}(ax)}{5x^5} + \frac{3}{40}a^5 \tan^{-1}\left(\sqrt{-1+ax}\right)
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 104, normalized size = 1.12

$$\frac{2ax + a^3x^3 - 3a^5x^5 + 8\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax) - 3a^5x^5 \sqrt{-1+a^2x^2} \text{ArcTan}\left(\sqrt{-1+a^2x^2}\right)}{40x^5 \sqrt{-1+ax} \sqrt{1+ax}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]/x^6, x]

[Out] -1/40*(2*a*x + a^3*x^3 - 3*a^5*x^5 + 8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] - 3*a^5*x^5*Sqrt[-1 + a^2*x^2]*ArcTan[Sqrt[-1 + a^2*x^2]])/(x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x])

Maple [A]

time = 1.49, size = 104, normalized size = 1.12

method	result
derivativedivides	$a^5 \left(-\frac{\operatorname{arccosh}(ax)}{5a^5x^5} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{40\sqrt{a^2x^2-1}} \left(3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) a^4x^4 - 3\sqrt{a^2x^2-1} a^2x^2 - 2\sqrt{a^2x^2-1} \right) \right)$
default	$a^5 \left(-\frac{\operatorname{arccosh}(ax)}{5a^5x^5} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{40\sqrt{a^2x^2-1}} \left(3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right) a^4x^4 - 3\sqrt{a^2x^2-1} a^2x^2 - 2\sqrt{a^2x^2-1} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-1/5/a^5/x^5*arccosh(a*x)-1/40*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(3*arctan(1/(a^2*x^2-1)^(1/2))*a^4*x^4-3*(a^2*x^2-1)^(1/2)*a^2*x^2-2*(a^2*x^2-1)^(1/2))/((a^2*x^2-1)^(1/2)/a^4/x^4)
```

Maxima [A]

time = 0.46, size = 63, normalized size = 0.68

$$-\frac{1}{40} \left(3a^4 \arcsin\left(\frac{1}{a|x|}\right) - \frac{3\sqrt{a^2x^2-1}a^2}{x^2} - \frac{2\sqrt{a^2x^2-1}}{x^4} \right) a - \frac{\operatorname{arccosh}(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/40*(3*a^4*arcsin(1/(a*abs(x)))) - 3*sqrt(a^2*x^2 - 1)*a^2/x^2 - 2*sqrt(a^2*x^2 - 1)/x^4)*a - 1/5*arccosh(a*x)/x^5
```

Fricas [A]

time = 0.36, size = 101, normalized size = 1.09

$$\frac{6a^5x^5 \arctan(-ax + \sqrt{a^2x^2-1}) + 8x^5 \log(-ax + \sqrt{a^2x^2-1}) + 8(x^5-1) \log(ax + \sqrt{a^2x^2-1}) + (3a^3x^3 + 2ax)\sqrt{a^2x^2-1}}{40x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(6*a^5*x^5*arctan(-a*x + sqrt(a^2*x^2 - 1)) + 8*x^5*log(-a*x + sqrt(a^2*x^2 - 1)) + 8*(x^5 - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + (3*a^3*x^3 + 2*a*x)*sqrt(a^2*x^2 - 1))/x^5
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)/x**6,x)

[Out] Integral(acosh(a*x)/x**6, x)

Giac [A]

time = 0.40, size = 85, normalized size = 0.91

$$\frac{3 a^6 \arctan\left(\sqrt{a^2 x^2 - 1}\right) + \frac{3(a^2 x^2 - 1)^{\frac{3}{2}} a^6 + 5 \sqrt{a^2 x^2 - 1} a^6}{a^4 x^4}}{40 a} - \frac{\log\left(ax + \sqrt{a^2 x^2 - 1}\right)}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x^6,x, algorithm="giac")

[Out] 1/40*(3*a^6*arctan(sqrt(a^2*x^2 - 1)) + (3*(a^2*x^2 - 1)^(3/2)*a^6 + 5*sqrt(a^2*x^2 - 1)*a^6)/(a^4*x^4))/a - 1/5*log(a*x + sqrt(a^2*x^2 - 1))/x^5

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)/x^6,x)

[Out] int(acosh(a*x)/x^6, x)

3.12 $\int x^4 \cosh^{-1}(ax)^2 dx$

Optimal. Leaf size=132

$$\frac{16x}{75a^4} + \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^5} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{75a^3}$$

[Out] 16/75*x/a^4+8/225*x^3/a^2+2/125*x^5+1/5*x^5*arccosh(a*x)^2-16/75*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-8/75*x^2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-2/25*x^4*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 0.31, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 5939, 5915, 8, 30}

$$-\frac{16\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{75a^5} + \frac{16x}{75a^4} - \frac{8x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{75a^3} + \frac{8x^3}{225a^2} + \frac{1}{5}x^5\cosh^{-1}(ax)^2 - \frac{2x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{25a} + \frac{2x^5}{125}$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcCosh[a*x]^2,x]

[Out] (16*x)/(75*a^4) + (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/(75*a^5) - (8*x^2*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/(75*a^3) - (2*x^4*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/(25*a) + (x^5*ArcCosh[a*x]^2)/5

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^ (n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(sqrt[1 + c*x]*sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^ (n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p_) * ((d2_) + (e2_.)*(x_))^(q_), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^q, x]

$e2*x)^{(p+1)}*((a+b*\text{ArcCosh}[c*x])^n/(2*e1*e2*(p+1))), x] - \text{Dist}[b*(n/(2*c*(p+1)))*\text{Simp}[(d1+e1*x)^p/(1+c*x)^p]*\text{Simp}[(d2+e2*x)^p/(-1+c*x)^p], \text{Int}[(1+c*x)^{(p+1/2)}*(-1+c*x)^{(p+1/2)}*(a+b*\text{ArcCosh}[c*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d1, e1, d2, e2, p\}, x] \&\& \text{EqQ}[e1, c*d1] \&\& \text{EqQ}[e2, (-c)*d2] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[p, -1]$

Rule 5939

$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^{(n-1)}*((f*x)^m*((d1 + (e1*x)^p)*((d2 + (e2*x)^p)), x_Symbol] :> \text{Simp}[f*(f*x)^{(m-1)}*(d1 + e1*x)^{(p+1)}*(d2 + e2*x)^{(p+1)}*((a + b*\text{ArcCosh}[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (\text{Dist}[f^2*((m-1)/(c^2*(m + 2*p + 1))), \text{Int}[(f*x)^{(m-2)}*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*\text{ArcCosh}[c*x])^n, x], x] - \text{Dist}[b*f*(n/(c*(m + 2*p + 1)))*\text{Simp}[(d1 + e1*x)^p/(1 + c*x)^p]*\text{Simp}[(d2 + e2*x)^p/(-1 + c*x)^p], \text{Int}[(f*x)^{(m-1)}*(1 + c*x)^{(p+1/2)}*(-1 + c*x)^{(p+1/2)}*(a + b*\text{ArcCosh}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d1, e1, d2, e2, f, p\}, x] \&\& \text{EqQ}[e1, c*d1] \&\& \text{EqQ}[e2, (-c)*d2] \&\& \text{GtQ}[n, 0] \&\& \text{IGtQ}[m, 1] \&\& \text{NeQ}[m + 2*p + 1, 0]$

Rubi steps

$$\begin{aligned} \int x^4 \cosh^{-1}(ax)^2 dx &= \frac{1}{5}x^5 \cosh^{-1}(ax)^2 - \frac{1}{5}(2a) \int \frac{x^5 \cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\ &= -\frac{2x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax)^2 + \frac{2 \int x^4 dx}{25} - \frac{8 \int \frac{x}{\sqrt{-1+ax}} dx}{25} \\ &= \frac{2x^5}{125} - \frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{75a^3} - \frac{2x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{25a} \\ &= \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{75a^5} - \frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{75a^3} \\ &= \frac{16x}{75a^4} + \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{75a^5} - \frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{75a^3} \end{aligned}$$

Mathematica [A]

time = 0.08, size = 80, normalized size = 0.61

$$\frac{\frac{240x}{a^4} + \frac{40x^3}{a^2} + 18x^5 - \frac{30\sqrt{-1+ax} \sqrt{1+ax} (8+4a^2x^2+3a^4x^4) \cosh^{-1}(ax)}{a^5} + 225x^5 \cosh^{-1}(ax)^2}{1125}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*ArcCosh[a*x]^2,x]

[Out] $((240*x)/a^4 + (40*x^3)/a^2 + 18*x^5 - (30*\sqrt{-1 + a*x})*\sqrt{1 + a*x}*(8 + 4*a^2*x^2 + 3*a^4*x^4)*\text{ArcCosh}[a*x])/a^5 + 225*x^5*\text{ArcCosh}[a*x]^2)/1125$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccosh(a*x)^2,x)`

[Out] `int(x^4*arccosh(a*x)^2,x)`

Maxima [A]

time = 0.26, size = 99, normalized size = 0.75

$$\frac{1}{5}x^5 \operatorname{arccosh}(ax)^2 - \frac{2}{75} \left(\frac{3\sqrt{a^2x^2-1}x^4}{a^2} + \frac{4\sqrt{a^2x^2-1}x^2}{a^4} + \frac{8\sqrt{a^2x^2-1}}{a^6} \right) a \operatorname{arccosh}(ax) + \frac{2(9a^4x^5 + 20a^2x^3 + 120x)}{1125a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^2,x, algorithm="maxima")`

[Out] $1/5*x^5*\operatorname{arccosh}(a*x)^2 - 2/75*(3*\sqrt{a^2*x^2 - 1}*x^4/a^2 + 4*\sqrt{a^2*x^2 - 1}*x^2/a^4 + 8*\sqrt{a^2*x^2 - 1}/a^6)*a*\operatorname{arccosh}(a*x) + 2/1125*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)/a^4$

Fricas [A]

time = 0.35, size = 99, normalized size = 0.75

$$\frac{225a^5x^5 \log\left(ax + \sqrt{a^2x^2-1}\right)^2 + 18a^5x^5 + 40a^3x^3 - 30(3a^4x^4 + 4a^2x^2 + 8)\sqrt{a^2x^2-1} \log\left(ax + \sqrt{a^2x^2-1}\right) + 240ax}{1125a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^2,x, algorithm="fricas")`

[Out] $1/1125*(225*a^5*x^5*\log(a*x + \sqrt{a^2*x^2 - 1})^2 + 18*a^5*x^5 + 40*a^3*x^3 - 30*(3*a^4*x^4 + 4*a^2*x^2 + 8)*\sqrt{a^2*x^2 - 1}*\log(a*x + \sqrt{a^2*x^2 - 1}) + 240*a*x)/a^5$

Sympy [A]

time = 0.47, size = 122, normalized size = 0.92

$$\begin{cases} \frac{x^5 \operatorname{acosh}^2(ax)}{5} + \frac{2x^5}{125} - \frac{2x^4 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{25a} + \frac{8x^3}{225a^2} - \frac{8x^2 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{75a^3} + \frac{16x}{75a^4} - \frac{16 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{75a^5} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^5}{20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*acosh(a*x)**2,x)

[Out] Piecewise((x**5*acosh(a*x)**2/5 + 2*x**5/125 - 2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)/(25*a) + 8*x**3/(225*a**2) - 8*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/(75*a**3) + 16*x/(75*a**4) - 16*sqrt(a**2*x**2 - 1)*acosh(a*x)/(75*a**5), Ne(a, 0)), (-pi**2*x**5/20, True))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccosh(a*x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*acosh(a*x)^2,x)

[Out] int(x^4*acosh(a*x)^2, x)

3.13 $\int x^3 \cosh^{-1}(ax)^2 dx$

Optimal. Leaf size=106

$$\frac{3x^2}{32a^2} + \frac{x^4}{32} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{8a} - \frac{3\cosh^{-1}(ax)^2}{32a^4} + \frac{1}{4}x^4 \cosh^{-1}(ax)$$

[Out] $3/32*x^2/a^2+1/32*x^4-3/32*\operatorname{arccosh}(a*x)^2/a^4+1/4*x^4*\operatorname{arccosh}(a*x)^2-3/16*x*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-1/8*x^3*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.28, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,

Rules used = {5883, 5939, 5893, 30}

$$-\frac{3\cosh^{-1}(ax)^2}{32a^4} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{16a^3} + \frac{3x^2}{32a^2} + \frac{1}{4}x^4\cosh^{-1}(ax)^2 - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{8a} + \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcCosh[a*x]^2,x]

[Out] $(3*x^2)/(32*a^2) + x^4/32 - (3*x*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x])/(16*a^3) - (x^3*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x])/(8*a) - (3*\operatorname{ArcCosh}[a*x]^2)/(32*a^4) + (x^4*\operatorname{ArcCosh}[a*x]^2)/4$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5883

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol] := Simp[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5893

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)/(Sqrt[(d1_) + (e1_)*(x_)]*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n+1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n+1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned} \int x^3 \cosh^{-1}(ax)^2 dx &= \frac{1}{4}x^4 \cosh^{-1}(ax)^2 - \frac{1}{2}a \int \frac{x^4 \cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\ &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{8a} + \frac{1}{4}x^4 \cosh^{-1}(ax)^2 + \frac{\int x^3 dx}{8} - \frac{3 \int \frac{x^2 \cos}{\sqrt{-1+ax}}}{8a} \\ &= \frac{x^4}{32} - \frac{3x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{16a^3} - \frac{x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{8a} + \frac{x^4}{32} \\ &= \frac{3x^2}{32a^2} + \frac{x^4}{32} - \frac{3x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{16a^3} - \frac{x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{8a} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 77, normalized size = 0.73

$$\frac{a^2 x^2 (3 + a^2 x^2) - 2 a x \sqrt{-1 + a x} \sqrt{1 + a x} (3 + 2 a^2 x^2) \cosh^{-1}(a x) + (-3 + 8 a^4 x^4) \cosh^{-1}(a x)^2}{32 a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCosh[a*x]^2,x]

[Out] (a^2*x^2*(3 + a^2*x^2) - 2*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(3 + 2*a^2*x^2)*ArcCosh[a*x] + (-3 + 8*a^4*x^4)*ArcCosh[a*x]^2)/(32*a^4)

Maple [A]

time = 1.68, size = 81, normalized size = 0.76

$$\frac{\left(\frac{8 \operatorname{arccosh}(ax)^2 + 1}{128}\right) \left(\cosh^2(2 \operatorname{arccosh}(ax))\right) + \left(\frac{4 \operatorname{arccosh}(ax)^2 - \operatorname{arccosh}(ax) \sinh(2 \operatorname{arccosh}(ax)) + 2}{32}\right) \cosh(2 \operatorname{arccosh}(ax)) - \frac{\operatorname{arccosh}(ax)^2}{32} - \frac{\operatorname{arccosh}(ax)}{32}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arccosh(a*x)^2,x)`

[Out] $1/a^4*(1/128*(8*\operatorname{arccosh}(a*x)^2+1)*\cosh(2*\operatorname{arccosh}(a*x))^2+1/32*(4*\operatorname{arccosh}(a*x)^2-\operatorname{arccosh}(a*x)*\sinh(2*\operatorname{arccosh}(a*x))+2)*\cosh(2*\operatorname{arccosh}(a*x))-1/32*\operatorname{arccosh}(a*x)^2-1/8*\operatorname{arccosh}(a*x)*\sinh(2*\operatorname{arccosh}(a*x))-1/256)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccosh(a*x)^2,x, algorithm="maxima")`

[Out] $1/4*x^4*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^2 - \operatorname{integrate}(1/2*(a^3*x^6 + \sqrt{a*x + 1}*\sqrt{a*x - 1}*a^2*x^5 - a*x^4)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))/ (a^3*x^3 + (a^2*x^2 - 1)*\sqrt{a*x + 1}*\sqrt{a*x - 1} - a*x), x)$

Fricas [A]

time = 0.38, size = 92, normalized size = 0.87

$$\frac{a^4x^4 + 3a^2x^2 + (8a^4x^4 - 3)\log\left(ax + \sqrt{a^2x^2 - 1}\right)^2 - 2(2a^3x^3 + 3ax)\sqrt{a^2x^2 - 1}\log\left(ax + \sqrt{a^2x^2 - 1}\right)}{32a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccosh(a*x)^2,x, algorithm="fricas")`

[Out] $1/32*(a^4*x^4 + 3*a^2*x^2 + (8*a^4*x^4 - 3)*\log(a*x + \sqrt{a^2*x^2 - 1}))^2 - 2*(2*a^3*x^3 + 3*a*x)*\sqrt{a^2*x^2 - 1}*\log(a*x + \sqrt{a^2*x^2 - 1}))/a^4$

Sympy [A]

time = 0.33, size = 99, normalized size = 0.93

$$\begin{cases} \frac{x^4 \operatorname{acosh}^2(ax)}{4} + \frac{x^4}{32} - \frac{x^3 \sqrt{a^2x^2 - 1} \operatorname{acosh}(ax)}{8a} + \frac{3x^2}{32a^2} - \frac{3x \sqrt{a^2x^2 - 1} \operatorname{acosh}(ax)}{16a^3} - \frac{3 \operatorname{acosh}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acosh(a*x)**2,x)`

[Out] `Piecewise((x**4*acosh(a*x)**2/4 + x**4/32 - x**3*sqrt(a**2*x**2 - 1)*acosh(a*x)/(8*a) + 3*x**2/(32*a**2) - 3*x*sqrt(a**2*x**2 - 1)*acosh(a*x)/(16*a**3) - 3*acosh(a*x)**2/(32*a**4), Ne(a, 0)), (-pi**2*x**4/16, True))`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccosh(a*x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*acosh(a*x)^2,x)

[Out] int(x^3*acosh(a*x)^2, x)

3.14 $\int x^2 \cosh^{-1}(ax)^2 dx$

Optimal. Leaf size=90

$$\frac{4x}{9a^2} + \frac{2x^3}{27} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a} + \frac{1}{3}x^3\cosh^{-1}(ax)^2$$

[Out] $4/9*x/a^2+2/27*x^3+1/3*x^3*\operatorname{arccosh}(a*x)^2-4/9*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-2/9*x^2*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.20, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 5939, 5915, 8, 30}

$$-\frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a^3} + \frac{4x}{9a^2} + \frac{1}{3}x^3\cosh^{-1}(ax)^2 - \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{9a} + \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCosh[a*x]^2,x]`

[Out] $(4*x)/(9*a^2) + (2*x^3)/27 - (4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x])/(9*a^3) - (2*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x])/(9*a) + (x^3*\operatorname{ArcCosh}[a*x]^2)/3$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 5883

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1+c*x]*Sqrt[-1+c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

Rule 5915

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d1_.) + (e1_.)*(x_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[(d1 + e1*x)^(p+1)*(d2 + e2*x)^(p+1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p+1))), x] - Dist[b*(n/(2`

c(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned} \int x^2 \cosh^{-1}(ax)^2 dx &= \frac{1}{3}x^3 \cosh^{-1}(ax)^2 - \frac{1}{3}(2a) \int \frac{x^3 \cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\ &= -\frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{9a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^2 + \frac{2 \int x^2 dx}{9} - \frac{4 \int \frac{x}{\sqrt{-1+ax}} dx}{9} \\ &= \frac{2x^3}{27} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{9a^3} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{9a} + \frac{2x^2}{9} \\ &= \frac{4x}{9a^2} + \frac{2x^3}{27} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{9a^3} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{9a} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 64, normalized size = 0.71

$$\frac{1}{27} \left(2x \left(\frac{6}{a^2} + x^2 \right) - \frac{6\sqrt{-1+ax} \sqrt{1+ax} (2 + a^2x^2) \cosh^{-1}(ax)}{a^3} + 9x^3 \cosh^{-1}(ax)^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcCosh[a*x]^2,x]

[Out] (2*x*(6/a^2 + x^2) - (6*sqrt[-1 + a*x]*sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x])/a^3 + 9*x^3*ArcCosh[a*x]^2)/27

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arccosh(a*x)^2,x)**[Out]** int(x^2*arccosh(a*x)^2,x)**Maxima [A]**

time = 0.26, size = 70, normalized size = 0.78

$$\frac{1}{3} x^3 \operatorname{arccosh}(ax)^2 - \frac{2}{9} a \left(\frac{\sqrt{a^2 x^2 - 1} x^2}{a^2} + \frac{2 \sqrt{a^2 x^2 - 1}}{a^4} \right) \operatorname{arccosh}(ax) + \frac{2(a^2 x^3 + 6x)}{27 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^2,x, algorithm="maxima")**[Out]** 1/3*x^3*arccosh(a*x)^2 - 2/9*a*(sqrt(a^2*x^2 - 1)*x^2/a^2 + 2*sqrt(a^2*x^2 - 1)/a^4)*arccosh(a*x) + 2/27*(a^2*x^3 + 6*x)/a^2**Fricas [A]**

time = 0.37, size = 82, normalized size = 0.91

$$\frac{9 a^3 x^3 \log \left(a x + \sqrt{a^2 x^2 - 1} \right)^2 + 2 a^3 x^3 - 6 (a^2 x^2 + 2) \sqrt{a^2 x^2 - 1} \log \left(a x + \sqrt{a^2 x^2 - 1} \right) + 12 a x}{27 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^2,x, algorithm="fricas")**[Out]** 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a^3*x^3 - 6*(a^2*x^2 + 2)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + 12*a*x)/a^3**Sympy [A]**

time = 0.19, size = 85, normalized size = 0.94

$$\begin{cases} \frac{x^3 \operatorname{acosh}^2(ax)}{3} + \frac{2x^3}{27} - \frac{2x^2 \sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{9a} + \frac{4x}{9a^2} - \frac{4 \sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{9a^3} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^3}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*acosh(a*x)**2,x)

```
[Out] Piecewise((x**3*acosh(a*x)**2/3 + 2*x**3/27 - 2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/(9*a) + 4*x/(9*a**2) - 4*sqrt(a**2*x**2 - 1)*acosh(a*x)/(9*a**3), Ne(a, 0)), (-pi**2*x**3/12, True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(a*x)^2,x)
```

```
[Out] int(x^2*acosh(a*x)^2, x)
```

3.15 $\int x \cosh^{-1}(ax)^2 dx$

Optimal. Leaf size=64

$$\frac{x^2}{4} - \frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a} - \frac{\cosh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^2$$

[Out] $1/4*x^2-1/4*arccosh(a*x)^2/a^2+1/2*x^2*arccosh(a*x)^2-1/2*x*arccosh(a*x)*(a*x-1)^{(1/2)*(a*x+1)^{(1/2)}/a}$

Rubi [A]

time = 0.16, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 5939, 5893, 30}

$$-\frac{\cosh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^2 - \frac{x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{2a} + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x*ArcCosh[a*x]^2,x]

[Out] $x^2/4 - (x*\text{Sqrt}[-1 + a*x]*\text{Sqrt}[1 + a*x]*\text{ArcCosh}[a*x])/(2*a) - \text{ArcCosh}[a*x]^2/(4*a^2) + (x^2*\text{ArcCosh}[a*x]^2)/2$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^ (n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^ (n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(q_.), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(q + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^q*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-
1 + c*x)^q], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(q + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int x \cosh^{-1}(ax)^2 dx &= \frac{1}{2}x^2 \cosh^{-1}(ax)^2 - a \int \frac{x^2 \cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{2a} + \frac{1}{2}x^2 \cosh^{-1}(ax)^2 + \frac{\int x dx}{2} - \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{2a} \\
&= \frac{x^2}{4} - \frac{x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{2a} - \frac{\cosh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^2
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 58, normalized size = 0.91

$$\frac{a^2 x^2 - 2ax\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax) + (-1 + 2a^2 x^2) \cosh^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcCosh[a*x]^2,x]

[Out] (a^2*x^2 - 2*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + (-1 + 2*a^2*x^2)*ArcCosh[a*x]^2)/(4*a^2)

Maple [A]

time = 1.64, size = 58, normalized size = 0.91

method	result	size
derivativedivides	$\frac{a^2 x^2 \operatorname{arccosh}(ax)^2}{2} - \frac{ax \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1}}{a^2} - \frac{\operatorname{arccosh}(ax)^2}{4} + \frac{a^2 x^2}{4}$	58
default	$\frac{a^2 x^2 \operatorname{arccosh}(ax)^2}{2} - \frac{ax \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1}}{a^2} - \frac{\operatorname{arccosh}(ax)^2}{4} + \frac{a^2 x^2}{4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccosh(a*x)^2-1/2*a*x*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/4*arccosh(a*x)^2+1/4*a^2*x^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 - integrate((a^3*x^4 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^3 - a*x^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A]

time = 0.33, size = 73, normalized size = 1.14

$$\frac{a^2 x^2 - 2 \sqrt{a^2 x^2 - 1} a x \log \left(a x + \sqrt{a^2 x^2 - 1} \right) + (2 a^2 x^2 - 1) \log \left(a x + \sqrt{a^2 x^2 - 1} \right)^2}{4 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 - 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1)) + (2*a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2)/a^2
```

Sympy [A]

time = 0.13, size = 60, normalized size = 0.94

$$\begin{cases} \frac{x^2 \operatorname{acosh}^2(ax)}{2} + \frac{x^2}{4} - \frac{x \sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{2a} - \frac{\operatorname{acosh}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^2}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acosh(a*x)**2/2 + x**2/4 - x*sqrt(a**2*x**2 - 1)*acosh(a*x)/(2*a) - acosh(a*x)**2/(4*a**2), Ne(a, 0)), (-pi**2*x**2/8, True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^2,x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acosh(a*x)^2,x)`

[Out] `int(x*acosh(a*x)^2, x)`

3.16 $\int \cosh^{-1}(ax)^2 dx$

Optimal. Leaf size=39

$$2x - \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a} + x\cosh^{-1}(ax)^2$$

[Out] 2*x+x*arccosh(a*x)^2-2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 0.08, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5879, 5915, 8}

$$x\cosh^{-1}(ax)^2 - \frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a} + 2x$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^2,x]

[Out] 2*x - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + x*ArcCosh[a*x]^2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n, x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]))], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n*(x_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(q_), x_Symbol] := Simp[(d1 + e1*x)^(p+1)*(d2 + e2*x)^(q+1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p+1))), x] - Dist[b*(n/(2*c*(p+1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-1 + c*x)^q], Int[(1 + c*x)^(p+1/2)*(-1 + c*x)^(q+1/2)*(a + b*ArcCosh[c*x])^(n-1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(ax)^2 dx &= x \cosh^{-1}(ax)^2 - (2a) \int \frac{x \cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{2\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} + x \cosh^{-1}(ax)^2 + 2 \int 1 dx \\
&= 2x - \frac{2\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} + x \cosh^{-1}(ax)^2
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 39, normalized size = 1.00

$$2x - \frac{2\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} + x \cosh^{-1}(ax)^2$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^2,x]``[Out] 2*x - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + x*ArcCosh[a*x]^2`**Maple [A]**

time = 1.77, size = 39, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\operatorname{arccosh}(ax)^2 ax - 2 \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1} + 2ax}{a}$	39
default	$\frac{\operatorname{arccosh}(ax)^2 ax - 2 \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1} + 2ax}{a}$	39

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^2,x,method=_RETURNVERBOSE)``[Out] 1/a*(arccosh(a*x)^2*a*x-2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2*a*x)`**Maxima [A]**

time = 0.26, size = 32, normalized size = 0.82

$$x \operatorname{arccosh}(ax)^2 + 2x - \frac{2\sqrt{a^2x^2-1} \operatorname{arccosh}(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^2,x, algorithm="maxima")``[Out] x*arccosh(a*x)^2 + 2*x - 2*sqrt(a^2*x^2 - 1)*arccosh(a*x)/a`

Fricas [A]

time = 0.34, size = 59, normalized size = 1.51

$$\frac{ax \log \left(ax + \sqrt{a^2 x^2 - 1} \right)^2 + 2ax - 2\sqrt{a^2 x^2 - 1} \log \left(ax + \sqrt{a^2 x^2 - 1} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^2,x, algorithm="fricas")``[Out] (a*x*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a*x - 2*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a`**Sympy [A]**

time = 0.08, size = 39, normalized size = 1.00

$$\begin{cases} x \operatorname{acosh}^2(ax) + 2x - \frac{2\sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{a} & \text{for } a \neq 0 \\ -\frac{\pi^2 x}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(acosh(a*x)**2,x)``[Out] Piecewise((x*acosh(a*x)**2 + 2*x - 2*sqrt(a**2*x**2 - 1)*acosh(a*x)/a, Ne(a, 0)), (-pi**2*x/4, True))`**Giac [A]**

time = 0.40, size = 62, normalized size = 1.59

$$x \log \left(ax + \sqrt{a^2 x^2 - 1} \right)^2 + 2a \left(\frac{x}{a} - \frac{\sqrt{a^2 x^2 - 1} \log \left(ax + \sqrt{a^2 x^2 - 1} \right)}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^2,x, algorithm="giac")``[Out] x*log(a*x + sqrt(a^2*x^2 - 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))/a^2)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{acosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(acosh(a*x)^2,x)``[Out] int(acosh(a*x)^2, x)`

$$3.17 \quad \int \frac{\cosh^{-1}(ax)^2}{x} dx$$

Optimal. Leaf size=62

$$-\frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \cosh^{-1}(ax) \text{PolyLog}\left(2, -e^{2 \cosh^{-1}(ax)}\right) - \frac{1}{2} \text{PolyLog}\left(3, -e^{2 \cosh^{-1}(ax)}\right)$$

```
[Out] -1/3*arccosh(a*x)^3+arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2))*(a*x+1)^(1/2))^2
)+arccosh(a*x)*polylog(2,-(a*x+(a*x-1)^(1/2))*(a*x+1)^(1/2))^2)-1/2*polylog(
3,-(a*x+(a*x-1)^(1/2))*(a*x+1)^(1/2))^2)
```

Rubi [A]

time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5882, 3799, 2221, 2611, 2320, 6724}

$$\cosh^{-1}(ax) \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right) - \frac{1}{2} \text{Li}_3\left(-e^{2 \cosh^{-1}(ax)}\right) - \frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log\left(e^{2 \cosh^{-1}(ax)} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/x,x]
```

```
[Out] -1/3*ArcCosh[a*x]^3 + ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])] + ArcCosh[
a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])] - PolyLog[3, -E^(2*ArcCosh[a*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

f, g, n}, x] && GtQ[m, 0]

Rule 3799

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(c + d*x)^m*(E^(2*((-I)*e + f*fz*x)))/(1 + E^(2*((-I)*e + f*fz*x)))]], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 5882

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b, Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 6724

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^{-1}(ax)^2}{x} dx &= \text{Subst} \left(\int x^2 \tanh(x) dx, x, \cosh^{-1}(ax) \right) \\
 &= -\frac{1}{3} \cosh^{-1}(ax)^3 + 2 \text{Subst} \left(\int \frac{e^{2x} x^2}{1 + e^{2x}} dx, x, \cosh^{-1}(ax) \right) \\
 &= -\frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) - 2 \text{Subst} \left(\int x \log(1 + e^{2x}) dx, x, \cosh^{-1}(ax) \right) \\
 &= -\frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + \cosh^{-1}(ax) \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right) \\
 &= -\frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + \cosh^{-1}(ax) \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right) \\
 &= -\frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + \cosh^{-1}(ax) \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 63, normalized size = 1.02

$$\frac{1}{3} \cosh^{-1}(ax)^3 + \cosh^{-1}(ax)^2 \log \left(1 + e^{-2 \cosh^{-1}(ax)} \right) - \cosh^{-1}(ax) \text{PolyLog} \left(2, -e^{-2 \cosh^{-1}(ax)} \right) - \frac{1}{2} \text{PolyLog} \left(3, -e^{-2 \cosh^{-1}(ax)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^2/x,x]

[Out] ArcCosh[a*x]^3/3 + ArcCosh[a*x]^2*Log[1 + E^(-2*ArcCosh[a*x])] - ArcCosh[a*x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] - PolyLog[3, -E^(-2*ArcCosh[a*x])]/2

Maple [A]

time = 1.81, size = 98, normalized size = 1.58

method	result
derivativedivides	$-\frac{\operatorname{arccosh}(ax)^3}{3} + \operatorname{arccosh}(ax)^2 \ln\left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2\right) + \operatorname{arccosh}(ax) \operatorname{polylog}\left(2, -\frac{ax + \sqrt{ax-1} \sqrt{ax+1}}{ax + 1}\right) - \frac{1}{2} \operatorname{polylog}\left(3, -\frac{ax + \sqrt{ax-1} \sqrt{ax+1}}{ax + 1}\right)$
default	$-\frac{\operatorname{arccosh}(ax)^3}{3} + \operatorname{arccosh}(ax)^2 \ln\left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2\right) + \operatorname{arccosh}(ax) \operatorname{polylog}\left(2, -\frac{ax + \sqrt{ax-1} \sqrt{ax+1}}{ax + 1}\right) - \frac{1}{2} \operatorname{polylog}\left(3, -\frac{ax + \sqrt{ax-1} \sqrt{ax+1}}{ax + 1}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^2/x,x,method=_RETURNVERBOSE)

[Out] -1/3*arccosh(a*x)^3+arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+arccosh(a*x)*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-1/2*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x,x, algorithm="maxima")

[Out] integrate(arccosh(a*x)^2/x, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^2/x, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**2/x,x)`

[Out] `Integral(acosh(a*x)**2/x, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^2/x,x, algorithm="giac")`

[Out] `integrate(arccosh(a*x)^2/x, x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^2/x,x)`

[Out] `int(acosh(a*x)^2/x, x)`

3.18 $\int \frac{\cosh^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=60

$$-\frac{\cosh^{-1}(ax)^2}{x} + 4a \cosh^{-1}(ax) \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 2ia \operatorname{PolyLog}\left(2, -ie^{\cosh^{-1}(ax)}\right) + 2ia \operatorname{PolyLog}\left(2, ie^{\cosh^{-1}(ax)}\right)$$

[Out] $-\operatorname{arccosh}(a*x)^2/x + 4*a*\operatorname{arccosh}(a*x)*\operatorname{arctan}(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}) - 2*I*a*\operatorname{polylog}(2, -I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})) + 2*I*a*\operatorname{polylog}(2, I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}))$

Rubi [A]

time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 5947, 4265, 2317, 2438}

$$4a \cosh^{-1}(ax) \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 2ia \operatorname{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 2ia \operatorname{Li}_2\left(ie^{\cosh^{-1}(ax)}\right) - \frac{\cosh^{-1}(ax)^2}{x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]^2/x^2, x]`

[Out] $-(\operatorname{ArcCosh}[a*x]^2/x) + 4*a*\operatorname{ArcCosh}[a*x]*\operatorname{ArcTan}[E^{\operatorname{ArcCosh}[a*x]}] - (2*I)*a*\operatorname{PolyLog}[2, (-I)*E^{\operatorname{ArcCosh}[a*x]}] + (2*I)*a*\operatorname{PolyLog}[2, I*E^{\operatorname{ArcCosh}[a*x]}]$

Rule 2317

`Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2438

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 4265

`Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(I*k*Pi)]/(f*fz*I)], x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{x^2} dx &= -\frac{\cosh^{-1}(ax)^2}{x} + (2a) \int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= -\frac{\cosh^{-1}(ax)^2}{x} + (2a) \text{Subst}\left(\int x \operatorname{sech}(x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{\cosh^{-1}(ax)^2}{x} + 4a \cosh^{-1}(ax) \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (2ia) \text{Subst}\left(\int \log(1 - ie^x) dx, x, e^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^2}{x} + 4a \cosh^{-1}(ax) \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (2ia) \text{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^2}{x} + 4a \cosh^{-1}(ax) \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 2ia \operatorname{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 2ia \operatorname{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 92, normalized size = 1.53

$$-ia \left(\cosh^{-1}(ax) \left(-\frac{i \cosh^{-1}(ax)}{ax} + 2 \log(1 - ie^{-\cosh^{-1}(ax)}) - 2 \log(1 + ie^{-\cosh^{-1}(ax)}) \right) + 2 \operatorname{PolyLog}(2, -ie^{-\cosh^{-1}(ax)}) - 2 \operatorname{PolyLog}(2, ie^{-\cosh^{-1}(ax)}) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/x^2, x]
```

```
[Out] (-I)*a*(ArcCosh[a*x]*((( -I)*ArcCosh[a*x])/(a*x) + 2*Log[1 - I/E^ArcCosh[a*x]]
) - 2*Log[1 + I/E^ArcCosh[a*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[a*x]] - 2*P
olyLog[2, I/E^ArcCosh[a*x]])
```

Maple [A]

time = 4.60, size = 138, normalized size = 2.30

method	result
derivativedivides	$a \left(-\frac{\operatorname{arccosh}(ax)^2}{ax} - 2i \operatorname{arccosh}(ax) \ln(1 + i(ax + \sqrt{ax-1} \sqrt{ax+1})) + 2i \operatorname{arccosh}(ax) \right)$
default	$a \left(-\frac{\operatorname{arccosh}(ax)^2}{ax} - 2i \operatorname{arccosh}(ax) \ln(1 + i(ax + \sqrt{ax-1} \sqrt{ax+1})) + 2i \operatorname{arccosh}(ax) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^2/x^2,x,method=_RETURNVERBOSE)`

[Out] $a \left(-\operatorname{arccosh}(a*x)^2/a/x - 2*I*\operatorname{arccosh}(a*x)*\ln(1+I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})) + 2*I*\operatorname{arccosh}(a*x)*\ln(1-I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})) - 2*I*\operatorname{dilog}(1+I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})) + 2*I*\operatorname{dilog}(1-I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})) \right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^2/x^2,x, algorithm="maxima")`

[Out] $-\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^2/x + \operatorname{integrate}(2*(a^3*x^2 + \sqrt{a*x + 1}*\sqrt{a*x - 1})*a^2*x - a)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})/(a^3*x^4 - a*x^2 + (a^2*x^3 - x)*\sqrt{a*x + 1}*\sqrt{a*x - 1}), x$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^2/x^2,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)^2/x^2, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**2/x**2,x)`

[Out] Integral(acosh(a*x)**2/x**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x^2,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^2/x^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^2/x^2,x)

[Out] int(acosh(a*x)^2/x^2, x)

$$3.19 \quad \int \frac{\cosh^{-1}(ax)^2}{x^3} dx$$

Optimal. Leaf size=48

$$\frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{x} - \frac{\cosh^{-1}(ax)^2}{2x^2} - a^2 \log(x)$$

[Out] $-1/2*\operatorname{arccosh}(a*x)^2/x^2 - a^2*\ln(x) + a*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x$

Rubi [A]

time = 0.13, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5883, 5918, 29}

$$a^2(-\log(x)) - \frac{\cosh^{-1}(ax)^2}{2x^2} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]^2/x^3,x]`

[Out] $(a*\sqrt{-1+a*x}*\sqrt{1+a*x}*\operatorname{ArcCosh}[a*x])/x - \operatorname{ArcCosh}[a*x]^2/(2*x^2) - a^2*\operatorname{Log}[x]$

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 5883

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m+1)*((a+b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a+b*ArcCosh[c*x])^(n-1)/(Sqrt[1+c*x]*Sqrt[-1+c*x])), x], x] /; FreeQ[{a,b,c,d,m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

Rule 5918

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(q_.), x_Symbol] := Simp[(f*x)^(m+1)*(d1+e1*x)^(p+1)*(d2+e2*x)^(q+1)*((a+b*ArcCosh[c*x])^n/(d1*d2*f*(m+1))), x] + Dist[b*c*(n/(f*(m+1)))*Simp[(d1+e1*x)^p/(1+c*x)^p]*Simp[(d2+e2*x)^q/(-1+c*x)^q], Int[(f*x)^(m+1)*(1+c*x)^(p+1/2)*(-1+c*x)^(q+1/2)*(a+b*ArcCosh[c*x])^(n-1), x], x] /; FreeQ[{a,b,c,d1,e1,d2,e2,f,m,p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && EqQ[m+2*p+3, 0] && NeQ[p, -1]`

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{x^3} dx &= -\frac{\cosh^{-1}(ax)^2}{2x^2} + a \int \frac{\cosh^{-1}(ax)}{x^2 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{x} - \frac{\cosh^{-1}(ax)^2}{2x^2} - a^2 \int \frac{1}{x} dx \\
&= \frac{a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{x} - \frac{\cosh^{-1}(ax)^2}{2x^2} - a^2 \log(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 48, normalized size = 1.00

$$\frac{a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{x} - \frac{\cosh^{-1}(ax)^2}{2x^2} - a^2 \log(x)$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^2/x^3,x]``[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/x - ArcCosh[a*x]^2/(2*x^2) - a^2*Log[x]`Maple [A]

time = 3.67, size = 81, normalized size = 1.69

method	result
derivativedivides	$a^2 \left(2 \operatorname{arccosh}(ax) - \frac{\operatorname{arccosh}(ax) \left(2a^2x^2 - 2\sqrt{ax+1} \sqrt{ax-1} \operatorname{arccosh}(ax) \right)}{2a^2x^2} \right) - \ln \left(1 + (ax + \dots) \right)$
default	$a^2 \left(2 \operatorname{arccosh}(ax) - \frac{\operatorname{arccosh}(ax) \left(2a^2x^2 - 2\sqrt{ax+1} \sqrt{ax-1} \operatorname{arccosh}(ax) \right)}{2a^2x^2} \right) - \ln \left(1 + (ax + \dots) \right)$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^2/x^3,x,method=_RETURNVERBOSE)``[Out] a^2*(2*arccosh(a*x)-1/2*arccosh(a*x)*(2*a^2*x^2-2*(a*x+1)^(1/2)*(a*x-1)^(1/2))*a*x+arccosh(a*x))/a^2/x^2-ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)`Maxima [A]

time = 0.46, size = 39, normalized size = 0.81

$$-a^2 \log(x) + \frac{\sqrt{a^2x^2 - 1} a \operatorname{arccosh}(ax)}{x} - \frac{\operatorname{arccosh}(ax)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x^3,x, algorithm="maxima")

[Out] $-a^2 \log(x) + \sqrt{a^2 x^2 - 1} a \operatorname{arccosh}(a x) / x - 1/2 \operatorname{arccosh}(a x)^2 / x^2$

Fricas [A]

time = 0.34, size = 65, normalized size = 1.35

$$\frac{2 a^2 x^2 \log(x) - 2 \sqrt{a^2 x^2 - 1} a x \log\left(ax + \sqrt{a^2 x^2 - 1}\right) + \log\left(ax + \sqrt{a^2 x^2 - 1}\right)^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x^3,x, algorithm="fricas")

[Out] $-1/2*(2*a^2*x^2*\log(x) - 2*\sqrt{a^2*x^2 - 1}*a*x*\log(a*x + \sqrt{a^2*x^2 - 1})) + \log(a*x + \sqrt{a^2*x^2 - 1})^2/x^2$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**2/x**3,x)

[Out] Integral(acosh(a*x)**2/x**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(42) = 84$.
time = 0.43, size = 98, normalized size = 2.04

$$\left(a \log\left(\left|-x|a| + \sqrt{a^2 x^2 - 1}\right|\right) - a \log(|x|) + \frac{2|a| \log\left(ax + \sqrt{a^2 x^2 - 1}\right)}{\left(x|a| - \sqrt{a^2 x^2 - 1}\right)^2 + 1} \right) a - \frac{\log\left(ax + \sqrt{a^2 x^2 - 1}\right)^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x^3,x, algorithm="giac")

[Out] $(a*\log(\operatorname{abs}(-x*\operatorname{abs}(a) + \sqrt{a^2*x^2 - 1}))) - a*\log(\operatorname{abs}(x)) + 2*\operatorname{abs}(a)*\log(a*x + \sqrt{a^2*x^2 - 1})/((x*\operatorname{abs}(a) - \sqrt{a^2*x^2 - 1})^2 + 1))*a - 1/2*\log(a*x + \sqrt{a^2*x^2 - 1})^2/x^2$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(ax)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/x^3,x)
```

```
[Out] int(acosh(a*x)^2/x^3, x)
```

3.20 $\int \frac{\cosh^{-1}(ax)^2}{x^4} dx$

Optimal. Leaf size=114

$$\frac{a^2}{3x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\cosh^{-1}(ax)\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - \frac{1}{3}ia^3\text{PolyLog}\left(2, -I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})\right) + \frac{1}{3}I*a^3*\text{polylog}\left(2, I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})\right) + \frac{1}{3}a*\text{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x^2$$

[Out] 1/3*a^2/x-1/3*arccosh(a*x)^2/x^3+2/3*a^3*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-1/3*I*a^3*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+1/3*I*a^3*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+1/3*a*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^2

Rubi [A]

time = 0.26, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5883, 5933, 5947, 4265, 2317, 2438, 30}

$$\frac{2}{3}a^3\cosh^{-1}(ax)\text{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - \frac{1}{3}ia^3\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + \frac{1}{3}ia^3\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right) + \frac{a^2}{3x} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^2/x^4,x]

[Out] a^2/(3*x) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(3*x^2) - ArcCosh[a*x]^2/(3*x^3) + (2*a^3*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/3 - (I/3)*a^3*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (I/3)*a^3*PolyLog[2, I*E^ArcCosh[a*x]]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2317

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4265

Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/E^((

```

I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
- E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

Rule 5883

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]

```

Rule 5933

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_.))^(m_.)*((d1_) + (e
1_.)*(x_.))^(p_.)*((d2_) + (e2_.)*(x_.))^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(q + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^q*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-1 + c*x)^q],
Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(q + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]

```

Rule 5947

```

Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{x^4} dx &= -\frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a) \int \frac{\cosh^{-1}(ax)}{x^3 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} - \frac{1}{3}a^2 \int \frac{1}{x^2} dx + \frac{1}{3}a^3 \int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}} dx \\
&= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^3 \text{Subst} \left(\int x \text{sech}(x) dx, \right. \\
&= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3 \cosh^{-1}(ax) \tan^{-1} \left(e^{\cosh^{-1}(ax)} \right) \\
&= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3 \cosh^{-1}(ax) \tan^{-1} \left(e^{\cosh^{-1}(ax)} \right) \\
&= \frac{a^2}{3x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{3x^2} - \frac{\cosh^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3 \cosh^{-1}(ax) \tan^{-1} \left(e^{\cosh^{-1}(ax)} \right)
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 144, normalized size = 1.26

$$\frac{1}{3}a^3 \left(\frac{1}{ax} + \frac{\sqrt{\frac{-1+ax}{1+ax}} (1+ax) \cosh^{-1}(ax)}{a^2 x^2} - \frac{\cosh^{-1}(ax)^2}{a^3 x^3} - i \cosh^{-1}(ax) \log(1 - ie^{-\cosh^{-1}(ax)}) + i \cosh^{-1}(ax) \log(1 + ie^{-\cosh^{-1}(ax)}) - i \text{PolyLog}(2, -ie^{-\cosh^{-1}(ax)}) + i \text{PolyLog}(2, ie^{-\cosh^{-1}(ax)}) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^2/x^4,x]

[Out] (a^3*(1/(a*x) + (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a^2*x^2) - ArcCosh[a*x]^2/(a^3*x^3) - I*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] + I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - I*PolyLog[2, (-I)/E^ArcCosh[a*x]] + I*PolyLog[2, I/E^ArcCosh[a*x]]))/3

Maple [A]

time = 4.67, size = 171, normalized size = 1.50

method	result
derivativedivides	$a^3 \left(-\frac{-ax \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1} + \operatorname{arccosh}(ax)^2 - a^2 x^2}{3a^3 x^3} - \frac{i \operatorname{arccosh}(ax) \ln \left(1+i \left(ax + \sqrt{ax-1} \right) \right)}{3} \right)$
default	$a^3 \left(-\frac{-ax \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1} + \operatorname{arccosh}(ax)^2 - a^2 x^2}{3a^3 x^3} - \frac{i \operatorname{arccosh}(ax) \ln \left(1+i \left(ax + \sqrt{ax-1} \right) \right)}{3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^2/x^4,x,method=_RETURNVERBOSE)

[Out] $a^3*(-1/3*(-a*x*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}+\operatorname{arccosh}(a*x)^2-a^2*x^2)/a^3/x^3-1/3*I*\operatorname{arccosh}(a*x)*\ln(1+I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}))+1/3*I*\operatorname{arccosh}(a*x)*\ln(1-I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}))-1/3*I*\operatorname{dilog}(1+I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}))+1/3*I*\operatorname{dilog}(1-I*(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^2/x^4,x, algorithm="maxima")`

[Out] $-1/3*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^2/x^3 + \operatorname{integrate}(2/3*(a^3*x^2 + \sqrt{a*x + 1}*\sqrt{a*x - 1}*a^2*x - a)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})/(a^3*x^6 - a*x^4 + (a^2*x^5 - x^3)*\sqrt{a*x + 1}*\sqrt{a*x - 1}), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^2/x^4,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)^2/x^4, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**2/x**4,x)`

[Out] `Integral(acosh(a*x)**2/x**4, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^2/x^4,x, algorithm="giac")`

[Out] integrate(arccosh(a*x)^2/x^4, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^2/x^4,x)

[Out] int(acosh(a*x)^2/x^4, x)

3.21 $\int \frac{\cosh^{-1}(ax)^2}{x^5} dx$

Optimal. Leaf size=95

$$\frac{a^2}{12x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x} - \frac{\cosh^{-1}(ax)^2}{4x^4} - \frac{1}{3}a^4\log(x)$$

[Out] 1/12*a^2/x^2-1/4*arccosh(a*x)^2/x^4-1/3*a^4*ln(x)+1/6*a*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^3+1/3*a^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x

Rubi [A]

time = 0.23, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5883, 5933, 5918, 29, 30}

$$-\frac{1}{3}a^4\log(x) + \frac{a^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{3x} + \frac{a^2}{12x^2} - \frac{\cosh^{-1}(ax)^2}{4x^4} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^2/x^5,x]

[Out] a^2/(12*x^2) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(6*x^3) + (a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(3*x) - ArcCosh[a*x]^2/(4*x^4) - (a^4*Log[x])/3

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^ (n_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5918

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^ (n_.)*((f_.)*(x_))^(m_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m+1)

```

*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
&& EqQ[m + 2*p + 3, 0] && NeQ[p, -1]

```

Rule 5933

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^n_.)*((f_.)*(x_.))^(m_.)*((d1_.) + (e
1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^2}{x^5} dx &= -\frac{\cosh^{-1}(ax)^2}{4x^4} + \frac{1}{2}a \int \frac{\cosh^{-1}(ax)}{x^4\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} - \frac{\cosh^{-1}(ax)^2}{4x^4} - \frac{1}{6}a^2 \int \frac{1}{x^3} dx + \frac{1}{3}a^3 \int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= \frac{a^2}{12x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x} - \frac{1}{4x^4} \\
&= \frac{a^2}{12x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{3x} - \frac{1}{4x^4}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 69, normalized size = 0.73

$$\frac{a^2x^2 + 2ax\sqrt{-1+ax}\sqrt{1+ax}(1 + 2a^2x^2)\cosh^{-1}(ax) - 3\cosh^{-1}(ax)^2 - 4a^4x^4\log(x)}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^2/x^5,x]

[Out] (a^2*x^2 + 2*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(1 + 2*a^2*x^2)*ArcCosh[a*x] - 3*ArcCosh[a*x]^2 - 4*a^4*x^4*Log[x])/(12*x^4)

Maple [A]

time = 4.15, size = 123, normalized size = 1.29

method	result
derivativedivides	$a^4 \left(\frac{2 \operatorname{arccosh}(ax)}{3} - \frac{-4 \operatorname{arccosh}(ax) \sqrt{ax+1} \sqrt{ax-1}}{12a^4x^4} a^3x^3 + 4 \operatorname{arccosh}(ax) a^4x^4 - 2ax \operatorname{arccosh}(ax) \sqrt{ax-1} \right)$
default	$a^4 \left(\frac{2 \operatorname{arccosh}(ax)}{3} - \frac{-4 \operatorname{arccosh}(ax) \sqrt{ax+1} \sqrt{ax-1}}{12a^4x^4} a^3x^3 + 4 \operatorname{arccosh}(ax) a^4x^4 - 2ax \operatorname{arccosh}(ax) \sqrt{ax-1} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(2/3*arccosh(a*x)-1/12*(-4*arccosh(a*x)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3+4*arccosh(a*x)*a^4*x^4-2*a*x*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*arccosh(a*x)^2-a^2*x^2)/a^4/x^4-1/3*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))
```

Maxima [A]

time = 0.47, size = 72, normalized size = 0.76

$$-\frac{1}{12} \left(4a^2 \log(x) - \frac{1}{x^2} \right) a^2 + \frac{1}{6} \left(\frac{2\sqrt{a^2x^2-1}a^2}{x} + \frac{\sqrt{a^2x^2-1}}{x^3} \right) a \operatorname{arccosh}(ax) - \frac{\operatorname{arccosh}(ax)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^2*log(x) - 1/x^2)*a^2 + 1/6*(2*sqrt(a^2*x^2 - 1)*a^2/x + sqrt(a^2*x^2 - 1)/x^3)*a*arccosh(a*x) - 1/4*arccosh(a*x)^2/x^4
```

Fricas [A]

time = 0.38, size = 85, normalized size = 0.89

$$\frac{4a^4x^4 \log(x) - a^2x^2 - 2(2a^3x^3 + ax)\sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1}) + 3 \log(ax + \sqrt{a^2x^2-1})^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^4*x^4*log(x) - a^2*x^2 - 2*(2*a^3*x^3 + a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + 3*log(a*x + sqrt(a^2*x^2 - 1))^2)/x^4
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**2/x**5,x)**[Out]** Integral(acosh(a*x)**2/x**5, x)**Giac [A]**

time = 0.45, size = 147, normalized size = 1.55

$$-\frac{1}{12} \left(2a^3 \log(x^2) - 4a^3 \log(|-x|a| + \sqrt{a^2x^2 - 1}|) - \frac{8 \left(3 \left(|x|a| - \sqrt{a^2x^2 - 1} \right)^2 + 1 \right) a^2 |a| \log(ax + \sqrt{a^2x^2 - 1})}{\left(\left(|x|a| - \sqrt{a^2x^2 - 1} \right)^2 + 1 \right)^3} - \frac{2a^3x^2 + a}{x^2} \right) a - \frac{\log(ax + \sqrt{a^2x^2 - 1})^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^2/x^5,x, algorithm="giac")

[Out] $-1/12*(2*a^3*\log(x^2) - 4*a^3*\log(\operatorname{abs}(-x*\operatorname{abs}(a) + \sqrt{a^2*x^2 - 1}))) - 8*(3*(x*\operatorname{abs}(a) - \sqrt{a^2*x^2 - 1})^2 + 1)*a^2*\operatorname{abs}(a)*\log(ax + \sqrt{a^2*x^2 - 1})/((x*\operatorname{abs}(a) - \sqrt{a^2*x^2 - 1})^2 + 1)^3 - (2*a^3*x^2 + a)/x^2)*a - 1/4*\log(ax + \sqrt{a^2*x^2 - 1})^2/x^4$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^2/x^5,x)**[Out]** int(acosh(a*x)^2/x^5, x)

3.22 $\int x^4 \cosh^{-1}(ax)^3 dx$

Optimal. Leaf size=231

$$\frac{4144\sqrt{-1+ax}\sqrt{1+ax}}{5625a^5} - \frac{272x^2\sqrt{-1+ax}\sqrt{1+ax}}{5625a^3} - \frac{6x^4\sqrt{-1+ax}\sqrt{1+ax}}{625a} + \frac{16x\cosh^{-1}(ax)}{25a^4} + \frac{8x^3}{25a^5}$$

[Out] $16/25*x*\operatorname{arccosh}(a*x)/a^4+8/75*x^3*\operatorname{arccosh}(a*x)/a^2+6/125*x^5*\operatorname{arccosh}(a*x)+1/5*x^5*\operatorname{arccosh}(a*x)^3-4144/5625*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^5-272/5625*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-6/625*x^4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-8/25*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^5-4/25*x^2*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-3/25*x^4*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.49, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5883, 5939, 5915, 5879, 75, 102, 12}

$$\frac{4144\sqrt{ax-1}\sqrt{ax+1}}{5625a^5} - \frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{25a^4} + \frac{16x\cosh^{-1}(ax)}{25a^4} - \frac{272x^2\sqrt{ax-1}\sqrt{ax+1}}{5625a^3} - \frac{4x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{25a^3} + \frac{8x^2\cosh^{-1}(ax)}{75a^2} + \frac{1}{5}x^2\cosh^{-1}(ax)^3 + \frac{6}{125}x^2\cosh^{-1}(ax) - \frac{6x^4\sqrt{ax-1}\sqrt{ax+1}}{625a} - \frac{3x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{25a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4*\operatorname{ArcCosh}[a*x]^3, x]$

[Out] $(-4144*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(5625*a^5) - (272*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(5625*a^3) - (6*x^4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(625*a) + (16*x*\operatorname{ArcCosh}[a*x])/(25*a^4) + (8*x^3*\operatorname{ArcCosh}[a*x])/(75*a^2) + (6*x^5*\operatorname{ArcCosh}[a*x])/125 - (8*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(25*a^5) - (4*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(25*a^3) - (3*x^4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(25*a) + (x^5*\operatorname{ArcCosh}[a*x]^3)/5$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 75

$\operatorname{Int}[(a_*) + (b_*)(x_)*((c_*) + (d_*)(x_))^{(n_*)}*((e_*) + (f_*)(x_))^{(p_*)}, x_Symbol] \rightarrow \operatorname{Simp}[b*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)})/(d*f*(n + p + 2)), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \ \&\& \ \operatorname{NeQ}[n + p + 2, 0] \ \&\& \ \operatorname{EqQ}[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]$

Rule 102

$\operatorname{Int}[(a_*) + (b_*)(x_))^{(m_*)}*((c_*) + (d_*)(x_))^{(n_*)}*((e_*) + (f_*)(x_))^{(p_*)}, x_Symbol] \rightarrow \operatorname{Simp}[b*(a + b*x)^{(m - 1)}*(c + d*x)^{(n + 1)}*((e + f*x$


```
)^(p + 1)/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
+ b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))] + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.), x_Symbol] := Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.)*(x_)*((d1_.) + (e1_.)*(x_))^(p
_)*((d2_.) + (e2_.)*(x_))^(p_), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.)*((f_.)*(x_))^(m_.)*((d1_.) + (e
1_.)*(x_))^(p_)*((d2_.) + (e2_.)*(x_))^(p_), x_Symbol] := Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4 \cosh^{-1}(ax)^3 dx &= \frac{1}{5}x^5 \cosh^{-1}(ax)^3 - \frac{1}{5}(3a) \int \frac{x^5 \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{3x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax)^3 + \frac{6}{25} \int x^4 \cosh^{-1}(ax) dx \\
&= \frac{6}{125}x^5 \cosh^{-1}(ax) - \frac{4x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{25a^3} - \frac{3x^4 \sqrt{-1+ax} \sqrt{1+ax}}{25a} \\
&= -\frac{6x^4 \sqrt{-1+ax} \sqrt{1+ax}}{625a} + \frac{8x^3 \cosh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5 \cosh^{-1}(ax) - \frac{8\sqrt{-1+ax} \sqrt{1+ax}}{75a} \\
&= -\frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax}}{225a^3} - \frac{6x^4 \sqrt{-1+ax} \sqrt{1+ax}}{625a} + \frac{16x \cosh^{-1}(ax)}{25a^4} + \frac{8x^3 \cosh^{-1}(ax)}{75a} \\
&= -\frac{16\sqrt{-1+ax} \sqrt{1+ax}}{25a^5} - \frac{272x^2 \sqrt{-1+ax} \sqrt{1+ax}}{5625a^3} - \frac{6x^4 \sqrt{-1+ax} \sqrt{1+ax}}{625a} \\
&= -\frac{32\sqrt{-1+ax} \sqrt{1+ax}}{45a^5} - \frac{272x^2 \sqrt{-1+ax} \sqrt{1+ax}}{5625a^3} - \frac{6x^4 \sqrt{-1+ax} \sqrt{1+ax}}{625a} \\
&= -\frac{4144\sqrt{-1+ax} \sqrt{1+ax}}{5625a^5} - \frac{272x^2 \sqrt{-1+ax} \sqrt{1+ax}}{5625a^3} - \frac{6x^4 \sqrt{-1+ax} \sqrt{1+ax}}{625a}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 130, normalized size = 0.56

$$\frac{-2\sqrt{-1+ax} \sqrt{1+ax} (2072 + 136a^2x^2 + 27a^4x^4) + 30ax(120 + 20a^2x^2 + 9a^4x^4) \cosh^{-1}(ax) - 225\sqrt{-1+ax} \sqrt{1+ax} (8 + 4a^2x^2 + 3a^4x^4) \cosh^{-1}(ax)^2 + 1125a^5x^5 \cosh^{-1}(ax)^3}{5625a^5}$$

Antiderivative was successfully verified.

`[In] Integrate[x^4*ArcCosh[a*x]^3,x]`

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2072 + 136*a^2*x^2 + 27*a^4*x^4) + 30*a*x
*(120 + 20*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x] - 225*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^2 + 1125*a^5*x^5*ArcCosh[a*x]
^3)/(5625*a^5)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^4*arccosh(a*x)^3,x)`

[Out] $\int (x^4 \operatorname{arccosh}(ax))^3 dx$

Maxima [A]

time = 0.28, size = 165, normalized size = 0.71

$$\frac{1}{5} x^5 \operatorname{arccosh}(ax)^3 - \frac{1}{25} \left(\frac{3\sqrt{a^2x^2-1}x^4}{a^2} + \frac{4\sqrt{a^2x^2-1}x^2}{a^4} + \frac{8\sqrt{a^2x^2-1}}{a^6} \right) a \operatorname{arccosh}(ax)^2 - \frac{2}{5625} a \left(\frac{27\sqrt{a^2x^2-1}a^2x^4 + 136\sqrt{a^2x^2-1}x^2 + 2072\sqrt{a^2x^2-1}}{a^4} - \frac{15(9a^4x^5 + 20a^2x^3 + 120x) \operatorname{arccosh}(ax)}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^3,x, algorithm="maxima")`

[Out] $\frac{1}{5} x^5 \operatorname{arccosh}(ax)^3 - \frac{1}{25} (3 \sqrt{a^2x^2-1} x^4/a^2 + 4 \sqrt{a^2x^2-1} x^2/a^4 + 8 \sqrt{a^2x^2-1}/a^6) a \operatorname{arccosh}(ax)^2 - \frac{2}{5625} a \left(\frac{27 \sqrt{a^2x^2-1} a^2 x^4 + 136 \sqrt{a^2x^2-1} x^2 + 2072 \sqrt{a^2x^2-1}}{a^4} - \frac{15 (9 a^4 x^5 + 20 a^2 x^3 + 120 x) \operatorname{arccosh}(ax)}{a^5} \right)$

Fricas [A]

time = 0.38, size = 151, normalized size = 0.65

$$\frac{1125 a^5 x^5 \log(ax + \sqrt{a^2x^2-1})^3 - 225 (3 a^4 x^4 + 4 a^2 x^2 + 8) \sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1})^2 + 30 (9 a^5 x^5 + 20 a^3 x^3 + 120 a x) \log(ax + \sqrt{a^2x^2-1}) - 2 (27 a^4 x^4 + 136 a^2 x^2 + 2072) \sqrt{a^2x^2-1}}{5625 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^3,x, algorithm="fricas")`

[Out] $\frac{1}{5625} (1125 a^5 x^5 \log(ax + \sqrt{a^2x^2-1})^3 - 225 (3 a^4 x^4 + 4 a^2 x^2 + 8) \sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1})^2 + 30 (9 a^5 x^5 + 20 a^3 x^3 + 120 a x) \log(ax + \sqrt{a^2x^2-1}) - 2 (27 a^4 x^4 + 136 a^2 x^2 + 2072) \sqrt{a^2x^2-1}) / a^5$

Sympy [C] Result contains complex when optimal does not.

time = 0.70, size = 206, normalized size = 0.89

$$\begin{cases} \frac{x^5 \operatorname{acosh}^3(ax)}{5} + \frac{6x^5 \operatorname{acosh}(ax)}{125} - \frac{3x^4 \sqrt{a^2x^2-1} \operatorname{acosh}^2(ax)}{25a} - \frac{6x^4 \sqrt{a^2x^2-1}}{625a} + \frac{8x^3 \operatorname{acosh}(ax)}{75a^2} - \frac{4x^2 \sqrt{a^2x^2-1} \operatorname{acosh}^2(ax)}{25a^3} - \frac{272x^2 \sqrt{a^2x^2-1}}{5625a^3} + \frac{16x \operatorname{acosh}(ax)}{25a^4} - \frac{8\sqrt{a^2x^2-1} \operatorname{acosh}^2(ax)}{25a^4} - \frac{4144\sqrt{a^2x^2-1}}{5625a^5} & \text{for } a \neq 0 \\ -\frac{ix^5}{40} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acosh(a*x)**3,x)`

[Out] `Piecewise((x**5*acosh(a*x)**3/5 + 6*x**5*acosh(a*x)/125 - 3*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(25*a) - 6*x**4*sqrt(a**2*x**2 - 1)/(625*a) + 8*x**3*acosh(a*x)/(75*a**2) - 4*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(25*a**3) - 272*x**2*sqrt(a**2*x**2 - 1)/(5625*a**3) + 16*x*acosh(a*x)/(25*a**4) - 8*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(25*a**5) - 4144*sqrt(a**2*x**2 - 1)/(5625*a**5), Ne(a, 0)), (-I*pi**3*x**5/40, True))`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int x^4 \operatorname{acosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acosh(a*x)^3,x)
```

```
[Out] int(x^4*acosh(a*x)^3, x)
```

3.23 $\int x^3 \cosh^{-1}(ax)^3 dx$

Optimal. Leaf size=183

$$\frac{45x\sqrt{-1+ax}\sqrt{1+ax}}{256a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}}{128a} - \frac{45\cosh^{-1}(ax)}{256a^4} + \frac{9x^2\cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\cosh^{-1}(ax)$$

[Out] $-45/256*\operatorname{arccosh}(a*x)/a^4+9/32*x^2*\operatorname{arccosh}(a*x)/a^2+3/32*x^4*\operatorname{arccosh}(a*x)-3/32*\operatorname{arccosh}(a*x)^3/a^4+1/4*x^4*\operatorname{arccosh}(a*x)^3-45/256*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-3/128*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-9/32*x*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-3/16*x^3*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.42, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$,

Rules used = {5883, 5939, 5893, 92, 54, 102, 12}

$$\frac{3\cosh^{-1}(ax)^3}{32a^4} - \frac{45\cosh^{-1}(ax)}{256a^4} - \frac{45x\sqrt{ax-1}\sqrt{ax+1}}{256a^3} - \frac{9x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{32a^3} + \frac{9x^2\cosh^{-1}(ax)}{32a^2} + \frac{1}{4}x^4\cosh^{-1}(ax)^3 + \frac{3}{32}x^4\cosh^{-1}(ax) - \frac{3x^3\sqrt{ax-1}\sqrt{ax+1}}{128a} - \frac{3x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{16a}$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcCosh[a*x]^3,x]

[Out] $(-45*x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(256*a^3) - (3*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(128*a) - (45*\operatorname{ArcCosh}[a*x])/(256*a^4) + (9*x^2*\operatorname{ArcCosh}[a*x])/(32*a^2) + (3*x^4*\operatorname{ArcCosh}[a*x])/32 - (9*x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(32*a^3) - (3*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(16*a) - (3*\operatorname{ArcCosh}[a*x]^3)/(32*a^4) + (x^4*\operatorname{ArcCosh}[a*x]^3)/4$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 54

Int[1/(Sqrt[(a_)+(b_.)*(x_)]*Sqrt[(c_)+(d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a+c, 0] && EqQ[b-d, 0] && GtQ[a, 0]

Rule 92

Int[((a_.)+(b_.)*(x_))^(2*((c_.)+(d_.)*(x_))^(n_.))*((e_.)+(f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a+b*x)*(c+d*x)^(n+1)*((e+f*x)^(p+1)/(d*f*(n+p+3))), x] + Dist[1/(d*f*(n+p+3)), Int[(c+d*x)^n*(e+f*x)^p*Simp[a^2*d*f*(n+p+3)-b*(b*c*e+a*(d*e*(n+1)+c*f*(p+1)))+b*(a*d*f*(n+p+4)-b*(d*e*(n+2)+c*f*(p+2)))*x, x], x] /; FreeQ

[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

Rule 102

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
\int x^3 \cosh^{-1}(ax)^3 dx &= \frac{1}{4}x^4 \cosh^{-1}(ax)^3 - \frac{1}{4}(3a) \int \frac{x^4 \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4 \cosh^{-1}(ax)^3 + \frac{3}{8} \int x^3 \cosh^{-1}(ax) dx \\
&= \frac{3}{32}x^4 \cosh^{-1}(ax) - \frac{9x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{32a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax}}{16a} \\
&= -\frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax}}{128a} + \frac{9x^2 \cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4 \cosh^{-1}(ax) - \frac{9x \sqrt{-1+ax} \sqrt{1+ax}}{64a^3} \\
&= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax}}{128a} + \frac{9x^2 \cosh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4 \cosh^{-1}(ax) \\
&= -\frac{45x \sqrt{-1+ax} \sqrt{1+ax}}{256a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax}}{128a} - \frac{9 \cosh^{-1}(ax)}{64a^4} + \frac{9x^2 \cosh^{-1}(ax)}{32a^2} \\
&= -\frac{45x \sqrt{-1+ax} \sqrt{1+ax}}{256a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax}}{128a} - \frac{45 \cosh^{-1}(ax)}{256a^4} + \frac{9x^2 \cosh^{-1}(ax)}{32a^2}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 143, normalized size = 0.78

$$\frac{-3ax\sqrt{-1+ax}\sqrt{1+ax}(15+2a^2x^2)+24a^2x^2(3+a^2x^2)\cosh^{-1}(ax)-24ax\sqrt{-1+ax}\sqrt{1+ax}(3+2a^2x^2)\cosh^{-1}(ax)^2+8(-3+8a^4x^4)\cosh^{-1}(ax)^3-45\log(ax+\sqrt{-1+ax}\sqrt{1+ax})}{256a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCosh[a*x]^3,x]

[Out] $(-3*a*x*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x]*(15+2*a^2*x^2)+24*a^2*x^2*(3+a^2*x^2)*\text{ArcCosh}[a*x]-24*a*x*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x]*(3+2*a^2*x^2)*\text{ArcCosh}[a*x]^2+8*(-3+8*a^4*x^4)*\text{ArcCosh}[a*x]^3-45*\text{Log}[a*x+\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x]])/(256*a^4)$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arccosh(a*x)^3,x)**[Out]** int(x^3*arccosh(a*x)^3,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^3*arccosh(a*x)^3,x, algorithm="maxima")`

```
[Out] 1/4*x^4*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3 - integrate(3/4*(a^3*x^6 +
sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^5 - a*x^4)*log(a*x + sqrt(a*x + 1)*sqrt(
a*x - 1))^2/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A]

time = 0.33, size = 142, normalized size = 0.78

$$\frac{8(8a^4x^4 - 3)\log(ax + \sqrt{a^2x^2 - 1})^3 - 24(2a^3x^3 + 3ax)\sqrt{a^2x^2 - 1}\log(ax + \sqrt{a^2x^2 - 1})^2 + 3(8a^4x^4 + 24a^2x^2 - 15)\log(ax + \sqrt{a^2x^2 - 1}) - 3(2a^3x^3 + 15ax)\sqrt{a^2x^2 - 1}}{256a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^3*arccosh(a*x)^3,x, algorithm="fricas")`

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 24*(2*a^3*x^3 + 3
*a*x)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 3*(8*a^4*x^4 + 24*
a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 - 1)) - 3*(2*a^3*x^3 + 15*a*x)*sqrt(a^
2*x^2 - 1))/a^4
```

Sympy [C] Result contains complex when optimal does not.

time = 0.47, size = 170, normalized size = 0.93

$$\begin{cases} \frac{x^4 \operatorname{acosh}^3(ax)}{4} + \frac{3x^4 \operatorname{acosh}(ax)}{32} - \frac{3x^3 \sqrt{a^2x^2 - 1} \operatorname{acosh}^2(ax)}{16a} - \frac{3x^3 \sqrt{a^2x^2 - 1}}{128a} + \frac{9x^2 \operatorname{acosh}(ax)}{32a^2} - \frac{9x \sqrt{a^2x^2 - 1} \operatorname{acosh}^2(ax)}{32a^3} - \frac{45x \sqrt{a^2x^2 - 1}}{256a^3} - \frac{3 \operatorname{acosh}^3(ax)}{32a^4} - \frac{45 \operatorname{acosh}(ax)}{256a^4} & \text{for } a \neq 0 \\ -\frac{i\pi^3 x^4}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**3*acosh(a*x)**3,x)`

```
[Out] Piecewise((x**4*acosh(a*x)**3/4 + 3*x**4*acosh(a*x)/32 - 3*x**3*sqrt(a**2*x
**2 - 1)*acosh(a*x)**2/(16*a) - 3*x**3*sqrt(a**2*x**2 - 1)/(128*a) + 9*x**2
*acosh(a*x)/(32*a**2) - 9*x*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(32*a**3) - 4
5*x*sqrt(a**2*x**2 - 1)/(256*a**3) - 3*acosh(a*x)**3/(32*a**4) - 45*acosh(a
*x)/(256*a**4), Ne(a, 0)), (-I*pi**3*x**4/32, True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x^3*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a*x)^3,x)
```

```
[Out] int(x^3*acosh(a*x)^3, x)
```

3.24 $\int x^2 \cosh^{-1}(ax)^3 dx$

Optimal. Leaf size=155

$$-\frac{40\sqrt{-1+ax}\sqrt{1+ax}}{27a^3} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{27a} + \frac{4x \cosh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \cosh^{-1}(ax) - \frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a^3}$$

[Out] $4/3*x*\operatorname{arccosh}(a*x)/a^2+2/9*x^3*\operatorname{arccosh}(a*x)+1/3*x^3*\operatorname{arccosh}(a*x)^3-40/27*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-2/27*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-2/3*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-1/3*x^2*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.31, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5883, 5939, 5915, 5879, 75, 102, 12}

$$-\frac{40\sqrt{ax-1}\sqrt{ax+1}}{27a^3} - \frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{3a^3} + \frac{4x \cosh^{-1}(ax)}{3a^2} + \frac{1}{3}x^3 \cosh^{-1}(ax)^3 + \frac{2}{9}x^3 \cosh^{-1}(ax) - \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{27a} - \frac{x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{3a}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCosh[a*x]^3,x]`

[Out] $(-40*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(27*a^3) - (2*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(27*a) + (4*x*\operatorname{ArcCosh}[a*x])/(3*a^2) + (2*x^3*\operatorname{ArcCosh}[a*x])/9 - (2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(3*a^3) - (x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^2)/(3*a) + (x^3*\operatorname{ArcCosh}[a*x]^3)/3$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 75

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]`

Rule 102

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}`

`}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]`

Rule 5879

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

Rule 5883

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

Rule 5915

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p_) * ((d2_) + (e2_.)*(x_))^(p_), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]`

Rule 5939

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e1_.)*(x_))^(p_) * ((d2_) + (e2_.)*(x_))^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]`

Rubi steps

$$\begin{aligned}
\int x^2 \cosh^{-1}(ax)^3 dx &= \frac{1}{3}x^3 \cosh^{-1}(ax)^3 - a \int \frac{x^3 \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^3 + \frac{2}{3} \int x^2 \cosh^{-1}(ax) dx - \\
&= \frac{2}{9}x^3 \cosh^{-1}(ax) - \frac{2\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{3a} \\
&= -\frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax}}{27a} + \frac{4x \cosh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \cosh^{-1}(ax) - \frac{2\sqrt{-1+ax} \sqrt{1+ax}}{3a^3} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax}}{3a^3} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax}}{27a} + \frac{4x \cosh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \cosh^{-1}(ax) \\
&= -\frac{40\sqrt{-1+ax} \sqrt{1+ax}}{27a^3} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax}}{27a} + \frac{4x \cosh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3 \cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 103, normalized size = 0.66

$$\frac{-2\sqrt{-1+ax} \sqrt{1+ax} (20 + a^2x^2) + 6ax(6 + a^2x^2) \cosh^{-1}(ax) - 9\sqrt{-1+ax} \sqrt{1+ax} (2 + a^2x^2) \cosh^{-1}(ax)^2 + 9a^3x^3 \cosh^{-1}(ax)^3}{27a^3}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*ArcCosh[a*x]^3,x]`

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(20 + a^2*x^2) + 6*a*x*(6 + a^2*x^2)*ArcCosh[a*x] - 9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^2 + 9*a^3*x^3*ArcCosh[a*x]^3)/(27*a^3)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*arccosh(a*x)^3,x)``[Out] int(x^2*arccosh(a*x)^3,x)`**Maxima [A]**

time = 0.26, size = 116, normalized size = 0.75

$$\frac{1}{3}x^3 \operatorname{arccosh}(ax)^3 - \frac{1}{3}a \left(\frac{\sqrt{a^2x^2-1}x^2}{a^2} + \frac{2\sqrt{a^2x^2-1}}{a^4} \right) \operatorname{arccosh}(ax)^2 - \frac{2}{27}a \left(\frac{\sqrt{a^2x^2-1}x^2 + \frac{20\sqrt{a^2x^2-1}}{a^2}}{a^2} - \frac{3(a^2x^3 + 6x) \operatorname{arccosh}(ax)}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^3,x, algorithm="maxima")

[Out] $\frac{1}{3}x^3\operatorname{arccosh}(ax)^3 - \frac{1}{3}a(\sqrt{a^2x^2 - 1})x^2/a^2 + 2\sqrt{a^2x^2 - 1}/a^4\operatorname{arccosh}(ax)^2 - \frac{2}{27}a((\sqrt{a^2x^2 - 1})x^2 + 20\sqrt{a^2x^2 - 1})/a^2 - 3(a^2x^3 + 6x)\operatorname{arccosh}(ax)/a^3$

Fricas [A]

time = 0.35, size = 124, normalized size = 0.80

$$\frac{9a^3x^3 \log(ax + \sqrt{a^2x^2 - 1})^3 - 9(a^2x^2 + 2)\sqrt{a^2x^2 - 1} \log(ax + \sqrt{a^2x^2 - 1})^2 + 6(a^3x^3 + 6ax) \log(ax + \sqrt{a^2x^2 - 1}) - 2(a^2x^2 + 20)\sqrt{a^2x^2 - 1}}{27a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^3,x, algorithm="fricas")

[Out] $\frac{1}{27}*(9*a^3*x^3*\log(ax + \sqrt{a^2*x^2 - 1})^3 - 9*(a^2*x^2 + 2)*\sqrt{a^2*x^2 - 1}*\log(ax + \sqrt{a^2*x^2 - 1})^2 + 6*(a^3*x^3 + 6*a*x)*\log(ax + \sqrt{a^2*x^2 - 1}) - 2*(a^2*x^2 + 20)*\sqrt{a^2*x^2 - 1})/a^3$

Sympy [C] Result contains complex when optimal does not.

time = 0.32, size = 138, normalized size = 0.89

$$\begin{cases} \frac{x^3 \operatorname{acosh}^3(ax)}{3} + \frac{2x^3 \operatorname{acosh}(ax)}{9} - \frac{x^2 \sqrt{a^2x^2 - 1} \operatorname{acosh}^2(ax)}{3a} - \frac{2x^2 \sqrt{a^2x^2 - 1}}{27a} + \frac{4x \operatorname{acosh}(ax)}{3a^2} - \frac{2\sqrt{a^2x^2 - 1} \operatorname{acosh}^2(ax)}{3a^3} - \frac{40\sqrt{a^2x^2 - 1}}{27a^3} & \text{for } a \neq 0 \\ -\frac{i\pi^3 x^3}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*acosh(a*x)**3,x)

[Out] Piecewise((x**3*acosh(a*x)**3/3 + 2*x**3*acosh(a*x)/9 - x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(3*a) - 2*x**2*sqrt(a**2*x**2 - 1)/(27*a) + 4*x*acosh(a*x)/(3*a**2) - 2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(3*a**3) - 40*sqrt(a**2*x**2 - 1)/(27*a**3), Ne(a, 0)), (-I*pi**3*x**3/24, True))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acosh(a*x)^3,x)`

[Out] `int(x^2*acosh(a*x)^3, x)`

3.25 $\int x \cosh^{-1}(ax)^3 dx$

Optimal. Leaf size=107

$$-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}}{8a} - \frac{3\cosh^{-1}(ax)}{8a^2} + \frac{3}{4}x^2\cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4a} - \frac{\cosh^{-1}(ax)^3}{4a^2}$$

[Out] $-3/8*\text{arccosh}(a*x)/a^2+3/4*x^2*\text{arccosh}(a*x)-1/4*\text{arccosh}(a*x)^3/a^2+1/2*x^2*a$
 $\text{rccosh}(a*x)^3-3/8*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-3/4*x*\text{arccosh}(a*x)^2*(a*x$
 $-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.24, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$,

Rules used = {5883, 5939, 5893, 92, 54}

$$-\frac{\cosh^{-1}(ax)^3}{4a^2} - \frac{3\cosh^{-1}(ax)}{8a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^3 + \frac{3}{4}x^2\cosh^{-1}(ax) - \frac{3x\sqrt{ax-1}\sqrt{ax+1}}{8a} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{ArcCosh}[a*x]^3, x]$

[Out] $(-3*x*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(8*a) - (3*\text{ArcCosh}[a*x])/(8*a^2) + (3*x$
 $^2*\text{ArcCosh}[a*x])/4 - (3*x*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x]*\text{ArcCosh}[a*x]^2)/(4*a$
 $) - \text{ArcCosh}[a*x]^3/(4*a^2) + (x^2*\text{ArcCosh}[a*x]^3)/2$

Rule 54

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*(x_)]*\text{Sqrt}[(c_) + (d_)*(x_)]), x_Symbol] \text{ :> } \text{Simp}[\text{ArcCosh}[b*(x/a)]/b, x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a + c, 0] \ \&\& \ \text{EqQ}[b - d, 0] \ \&\& \ \text{GtQ}[a, 0]$

Rule 92

$\text{Int}[(a_ + (b_)*(x_))^2*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x_Symbol] \text{ :> } \text{Simp}[b*(a + b*x)*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)})/(d*f*(n + p + 3)), x] + \text{Dist}[1/(d*f*(n + p + 3)), \text{Int}[(c + d*x)^n*(e + f*x)^p*\text{Simp}[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))]*x, x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \ \&\& \ \text{NeQ}[n + p + 3, 0]$

Rule 5883

$\text{Int}[(a_ + \text{ArcCosh}[(c_)*(x_)]*(b_))^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m + 1)}*((a + b*\text{ArcCosh}[c*x])^n/(d*(m + 1))), x] - \text{Dist}[b*c*(n/(d*(m + 1))), \text{Int}[(d*x)^{(m + 1)}*((a + b*\text{ArcCosh}[c*x])^{(n - 1)})/(\text{Sqrt}[1 + c*x]*\text{Sqrt}[-1 + c*x]), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\&$

NeQ[m, -1]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/(Sqrt[(d1_.) + (e1_.)*(x_.)]*Sqrt[(d2_.) + (e2_.)*(x_.)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
 \int x \cosh^{-1}(ax)^3 dx &= \frac{1}{2}x^2 \cosh^{-1}(ax)^3 - \frac{1}{2}(3a) \int \frac{x^2 \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{4a} + \frac{1}{2}x^2 \cosh^{-1}(ax)^3 + \frac{3}{2} \int x \cosh^{-1}(ax) dx - \frac{3}{2} \int x \cosh^{-1}(ax) dx \\
 &= \frac{3}{4}x^2 \cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{4a} - \frac{\cosh^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax) \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax}}{8a} + \frac{3}{4}x^2 \cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{4a} \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax}}{8a} - \frac{3 \cosh^{-1}(ax)}{8a^2} + \frac{3}{4}x^2 \cosh^{-1}(ax) - \frac{3x\sqrt{-1+ax} \sqrt{1+ax}}{4a}
 \end{aligned}$$

Mathematica [A]

time = 0.06, size = 113, normalized size = 1.06

$$\frac{6a^2x^2 \cosh^{-1}(ax) - 6ax\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2 + (-2+4a^2x^2) \cosh^{-1}(ax)^3 - 3(ax\sqrt{-1+ax} \sqrt{1+ax} + \log(ax + \sqrt{-1+ax} \sqrt{1+ax}))}{8a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcCosh[a*x]^3,x]

[Out] $(6*a^2*x^2*ArcCosh[a*x] - 6*a*x*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x]^2 + (-2 + 4*a^2*x^2)*ArcCosh[a*x]^3 - 3*(a*x*sqrt[-1 + a*x]*sqrt[1 + a*x] + Log[a*x + sqrt[-1 + a*x]*sqrt[1 + a*x]]))/(8*a^2)$

Maple [A]

time = 1.66, size = 88, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{a^2 x^2 \operatorname{arccosh}(ax)^3}{2} - \frac{3\sqrt{ax-1}\sqrt{ax+1} a x \operatorname{arccosh}(ax)^2}{4} - \frac{\operatorname{arccosh}(ax)^3}{a^2} + \frac{3a^2 x^2 \operatorname{arccosh}(ax)}{4} - \frac{3\sqrt{ax+1}\sqrt{ax-1}}{8}}$
default	$\frac{\frac{a^2 x^2 \operatorname{arccosh}(ax)^3}{2} - \frac{3\sqrt{ax-1}\sqrt{ax+1} a x \operatorname{arccosh}(ax)^2}{4} - \frac{\operatorname{arccosh}(ax)^3}{a^2} + \frac{3a^2 x^2 \operatorname{arccosh}(ax)}{4} - \frac{3\sqrt{ax+1}\sqrt{ax-1}}{8}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arccosh(a*x)^3,x,method=_RETURNVERBOSE)

[Out] $1/a^2*(1/2*a^2*x^2*arccosh(a*x)^3-3/4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}*a*x*arccosh(a*x)^2-1/4*arccosh(a*x)^3+3/4*a^2*x^2*arccosh(a*x)-3/8*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x-3/8*arccosh(a*x))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccosh(a*x)^3,x, algorithm="maxima")

[Out] $1/2*x^2*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3 - integrate(3/2*(a^3*x^4 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^3 - a*x^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)$

Fricas [A]

time = 0.35, size = 112, normalized size = 1.05

$$\frac{6\sqrt{a^2x^2-1}ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2-2(2a^2x^2-1)\log\left(ax+\sqrt{a^2x^2-1}\right)^3+3\sqrt{a^2x^2-1}ax-3(2a^2x^2-1)\log\left(ax+\sqrt{a^2x^2-1}\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccosh(a*x)^3,x, algorithm="fricas")

[Out] $-1/8*(6*sqrt(a^2*x^2 - 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2*(2*a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 3*sqrt(a^2*x^2 - 1)*a*x - 3*(2*a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)))/a^2$

Sympy [C] Result contains complex when optimal does not.

time = 0.19, size = 102, normalized size = 0.95

$$\begin{cases} \frac{x^2 \operatorname{acosh}^3(ax)}{2} + \frac{3x^2 \operatorname{acosh}(ax)}{4} - \frac{3x\sqrt{a^2x^2-1} \operatorname{acosh}^2(ax)}{4a} - \frac{3x\sqrt{a^2x^2-1}}{8a} - \frac{\operatorname{acosh}^3(ax)}{4a^2} - \frac{3\operatorname{acosh}(ax)}{8a^2} & \text{for } a \neq 0 \\ -\frac{i\pi^3x^2}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*acosh(a*x)**3,x)

[Out] Piecewise((x**2*acosh(a*x)**3/2 + 3*x**2*acosh(a*x)/4 - 3*x*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(4*a) - 3*x*sqrt(a**2*x**2 - 1)/(8*a) - acosh(a*x)**3/(4*a**2) - 3*acosh(a*x)/(8*a**2), Ne(a, 0)), (-I*pi**3*x**2/16, True))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccosh(a*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*acosh(a*x)^3,x)

[Out] int(x*acosh(a*x)^3, x)

3.26 $\int \cosh^{-1}(ax)^3 dx$

Optimal. Leaf size=68

$$-\frac{6\sqrt{-1+ax}\sqrt{1+ax}}{a} + 6x \cosh^{-1}(ax) - \frac{3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a} + x \cosh^{-1}(ax)^3$$

[Out] 6*x*arccosh(a*x)+x*arccosh(a*x)^3-6*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5879, 5915, 75}

$$-\frac{6\sqrt{ax-1}\sqrt{ax+1}}{a} + x \cosh^{-1}(ax)^3 - \frac{3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{a} + 6x \cosh^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^3,x]

[Out] (-6*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + 6*x*ArcCosh[a*x] - (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + x*ArcCosh[a*x]^3

Rule 75

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p_) * ((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(ax)^3 dx &= x \cosh^{-1}(ax)^3 - (3a) \int \frac{x \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{3\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{a} + x \cosh^{-1}(ax)^3 + 6 \int \cosh^{-1}(ax) dx \\
&= 6x \cosh^{-1}(ax) - \frac{3\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{a} + x \cosh^{-1}(ax)^3 - (6a) \int \frac{1}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{6\sqrt{-1+ax} \sqrt{1+ax}}{a} + 6x \cosh^{-1}(ax) - \frac{3\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{a} + x \cosh^{-1}(ax)^3
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 68, normalized size = 1.00

$$-\frac{6\sqrt{-1+ax} \sqrt{1+ax}}{a} + 6x \cosh^{-1}(ax) - \frac{3\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{a} + x \cosh^{-1}(ax)^3$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^3,x]`

```
[Out] (-6*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + 6*x*ArcCosh[a*x] - (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + x*ArcCosh[a*x]^3
```

Maple [A]

time = 1.78, size = 61, normalized size = 0.90

method	result	size
derivativdivides	$\frac{\operatorname{arccosh}(ax)^3 ax - 3 \operatorname{arccosh}(ax)^2 \sqrt{ax-1} \sqrt{ax+1} + 6ax \operatorname{arccosh}(ax) - 6 \sqrt{ax-1} \sqrt{ax+1}}{a}$	61
default	$\frac{\operatorname{arccosh}(ax)^3 ax - 3 \operatorname{arccosh}(ax)^2 \sqrt{ax-1} \sqrt{ax+1} + 6ax \operatorname{arccosh}(ax) - 6 \sqrt{ax-1} \sqrt{ax+1}}{a}$	61

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^3,x,method=_RETURNVERBOSE)`

```
[Out] 1/a*(arccosh(a*x)^3*a*x-3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+6*a*x*arccosh(a*x)-6*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

Maxima [A]

time = 0.25, size = 57, normalized size = 0.84

$$x \operatorname{arccosh}(ax)^3 - \frac{3 \sqrt{a^2 x^2 - 1} \operatorname{arccosh}(ax)^2}{a} + \frac{6 \left(ax \operatorname{arccosh}(ax) - \sqrt{a^2 x^2 - 1} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3,x, algorithm="maxima")

[Out] $x \operatorname{arccosh}(a x)^3 - 3 \sqrt{a^2 x^2 - 1} \operatorname{arccosh}(a x)^2 / a + 6 (a x \operatorname{arccosh}(a x) - \sqrt{a^2 x^2 - 1}) / a$

Fricas [A]

time = 0.41, size = 90, normalized size = 1.32

$$\frac{ax \log \left(ax + \sqrt{a^2 x^2 - 1} \right)^3 + 6 ax \log \left(ax + \sqrt{a^2 x^2 - 1} \right) - 3 \sqrt{a^2 x^2 - 1} \log \left(ax + \sqrt{a^2 x^2 - 1} \right)^2 - 6 \sqrt{a^2 x^2 - 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3,x, algorithm="fricas")

[Out] $(a x \log(a x + \sqrt{a^2 x^2 - 1})^3 + 6 a x \log(a x + \sqrt{a^2 x^2 - 1}) - 3 \sqrt{a^2 x^2 - 1} \log(a x + \sqrt{a^2 x^2 - 1})^2 - 6 \sqrt{a^2 x^2 - 1}) / a$

Sympy [C] Result contains complex when optimal does not.

time = 0.12, size = 63, normalized size = 0.93

$$\begin{cases} x \operatorname{acosh}^3(ax) + 6x \operatorname{acosh}(ax) - \frac{3\sqrt{a^2 x^2 - 1} \operatorname{acosh}^2(ax)}{a} - \frac{6\sqrt{a^2 x^2 - 1}}{a} & \text{for } a \neq 0 \\ -\frac{i\pi^3 x}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**3,x)

[Out] Piecewise((x*acosh(a*x)**3 + 6*x*acosh(a*x) - 3*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/a - 6*sqrt(a**2*x**2 - 1)/a, Ne(a, 0)), (-I*pi**3*x/8, True))

Giac [A]

time = 0.40, size = 98, normalized size = 1.44

$$x \log \left(ax + \sqrt{a^2 x^2 - 1} \right)^3 - 3 a \left(\frac{\sqrt{a^2 x^2 - 1} \log \left(ax + \sqrt{a^2 x^2 - 1} \right)^2}{a^2} - \frac{2 \left(x \log \left(ax + \sqrt{a^2 x^2 - 1} \right) - \frac{\sqrt{a^2 x^2 - 1}}{a} \right)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3,x, algorithm="giac")

[Out] $x \log(a x + \sqrt{a^2 x^2 - 1})^3 - 3 a (\sqrt{a^2 x^2 - 1} \log(a x + \sqrt{a^2 x^2 - 1})^2 / a^2 - 2 (x \log(a x + \sqrt{a^2 x^2 - 1}) - \sqrt{a^2 x^2 - 1}) / a) / a$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^3,x)`

[Out] `int(acosh(a*x)^3, x)`

$$3.27 \quad \int \frac{\cosh^{-1}(ax)^3}{x} dx$$

Optimal. Leaf size=87

$$-\frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{3}{2} \cosh^{-1}(ax)^2 \text{PolyLog}\left(2, -e^{2 \cosh^{-1}(ax)}\right) - \frac{3}{2} \cosh^{-1}(ax)$$

```
[Out] -1/4*arccosh(a*x)^4+arccosh(a*x)^3*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+3/2*arccosh(a*x)^2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3/2*arccosh(a*x)*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+3/4*polylog(4,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Rubi [A]

time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5882, 3799, 2221, 2611, 6744, 2320, 6724}

$$\frac{3}{2} \cosh^{-1}(ax)^2 \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right) - \frac{3}{2} \cosh^{-1}(ax) \text{Li}_3\left(-e^{2 \cosh^{-1}(ax)}\right) + \frac{3}{4} \text{Li}_4\left(-e^{2 \cosh^{-1}(ax)}\right) - \frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(e^{2 \cosh^{-1}(ax)} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x, x]
```

```
[Out] -1/4*ArcCosh[a*x]^4 + ArcCosh[a*x]^3*Log[1 + E^(2*ArcCosh[a*x])] + (3*ArcCosh[a*x]^2*PolyLog[2, -E^(2*ArcCosh[a*x])])/2 - (3*ArcCosh[a*x]*PolyLog[3, -E^(2*ArcCosh[a*x])])/2 + (3*PolyLog[4, -E^(2*ArcCosh[a*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F]), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x)))/(1 + E^(2*((-I)*e + f*fz*x)))], x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{x} dx &= \text{Subst}\left(\int x^3 \tanh(x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{1}{4} \cosh^{-1}(ax)^4 + 2\text{Subst}\left(\int \frac{e^{2x} x^3}{1 + e^{2x}} dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) - 3\text{Subst}\left(\int x^2 \log(1 + e^{2x}) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{3}{2} \cosh^{-1}(ax)^2 \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right) \\
&= -\frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{3}{2} \cosh^{-1}(ax)^2 \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right) \\
&= -\frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{3}{2} \cosh^{-1}(ax)^2 \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right) \\
&= -\frac{1}{4} \cosh^{-1}(ax)^4 + \cosh^{-1}(ax)^3 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + \frac{3}{2} \cosh^{-1}(ax)^2 \text{Li}_2\left(-e^{2 \cosh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 82, normalized size = 0.94

$$\frac{1}{4} \left(\cosh^{-1}(ax)^4 + 4 \cosh^{-1}(ax)^3 \log\left(1 + e^{-2 \cosh^{-1}(ax)}\right) - 6 \cosh^{-1}(ax)^2 \text{PolyLog}\left(2, -e^{-2 \cosh^{-1}(ax)}\right) - 6 \cosh^{-1}(ax) \text{PolyLog}\left(3, -e^{-2 \cosh^{-1}(ax)}\right) - 3 \text{PolyLog}\left(4, -e^{-2 \cosh^{-1}(ax)}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^3/x,x]

[Out] (ArcCosh[a*x]^4 + 4*ArcCosh[a*x]^3*Log[1 + E^(-2*ArcCosh[a*x])]) - 6*ArcCosh[a*x]^2*PolyLog[2, -E^(-2*ArcCosh[a*x])] - 6*ArcCosh[a*x]*PolyLog[3, -E^(-2*ArcCosh[a*x])] - 3*PolyLog[4, -E^(-2*ArcCosh[a*x])])/4

Maple [A]

time = 2.57, size = 132, normalized size = 1.52

method	result
derivativedivides	$-\frac{\text{arccosh}(ax)^4}{4} + \text{arccosh}(ax)^3 \ln\left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2\right) + \frac{3\text{arccosh}(ax)^2 \text{polylog}\left(2, -e^{-2 \cosh^{-1}(ax)}\right)}{2}$
default	$-\frac{\text{arccosh}(ax)^4}{4} + \text{arccosh}(ax)^3 \ln\left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2\right) + \frac{3\text{arccosh}(ax)^2 \text{polylog}\left(2, -e^{-2 \cosh^{-1}(ax)}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^3/x,x,method=_RETURNVERBOSE)

[Out] $-1/4*\operatorname{arccosh}(ax)^4+\operatorname{arccosh}(ax)^3*\ln(1+(ax+(ax-1)^{1/2}*(ax+1)^{1/2})^2)+3/2*\operatorname{arccosh}(ax)^2*\operatorname{polylog}(2,-(ax+(ax-1)^{1/2}*(ax+1)^{1/2})^2)-3/2*\operatorname{arccosh}(ax)*\operatorname{polylog}(3,-(ax+(ax-1)^{1/2}*(ax+1)^{1/2})^2)+3/4*\operatorname{polylog}(4,-(ax+(ax-1)^{1/2}*(ax+1)^{1/2})^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(ax)^3/x,x, algorithm="maxima")`

[Out] `integrate(arccosh(ax)^3/x, x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(ax)^3/x,x, algorithm="fricas")`

[Out] `integral(arccosh(ax)^3/x, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(ax)**3/x,x)`

[Out] `Integral(acosh(ax)**3/x, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(ax)^3/x,x, algorithm="giac")`

[Out] `integrate(arccosh(ax)^3/x, x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^3/x,x)

[Out] int(acosh(a*x)^3/x, x)

3.28 $\int \frac{\cosh^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=104

$$-\frac{\cosh^{-1}(ax)^3}{x} + 6a \cosh^{-1}(ax)^2 \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 6ia \cosh^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{\cosh^{-1}(ax)}\right) + 6ia \cosh^{-1}(ax)$$

```
[Out] -arccosh(a*x)^3/x+6*a*arccosh(a*x)^2*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
)-6*I*a*arccosh(a*x)*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+6*I*a*
arccosh(a*x)*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+6*I*a*polylog(3
,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-6*I*a*polylog(3,I*(a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2)))
```

Rubi [A]

time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5883, 5947, 4265, 2611, 2320, 6724}

$$6a \cosh^{-1}(ax)^2 \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 6ia \cosh^{-1}(ax) \operatorname{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 6ia \cosh^{-1}(ax) \operatorname{Li}_2\left(ie^{\cosh^{-1}(ax)}\right) + 6ia \operatorname{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right) - 6ia \operatorname{Li}_3\left(ie^{\cosh^{-1}(ax)}\right) - \frac{\cosh^{-1}(ax)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/x^2,x]
```

```
[Out] -(ArcCosh[a*x]^3/x) + 6*a*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]] - (6*I)*a*A
rcCosh[a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (6*I)*a*ArcCosh[a*x]*PolyLog[
2, I*E^ArcCosh[a*x]] + (6*I)*a*PolyLog[3, (-I)*E^ArcCosh[a*x]] - (6*I)*a*Po
lyLog[3, I*E^ArcCosh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

Rule 5883

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

```

Rule 5947

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && IntegerQ[m]

```

Rule 6724

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{x^2} dx &= -\frac{\cosh^{-1}(ax)^3}{x} + (3a) \int \frac{\cosh^{-1}(ax)^2}{x\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= -\frac{\cosh^{-1}(ax)^3}{x} + (3a) \text{Subst}\left(\int x^2 \text{sech}(x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{\cosh^{-1}(ax)^3}{x} + 6a \cosh^{-1}(ax)^2 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (6ia) \text{Subst}\left(\int x \log(1 - ie^x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{\cosh^{-1}(ax)^3}{x} + 6a \cosh^{-1}(ax)^2 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 6ia \cosh^{-1}(ax) \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^3}{x} + 6a \cosh^{-1}(ax)^2 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 6ia \cosh^{-1}(ax) \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^3}{x} + 6a \cosh^{-1}(ax)^2 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 6ia \cosh^{-1}(ax) \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 128, normalized size = 1.23

$$-\frac{\cosh^{-1}(ax)^3}{x} + 3ia(-\cosh^{-1}(ax)^2(\log(1 - ie^{-\cosh^{-1}(ax)}) - \log(1 + ie^{-\cosh^{-1}(ax)})) - 2\cosh^{-1}(ax)(\text{PolyLog}(2, -ie^{-\cosh^{-1}(ax)}) - \text{PolyLog}(2, ie^{-\cosh^{-1}(ax)})) - 2\text{PolyLog}(3, -ie^{-\cosh^{-1}(ax)}) + 2\text{PolyLog}(3, ie^{-\cosh^{-1}(ax)}))$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^3/x^2, x]`

```
[Out] -(ArcCosh[a*x]^3/x) + (3*I)*a*(-(ArcCosh[a*x]^2*(Log[1 - I/E^ArcCosh[a*x]] - Log[1 + I/E^ArcCosh[a*x]])) - 2*ArcCosh[a*x]*(PolyLog[2, (-I)/E^ArcCosh[a*x]] - PolyLog[2, I/E^ArcCosh[a*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[a*x]] + 2*PolyLog[3, I/E^ArcCosh[a*x]])
```

Maple [F]

time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\text{arccosh}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^3/x^2, x)``[Out] int(arccosh(a*x)^3/x^2, x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3/x^2,x, algorithm="maxima")

[Out] $-\log(ax + \sqrt{ax + 1})\sqrt{ax - 1})^3/x + \int (3(a^3x^2 + \sqrt{ax + 1})\sqrt{ax - 1}a^{2x} - a)\log(ax + \sqrt{ax + 1})\sqrt{ax - 1})^2/(a^3x^4 - ax^2 + (a^2x^3 - x)\sqrt{ax + 1})\sqrt{ax - 1}) dx$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3/x^2,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^3/x^2, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**3/x**2,x)

[Out] Integral(acosh(a*x)**3/x**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3/x^2,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^3/x^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^3/x^2,x)

[Out] int(acosh(a*x)^3/x^2, x)

$$3.29 \quad \int \frac{\cosh^{-1}(ax)^3}{x^3} dx$$

Optimal. Leaf size=98

$$\frac{3}{2}a^2 \cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2 \cosh^{-1}(ax) \log\left(1 + e^{2\cosh^{-1}(ax)}\right) -$$

[Out] $3/2*a^2*\operatorname{arccosh}(a*x)^2 - 1/2*\operatorname{arccosh}(a*x)^3/x^2 - 3*a^2*\operatorname{arccosh}(a*x)*\ln(1+(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2) - 3/2*a^2*\operatorname{polylog}(2, -(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2) + 3/2*a*\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/x$

Rubi [A]

time = 0.22, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5883, 5918, 5882, 3799, 2221, 2317, 2438}

$$-\frac{3}{2}a^2 \operatorname{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right) + \frac{3}{2}a^2 \cosh^{-1}(ax)^2 - 3a^2 \cosh^{-1}(ax) \log\left(e^{2\cosh^{-1}(ax)} + 1\right) - \frac{\cosh^{-1}(ax)^3}{2x^2} + \frac{3a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]^3/x^3,x]`

[Out] $(3*a^2*\operatorname{ArcCosh}[a*x]^2)/2 + (3*a*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x]^2)/(2*x) - \operatorname{ArcCosh}[a*x]^3/(2*x^2) - 3*a^2*\operatorname{ArcCosh}[a*x]*\operatorname{Log}[1 + E^{(2*\operatorname{ArcCosh}[a*x])}] - (3*a^2*\operatorname{PolyLog}[2, -E^{(2*\operatorname{ArcCosh}[a*x])}])/2$

Rule 2221

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

Rule 2317

`Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2438

`Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 3799


```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_]*(f_.)*(x_)] , x
_Symbol] := Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*(-I)*e + f*fz*x))/(1 + E^(2*(-I)*e + f*fz*x))], x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
&& EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^3}{x^3} dx &= -\frac{\cosh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a) \int \frac{\cosh^{-1}(ax)^2}{x^2 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{3a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - (3a^2) \int \frac{\cosh^{-1}(ax)}{x} dx \\
&= \frac{3a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - (3a^2) \text{Subst} \left(\int x \tanh(x) dx, x, \right. \\
&= \frac{3}{2}a^2 \cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - (6a^2) \text{Subst} \left(\right. \\
&= \frac{3}{2}a^2 \cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2 \cosh^{-1}(a \\
&= \frac{3}{2}a^2 \cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2 \cosh^{-1}(a \\
&= \frac{3}{2}a^2 \cosh^{-1}(ax)^2 + \frac{3a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x} - \frac{\cosh^{-1}(ax)^3}{2x^2} - 3a^2 \cosh^{-1}(a
\end{aligned}$$

Mathematica [A]

time = 0.57, size = 92, normalized size = 0.94

$$\frac{1}{2} \left(-\frac{\cosh^{-1}(ax)^3}{x^2} + 3a^2 \left(\cosh^{-1}(ax) \left(-\cosh^{-1}(ax) + \frac{\sqrt{-1+ax}}{1+ax} (1+ax) \cosh^{-1}(ax) - 2 \log(1 + e^{-2 \cosh^{-1}(ax)}) \right) + \text{PolyLog}(2, -e^{-2 \cosh^{-1}(ax)}) \right) \right)$$

Warning: Unable to verify antiderivative.

`[In] Integrate[ArcCosh[a*x]^3/x^3,x]`

```
[Out] (-ArcCosh[a*x]^3/x^2) + 3*a^2*(ArcCosh[a*x]*(-ArcCosh[a*x] + (Sqrt[(-1 + a
*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]))/(a*x) - 2*Log[1 + E^(-2*ArcCosh[a*x]
)]) + PolyLog[2, -E^(-2*ArcCosh[a*x])])
```

Maple [A]

time = 3.60, size = 116, normalized size = 1.18

method	result
derivativedivides	$ a^2 \left(-\frac{\operatorname{arccosh}(ax)^2 \left(-3\sqrt{ax+1} \sqrt{ax-1} ax + 3a^2 x^2 + \operatorname{arccosh}(ax) \right)}{2a^2 x^2} + 3\operatorname{arccosh}(ax)^2 - 3\operatorname{arccosh}(ax) \right) $

default	$a^2 \left(-\frac{\operatorname{arccosh}(ax)^2 \left(-3\sqrt{ax+1} \sqrt{ax-1} ax + 3a^2x^2 + \operatorname{arccosh}(ax) \right)}{2a^2x^2} + 3\operatorname{arccosh}(ax)^2 - 3\operatorname{arccosh}(ax) \right)$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^3/x^3,x,method=_RETURNVERBOSE)`

[Out] $a^2*(-1/2*\operatorname{arccosh}(a*x)^2*(-3*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x+3*a^2*x^2+\operatorname{arccosh}(a*x))/a^2/x^2+3*\operatorname{arccosh}(a*x)^2-3*\operatorname{arccosh}(a*x)*\ln(1+(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2)-3/2*\operatorname{polylog}(2,-(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^3/x^3,x, algorithm="maxima")`

[Out] $-1/2*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^3/x^2 + \operatorname{integrate}(3/2*(a^3*x^2 + \sqrt{a*x + 1}*\sqrt{a*x - 1}*a^2*x - a)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^2/(a^3*x^5 - a*x^3 + (a^2*x^4 - x^2)*\sqrt{a*x + 1}*\sqrt{a*x - 1}), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^3/x^3,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)^3/x^3, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**3/x**3,x)`

[Out] `Integral(acosh(a*x)**3/x**3, x)`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^3/x^3,x)

[Out] int(acosh(a*x)^3/x^3, x)

3.30 $\int \frac{\cosh^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=183

$$\frac{a^2 \cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3 \cosh^{-1}(ax)^2 \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - a^3$$

[Out] $a^2 \operatorname{arccosh}(ax)/x - 1/3 \operatorname{arccosh}(ax)^3/x^3 + a^3 \operatorname{arccosh}(ax)^2 \operatorname{arctan}(ax + (ax-1)^{1/2} \cdot (ax+1)^{1/2}) - a^3 \operatorname{arctan}((ax-1)^{1/2} \cdot (ax+1)^{1/2}) - I a^3 \operatorname{arc} \cosh(ax) \operatorname{polylog}(2, -I \cdot (ax + (ax-1)^{1/2} \cdot (ax+1)^{1/2})) + I a^3 \operatorname{arccosh}(ax) \operatorname{polylog}(2, I \cdot (ax + (ax-1)^{1/2} \cdot (ax+1)^{1/2})) + I a^3 \operatorname{polylog}(3, -I \cdot (ax + (ax-1)^{1/2} \cdot (ax+1)^{1/2})) - I a^3 \operatorname{polylog}(3, I \cdot (ax + (ax-1)^{1/2} \cdot (ax+1)^{1/2})) + 1/2 a \operatorname{arccosh}(ax)^2 \cdot (ax-1)^{1/2} \cdot (ax+1)^{1/2} / x^2$

Rubi [A]

time = 0.38, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5883, 5933, 5947, 4265, 2611, 2320, 6724, 94, 211}

$$-a^3 \operatorname{ArcTan}\left(\sqrt{ax-1}\sqrt{ax+1}\right) + a^3 \cosh^{-1}(ax)^2 \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - ia^3 \cosh^{-1}(ax) \operatorname{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + ia^3 \cosh^{-1}(ax) \operatorname{Li}_2\left(ie^{\cosh^{-1}(ax)}\right) + ia^3 \operatorname{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) - ia^3 \operatorname{Li}_2\left(ie^{\cosh^{-1}(ax)}\right) + \frac{a^2 \cosh^{-1}(ax)}{x} - \frac{\cosh^{-1}(ax)^3}{3x^3} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]^3/x^4, x]`

[Out] $(a^2 \operatorname{ArcCosh}[a*x])/x + (a \operatorname{Sqrt}[-1+a*x] \operatorname{Sqrt}[1+a*x] \operatorname{ArcCosh}[a*x]^2)/(2*x^2) - \operatorname{ArcCosh}[a*x]^3/(3*x^3) + a^3 \operatorname{ArcCosh}[a*x]^2 \operatorname{ArcTan}[E^{\operatorname{ArcCosh}[a*x]}] - a^3 \operatorname{ArcTan}[\operatorname{Sqrt}[-1+a*x] \operatorname{Sqrt}[1+a*x]] - I a^3 \operatorname{ArcCosh}[a*x] \operatorname{PolyLog}[2, (-I) E^{\operatorname{ArcCosh}[a*x]}] + I a^3 \operatorname{ArcCosh}[a*x] \operatorname{PolyLog}[2, I E^{\operatorname{ArcCosh}[a*x]}] + I a^3 \operatorname{PolyLog}[3, (-I) E^{\operatorname{ArcCosh}[a*x]}] - I a^3 \operatorname{PolyLog}[3, I E^{\operatorname{ArcCosh}[a*x]}]$

Rule 94

`Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]`

Rule 211

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 2320

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi`

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m, x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
- E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n*((d_.)*(x_))^(m), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n*((f_.)*(x_))^(m)*((d1_) + (e
1_.)*(x_))^(p)*((d2_) + (e2_.)*(x_))^(p), x_Symbol] := Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*(m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n*(x_)^(m))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
```

```
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^{-1}(ax)^3}{x^4} dx &= -\frac{\cosh^{-1}(ax)^3}{3x^3} + a \int \frac{\cosh^{-1}(ax)^2}{x^3 \sqrt{-1+ax} \sqrt{1+ax}} dx \\ &= \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} - a^2 \int \frac{\cosh^{-1}(ax)}{x^2} dx + \frac{1}{2}a^3 \int \frac{1}{x} dx \\ &= \frac{a^2 \cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + \frac{1}{2}a^3 \text{Subst} \left(\int \frac{1}{x} dx \right) \\ &= \frac{a^2 \cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3 \cosh^{-1}(ax)^2 \\ &= \frac{a^2 \cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3 \cosh^{-1}(ax)^2 \\ &= \frac{a^2 \cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3 \cosh^{-1}(ax)^2 \\ &= \frac{a^2 \cosh^{-1}(ax)}{x} + \frac{a\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{2x^2} - \frac{\cosh^{-1}(ax)^3}{3x^3} + a^3 \cosh^{-1}(ax)^2 \end{aligned}$$

Mathematica [A]

time = 0.40, size = 201, normalized size = 1.10

$$\frac{1}{6} \left(\frac{6a^2 \cosh^{-1}(ax)}{x} + \frac{3a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2}{x^2} - \frac{2 \cosh^{-1}(ax)^3}{x^3} - 3a^2 \left(-4 \text{ArcTan} \left(\tanh \left(\frac{1}{2} \cosh^{-1}(ax) \right) \right) + \cosh^{-1}(ax)^2 \log(1 - ic^{-\cosh^{-1}(ax)}) - \cosh^{-1}(ax)^2 \log(1 + ic^{-\cosh^{-1}(ax)}) + 2 \cosh^{-1}(ax) \text{PolyLog}(2, ic^{-\cosh^{-1}(ax)}) - 2 \cosh^{-1}(ax) \text{PolyLog}(2, ic^{-\cosh^{-1}(ax)}) + 2 \text{PolyLog}(3, ic^{-\cosh^{-1}(ax)}) - 2 \text{PolyLog}(3, ic^{-\cosh^{-1}(ax)}) \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/x^4, x]
```

```
[Out] ((6*a^2*ArcCosh[a*x])/x + (3*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^2)/x^2 - (2*ArcCosh[a*x]^3)/x^3 - (3*I)*a^3*((-4*I)*ArcTan[Tanh[ArcCosh[a*x]/2]] + ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, (-I)/E^ArcCosh[a*x]] - 2
```

$*\text{ArcCosh}[a*x]*\text{PolyLog}[2, I/E^{\text{ArcCosh}[a*x]}] + 2*\text{PolyLog}[3, (-I)/E^{\text{ArcCosh}[a*x]}] - 2*\text{PolyLog}[3, I/E^{\text{ArcCosh}[a*x]}])]/6$

Maple [F]

time = 3.05, size = 0, normalized size = 0.00

$$\int \frac{\text{arccosh}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^3/x^4,x)

[Out] int(arccosh(a*x)^3/x^4,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3/x^4,x, algorithm="maxima")

[Out] $-1/3*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^3/x^3 + \text{integrate}((a^3*x^2 + \sqrt{a*x + 1}*\sqrt{a*x - 1}*a^2*x - a)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^2/(a^3*x^6 - a*x^4 + (a^2*x^5 - x^3)*\sqrt{a*x + 1}*\sqrt{a*x - 1}), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^3/x^4,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^3/x^4, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{acosh}^3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**3/x**4,x)

[Out] Integral(acosh(a*x)**3/x**4, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^3/x^4,x, algorithm="giac")``[Out] integrate(arccosh(a*x)^3/x^4, x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(acosh(a*x)^3/x^4,x)``[Out] int(acosh(a*x)^3/x^4, x)`

3.31 $\int \frac{\cosh^{-1}(ax)^3}{x^5} dx$

Optimal. Leaf size=174

$$-\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x^4}$$

[Out] 1/4*a^2*arccosh(a*x)/x^2+1/2*a^4*arccosh(a*x)^2-1/4*arccosh(a*x)^3/x^4-a^4*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-1/2*a^4*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-1/4*a^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x+1/4*a*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^3+1/2*a^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x

Rubi [A]

time = 0.38, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5883, 5933, 5918, 5882, 3799, 2221, 2317, 2438, 97}

$$-\frac{1}{2}a^4\text{Li}_2(-e^{2\cosh^{-1}(ax)}) + \frac{1}{2}a^4\cosh^{-1}(ax)^2 - a^4\cosh^{-1}(ax)\log(e^{2\cosh^{-1}(ax)}+1) - \frac{a^3\sqrt{ax-1}\sqrt{ax+1}}{4x} + \frac{a^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} - \frac{\cosh^{-1}(ax)^3}{4x^4} + \frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{4x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^3/x^5,x]

[Out] -1/4*(a^3*Sqrt[-1+a*x]*Sqrt[1+a*x])/x + (a^2*ArcCosh[a*x])/(4*x^2) + (a^4*ArcCosh[a*x]^2)/2 + (a*Sqrt[-1+a*x]*Sqrt[1+a*x]*ArcCosh[a*x]^2)/(4*x^3) + (a^3*Sqrt[-1+a*x]*Sqrt[1+a*x]*ArcCosh[a*x]^2)/(2*x) - ArcCosh[a*x]^3/(4*x^4) - a^4*ArcCosh[a*x]*Log[1+E^(2*ArcCosh[a*x])] - (a^4*PolyLog[2,-E^(2*ArcCosh[a*x])])/2

Rule 97

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]

Rule 2221

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x))))], x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
&& EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
```

m + 1)), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^{-1}(ax)^3}{x^5} dx &= -\frac{\cosh^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a) \int \frac{\cosh^{-1}(ax)^2}{x^4\sqrt{-1+ax}\sqrt{1+ax}} dx \\
 &= \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} - \frac{\cosh^{-1}(ax)^3}{4x^4} - \frac{1}{2}a^2 \int \frac{\cosh^{-1}(ax)}{x^3} dx + \frac{1}{2}a^3 \int \frac{1}{x} dx \\
 &= \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2x} \\
 &= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{4x^3} + \frac{a^3}{2x} \\
 &= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{4x^3} \\
 &= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{4x^3} \\
 &= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{4x^3} \\
 &= -\frac{a^3\sqrt{-1+ax}\sqrt{1+ax}}{4x} + \frac{a^2\cosh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\cosh^{-1}(ax)^2 + \frac{a\sqrt{-1+ax}\sqrt{1+ax}}{4x^3}
 \end{aligned}$$

Mathematica [A]

time = 0.45, size = 220, normalized size = 1.26

$$\frac{a^2x^3 - a^2x^5 - ax(1+ax) \left(1 - ax + 2a^2x^2 + 2a^2x^3 \left(-1 + \sqrt{\frac{-1+ax}{1+ax}} \right) \right) \cosh^{-1}(ax)^2 - \sqrt{-1+ax}\sqrt{1+ax} \cosh^{-1}(ax)^3 - a^2x^2 \sqrt{\frac{-1+ax}{1+ax}} (1+ax) \cosh^{-1}(ax) \left(-1 + 4a^2x^2 \log(1 + e^{-2\cosh^{-1}(ax)}) \right) + 2a^2x^4 \sqrt{\frac{-1+ax}{1+ax}} (1+ax) \text{PolyLog}\left(2, -e^{-2\cosh^{-1}(ax)}\right)}{4x^4\sqrt{-1+ax}\sqrt{1+ax}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^3/x^5,x]

[Out] (a^3*x^3 - a^5*x^5 - a*x*(1 + a*x)*(1 - a*x + 2*a^2*x^2 + 2*a^3*x^3*(-1 + Sqrt[(-1 + a*x)/(1 + a*x)]))*ArcCosh[a*x]^2 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3 - a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]*(-1 + 4*a^2*x^2*Log[1 + E^(-2*ArcCosh[a*x])]) + 2*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*PolyLog[2, -E^(-2*ArcCosh[a*x])])/(4*x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x])

Maple [A]

time = 4.01, size = 191, normalized size = 1.10

method	result
derivativedivides	$a^4 \left(-\frac{-2\operatorname{arccosh}(ax)^2 \sqrt{ax-1} \sqrt{ax+1} a^3 x^3 + 2\operatorname{arccosh}(ax)^2 a^4 x^4 - \sqrt{ax-1} \sqrt{ax+1} ax \operatorname{arccosh}(ax)}{4a^4 x^4} \right)$
default	$a^4 \left(-\frac{-2\operatorname{arccosh}(ax)^2 \sqrt{ax-1} \sqrt{ax+1} a^3 x^3 + 2\operatorname{arccosh}(ax)^2 a^4 x^4 - \sqrt{ax-1} \sqrt{ax+1} ax \operatorname{arccosh}(ax)}{4a^4 x^4} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^3/x^5,x,method=_RETURNVERBOSE)`

[Out] $a^4 * (-1/4 * (-2 * \operatorname{arccosh}(a*x)^2 * (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)} * a^3 * x^3 + 2 * \operatorname{arccosh}(a*x)^2 * a^4 * x^4 - (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)} * a * x * \operatorname{arccosh}(a*x)^2 + (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a^3 * x^3 - a^4 * x^4 + \operatorname{arccosh}(a*x)^3 - a^2 * x^2 * \operatorname{arccosh}(a*x)) / a^4 / x^4 + \operatorname{arccosh}(a*x)^2 - \operatorname{arccosh}(a*x) * \ln(1 + (a*x + (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)})^2) - 1/2 * \operatorname{polylog}(2, -(a*x + (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)})^2))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^3/x^5,x, algorithm="maxima")`

[Out] $-1/4 * \log(a*x + \sqrt{a*x + 1} * \sqrt{a*x - 1})^3 / x^4 + \operatorname{integrate}(3/4 * (a^3 * x^2 + \sqrt{a*x + 1} * \sqrt{a*x - 1} * a^2 * x - a) * \log(a*x + \sqrt{a*x + 1} * \sqrt{a*x - 1})^2 / (a^3 * x^7 - a * x^5 + (a^2 * x^6 - x^4) * \sqrt{a*x + 1} * \sqrt{a*x - 1}), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^3/x^5,x, algorithm="fricas")`**[Out]** `integral(arccosh(a*x)^3/x^5, x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**3/x**5,x)`

[Out] `Integral(acosh(a*x)**3/x**5, x)`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^3/x^5,x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^3/x^5,x)`

[Out] `int(acosh(a*x)^3/x^5, x)`

3.32 $\int x^5 \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=306

$$\frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{576a^5} - \frac{65x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{864a^3} - \frac{x^5}{324}$$

[Out] 245/1152*x^2/a^4+65/3456*x^4/a^2+1/324*x^6-245/1152*arccosh(a*x)^2/a^6+5/16*x^2*arccosh(a*x)^2/a^4+5/48*x^4*arccosh(a*x)^2/a^2+1/18*x^6*arccosh(a*x)^2-5/96*arccosh(a*x)^4/a^6+1/6*x^6*arccosh(a*x)^4-245/576*x*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-65/864*x^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-1/54*x^5*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-5/24*x*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-5/36*x^3*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-1/9*x^5*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 1.39, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5883, 5939, 5893, 30}

$\frac{5 \cosh^{-1}(ax)^2}{96a^4} - \frac{245 \cosh^{-1}(ax)^2}{1152a^4} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{24a^5} - \frac{245x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{576a^5} + \frac{245x^2}{1152a^4} + \frac{5x^2 \cosh^{-1}(ax)^2}{16a^4} - \frac{5x^2 \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{36a^4} - \frac{65x^3 \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{864a^3} + \frac{65x^3}{3456a^3} + \frac{5x^4 \cosh^{-1}(ax)^2}{48a^2} + \frac{1}{324} x^6 \cosh^{-1}(ax) + \frac{1}{18} x^6 \cosh^{-1}(ax)^2 - \frac{x^6 \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{9a} - \frac{x^6 \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{36a} + \frac{x^6}{324}$

Antiderivative was successfully verified.

[In] Int[x^5*ArcCosh[a*x]^4,x]

[Out] (245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(576*a^5) - (65*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(864*a^3) - (x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(54*a) - (245*ArcCosh[a*x]^2)/(1152*a^6) + (5*x^2*ArcCosh[a*x]^2)/(16*a^4) + (5*x^4*ArcCosh[a*x]^2)/(48*a^2) + (x^6*ArcCosh[a*x]^2)/18 - (5*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(24*a^5) - (5*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(36*a^3) - (x^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(9*a) - (5*ArcCosh[a*x]^4)/(96*a^6) + (x^6*ArcCosh[a*x]^4)/6

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n)/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/(Sqrt[(d1_.) + (e1_.)*(x_.)]*Sqrt[(d2_.) + (e2_.)*(x_.)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^5 \cosh^{-1}(ax)^4 dx &= \frac{1}{6}x^6 \cosh^{-1}(ax)^4 - \frac{1}{3}(2a) \int \frac{x^6 \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^5 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6 \cosh^{-1}(ax)^4 + \frac{1}{3} \int x^5 \cosh^{-1}(ax)^2 dx - \\
&= \frac{1}{18}x^6 \cosh^{-1}(ax)^2 - \frac{5x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{36a^3} - \frac{x^5 \sqrt{-1+ax} \sqrt{1+ax}}{9a} \\
&= -\frac{x^5 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{54a} + \frac{5x^4 \cosh^{-1}(ax)^2}{48a^2} + \frac{1}{18}x^6 \cosh^{-1}(ax)^2 - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{54a} \\
&= \frac{x^6}{324} - \frac{65x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{864a^3} - \frac{x^5 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{54a} + \\
&= \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{576a^5} - \frac{65x^3 \sqrt{-1+ax} \sqrt{1+ax}}{864a^3} \\
&= \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{576a^5} - \frac{65x^3 \sqrt{-1+ax} \sqrt{1+ax}}{864a^3}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 175, normalized size = 0.57

$$\frac{a^2 x^2 (2205 + 195 a^2 x^2 + 32 a^4 x^4) - 6 a x \sqrt{-1 + a x} \sqrt{1 + a x} (735 + 130 a^2 x^2 + 32 a^4 x^4) \cosh^{-1}(a x) + 9(-245 + 360 a^2 x^2 + 120 a^4 x^4 + 64 a^6 x^6) \cosh^{-1}(a x)^2 - 144 a x \sqrt{-1 + a x} \sqrt{1 + a x} (15 + 10 a^2 x^2 + 8 a^4 x^4) \cosh^{-1}(a x)^3 + 108(-5 + 16 a^6 x^6) \cosh^{-1}(a x)^4}{10368 a^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*ArcCosh[a*x]^4, x]

[Out] (a^2*x^2*(2205 + 195*a^2*x^2 + 32*a^4*x^4) - 6*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(735 + 130*a^2*x^2 + 32*a^4*x^4)*ArcCosh[a*x] + 9*(-245 + 360*a^2*x^2 + 120*a^4*x^4 + 64*a^6*x^6)*ArcCosh[a*x]^2 - 144*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(15 + 10*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x]^3 + 108*(-5 + 16*a^6*x^6)*ArcCosh[a*x]^4)/(10368*a^6)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^5 \operatorname{arccosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*arccosh(a*x)^4, x)

[Out] int(x^5*arccosh(a*x)^4, x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*arccosh(a*x)^4, x, algorithm="maxima")

[Out] 1/6*x^6*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4 - integrate(2/3*(a^3*x^8 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^7 - a*x^6)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x)

Fricas [A]

time = 0.37, size = 208, normalized size = 0.68

$$\frac{32 a^6 x^6 + 195 a^4 x^4 + 108 (16 a^6 x^6 - 5) \log(ax + \sqrt{a^2 x^2 - 1})^4 - 144 (8 a^5 x^5 + 10 a^3 x^3 + 15 a x) \sqrt{a^2 x^2 - 1} \log(ax + \sqrt{a^2 x^2 - 1})^3 + 2205 a^2 x^2 + 9 (64 a^6 x^6 + 120 a^4 x^4 + 360 a^2 x^2 - 245) \log(ax + \sqrt{a^2 x^2 - 1})^2 - 6 (32 a^5 x^5 + 130 a^3 x^3 + 735 a x) \sqrt{a^2 x^2 - 1} \log(ax + \sqrt{a^2 x^2 - 1})}{10368 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*arccosh(a*x)^4, x, algorithm="fricas")

[Out] 1/10368*(32*a^6*x^6 + 195*a^4*x^4 + 108*(16*a^6*x^6 - 5)*log(a*x + sqrt(a^2*x^2 - 1))^4 - 144*(8*a^5*x^5 + 10*a^3*x^3 + 15*a*x)*sqrt(a^2*x^2 - 1)*log(

$a*x + \sqrt{a^2*x^2 - 1})^3 + 2205*a^2*x^2 + 9*(64*a^6*x^6 + 120*a^4*x^4 + 360*a^2*x^2 - 245)*\log(a*x + \sqrt{a^2*x^2 - 1})^2 - 6*(32*a^5*x^5 + 130*a^3*x^3 + 735*a*x)*\sqrt{a^2*x^2 - 1}*\log(a*x + \sqrt{a^2*x^2 - 1})/a^6$

Sympy [A]

time = 1.51, size = 275, normalized size = 0.90

$$\left(\frac{x^6 \operatorname{acosh}^4(ax)}{96} + \frac{x^6 \operatorname{acosh}^3(ax)}{18} + \frac{x^6}{324} - \frac{x^5 \sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{9a} - \frac{x^5 \sqrt{a^2 x^2 - 1} \operatorname{acosh}^2(ax)}{54a} + \frac{5x^4 \operatorname{acosh}^3(ax)}{48a^2} + \frac{65x^4}{3456a^2} - \frac{5x^3 \sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{36a^3} - \frac{65x^3 \sqrt{a^2 x^2 - 1} \operatorname{acosh}^2(ax)}{864a^3} + \frac{5x^2 \operatorname{acosh}^3(ax)}{16a^4} + \frac{245x^2}{1152a^4} - \frac{5x \sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{24a^5} - \frac{245x \sqrt{a^2 x^2 - 1} \operatorname{acosh}^2(ax)}{576a^5} - \frac{5 \operatorname{acosh}^4(ax)}{96a^6} - \frac{245 \operatorname{acosh}^3(ax)}{1152a^6} \right)$$
 for $a \neq 0$
otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*acosh(a*x)**4,x)

[Out] Piecewise((x**6*acosh(a*x)**4/6 + x**6*acosh(a*x)**2/18 + x**6/324 - x**5*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(9*a) - x**5*sqrt(a**2*x**2 - 1)*acosh(a*x)/(54*a) + 5*x**4*acosh(a*x)**2/(48*a**2) + 65*x**4/(3456*a**2) - 5*x**3*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(36*a**3) - 65*x**3*sqrt(a**2*x**2 - 1)*acosh(a*x)/(864*a**3) + 5*x**2*acosh(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4) - 5*x*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(24*a**5) - 245*x*sqrt(a**2*x**2 - 1)*acosh(a*x)/(576*a**5) - 5*acosh(a*x)**4/(96*a**6) - 245*acosh(a*x)**2/(1152*a**6), Ne(a, 0)), (pi**4*x**6/96, True))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*arccosh(a*x)^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*acosh(a*x)^4,x)

[Out] int(x^5*acosh(a*x)^4, x)

3.33 $\int x^4 \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=274

$$\frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{5625a^5} - \frac{1088x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{5625a^3}$$

[Out] 16576/5625*x/a^4+1088/16875*x^3/a^2+24/3125*x^5+32/25*x*arccosh(a*x)^2/a^4+16/75*x^3*arccosh(a*x)^2/a^2+12/125*x^5*arccosh(a*x)^2+1/5*x^5*arccosh(a*x)^4-16576/5625*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-1088/5625*x^2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-24/625*x^4*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-32/75*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-16/75*x^2*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-4/25*x^4*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 1.03, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5883, 5939, 5915, 5879, 8, 30}

$$\frac{32\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{75a^4} - \frac{16576\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{5625a^5} + \frac{16576x}{5625a^4} - \frac{32x\cosh^{-1}(ax)^2}{25a^4} - \frac{16x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{75a^5} - \frac{1088x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{5625a^3} + \frac{1088x^3}{16875a^2} + \frac{16x^3\cosh^{-1}(ax)^2}{75a^4} + \frac{1}{5}x^5\cosh^{-1}(ax)^4 + \frac{12}{125}x^5\cosh^{-1}(ax)^2 - \frac{4x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{25a} - \frac{24x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{625a} - \frac{24x^5}{3125}$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcCosh[a*x]^4, x]

[Out] (16576*x)/(5625*a^4) + (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/(5625*a^5) - (1088*x^2*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/(5625*a^3) - (24*x^4*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/(625*a) + (32*x*ArcCosh[a*x]^2)/(25*a^4) + (16*x^3*ArcCosh[a*x]^2)/(75*a^2) + (12*x^5*ArcCosh[a*x]^2)/125 - (32*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x]^3)/(75*a^5) - (16*x^2*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x]^3)/(75*a^3) - (4*x^4*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x]^3)/(25*a) + (x^5*ArcCosh[a*x]^4)/5

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5879

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1))/(sqrt[

$1 + c*x]*\text{Sqrt}[-1 + c*x]))$, x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((d_.)*(x_.))^ (m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]))], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)*((d1_.) + (e1_.)*(x_))^ (p_)*((d2_.) + (e2_.)*(x_))^ (p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_))^ (m_)*((d1_.) + (e1_.)*(x_))^ (p_)*((d2_.) + (e2_.)*(x_))^ (p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*(m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
\int x^4 \cosh^{-1}(ax)^4 dx &= \frac{1}{5}x^5 \cosh^{-1}(ax)^4 - \frac{1}{5}(4a) \int \frac{x^5 \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{4x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{25a} + \frac{1}{5}x^5 \cosh^{-1}(ax)^4 + \frac{12}{25} \int x^4 \cosh^{-1}(ax)^2 dx \\
&= \frac{12}{125}x^5 \cosh^{-1}(ax)^2 - \frac{16x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{75a^3} - \frac{4x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{25a} \\
&= -\frac{24x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{625a} + \frac{16x^3 \cosh^{-1}(ax)^2}{75a^2} + \frac{12}{125}x^5 \cosh^{-1}(ax)^2 - \frac{16x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{75a^3} \\
&= \frac{24x^5}{3125} - \frac{1088x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{5625a^3} - \frac{24x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{625a} \\
&= \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{5625a^5} - \frac{1088x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{5625a^3} \\
&= \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{5625a^5} - \frac{1088x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{5625a^3}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 158, normalized size = 0.58

$$\frac{8ax(31080 + 680a^2x^2 + 81a^4x^4) - 120\sqrt{-1+ax}\sqrt{1+ax}(2072 + 136a^2x^2 + 27a^4x^4)\cosh^{-1}(ax) + 900ax(120 + 20a^2x^2 + 9a^4x^4)\cosh^{-1}(ax)^2 - 4500\sqrt{-1+ax}\sqrt{1+ax}(8 + 4a^2x^2 + 3a^4x^4)\cosh^{-1}(ax)^3 + 16875a^5x^5\cosh^{-1}(ax)^4}{84375a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*ArcCosh[a*x]^4,x]

[Out] (8*a*x*(31080 + 680*a^2*x^2 + 81*a^4*x^4) - 120*sqrt[-1 + a*x]*sqrt[1 + a*x]*(2072 + 136*a^2*x^2 + 27*a^4*x^4)*ArcCosh[a*x] + 900*a*x*(120 + 20*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x]^2 - 4500*sqrt[-1 + a*x]*sqrt[1 + a*x]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^3 + 16875*a^5*x^5*ArcCosh[a*x]^4)/(84375*a^5)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arccosh(a*x)^4,x)**[Out]** int(x^4*arccosh(a*x)^4,x)

Maxima [A]

time = 0.28, size = 201, normalized size = 0.73

$$\frac{1}{5} x^5 \operatorname{arccosh}(ax)^4 - \frac{4}{75} \left(\frac{3\sqrt{a^2x^2-1}x^4}{a^2} + \frac{4\sqrt{a^2x^2-1}x^2}{a^4} + \frac{8\sqrt{a^2x^2-1}}{a^6} \right) a \operatorname{arccosh}(ax)^3 - \frac{4}{84375} \left(2a \left(\frac{15(27\sqrt{a^2x^2-1}a^2x^4 + 136\sqrt{a^2x^2-1}x^2 + 2072\sqrt{a^2x^2-1}) \operatorname{arccosh}(ax)}{a^5} - \frac{81a^4x^5 + 680a^2x^3 + 31080x}{a^5} \right) - \frac{225(9a^4x^5 + 20a^2x^3 + 120x) \operatorname{arccosh}(ax)^2}{a^5} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccosh(a*x)^4,x, algorithm="maxima")

[Out] $\frac{1}{5}x^5 \operatorname{arccosh}(ax)^4 - \frac{4}{75} \left(\frac{3\sqrt{a^2x^2-1}x^4}{a^2} + \frac{4\sqrt{a^2x^2-1}x^2}{a^4} + \frac{8\sqrt{a^2x^2-1}}{a^6} \right) a \operatorname{arccosh}(ax)^3 - \frac{4}{84375} \left(2a \left(\frac{15(27\sqrt{a^2x^2-1}a^2x^4 + 136\sqrt{a^2x^2-1}x^2 + 2072\sqrt{a^2x^2-1}) \operatorname{arccosh}(ax)}{a^5} - \frac{81a^4x^5 + 680a^2x^3 + 31080x}{a^5} \right) - \frac{225(9a^4x^5 + 20a^2x^3 + 120x) \operatorname{arccosh}(ax)^2}{a^5} \right) a$

Fricas [A]

time = 0.37, size = 189, normalized size = 0.69

$$\frac{16875a^5x^5 \log(ax + \sqrt{a^2x^2-1})^4 + 648a^5x^5 + 5440a^3x^3 - 4500(3a^4x^4 + 4a^2x^2 + 8)\sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1})^3 + 900(9a^5x^5 + 20a^3x^3 + 120ax) \log(ax + \sqrt{a^2x^2-1})^2 - 120(27a^4x^4 + 136a^2x^2 + 2072)\sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1}) - 248640ax}{84375a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccosh(a*x)^4,x, algorithm="fricas")

[Out] $\frac{1}{84375} (16875a^5x^5 \log(ax + \sqrt{a^2x^2-1})^4 + 648a^5x^5 + 5440a^3x^3 - 4500(3a^4x^4 + 4a^2x^2 + 8)\sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1})^3 + 900(9a^5x^5 + 20a^3x^3 + 120ax) \log(ax + \sqrt{a^2x^2-1})^2 - 120(27a^4x^4 + 136a^2x^2 + 2072)\sqrt{a^2x^2-1} \log(ax + \sqrt{a^2x^2-1}) + 248640ax) / a^5$

Sympy [A]

time = 1.02, size = 248, normalized size = 0.91

$$\begin{cases} \frac{x^5 \operatorname{acosh}(ax)}{5} + \frac{12x^3 \operatorname{acosh}^2(ax)}{125} + \frac{34x}{1875} - \frac{4x^4 \sqrt{a^2x^2-1} \operatorname{acosh}^2(ax)}{25a} - \frac{24x^2 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{625a} + \frac{16x^3 \operatorname{acosh}^3(ax)}{75a^2} + \frac{1088x}{16875a} - \frac{16x^4 \sqrt{a^2x^2-1} \operatorname{acosh}^3(ax)}{75a^3} - \frac{1088x^2 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{5625a^3} + \frac{32x \operatorname{acosh}^4(ax)}{25a^4} + \frac{16576x}{5625a^4} - \frac{32\sqrt{a^2x^2-1} \operatorname{acosh}^4(ax)}{75a^5} - \frac{16576\sqrt{a^2x^2-1} \operatorname{acosh}(ax)}{5625a^5} & \text{for } a \neq 0 \\ \frac{x^5}{80} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*acosh(a*x)**4,x)

[Out] $\text{Piecewise}((x^5 \operatorname{acosh}(ax))^4 / 5 + 12x^3 \operatorname{acosh}(ax)^2 / 125 + 24x^5 / 3125 - 4x^4 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)^3 / (25a) - 24x^4 \sqrt{a^2x^2-1} \operatorname{acosh}(ax) / (625a) + 16x^3 \operatorname{acosh}(ax)^2 / (75a^2) + 1088x^3 / (16875a^2) - 16x^3 \sqrt{a^2x^2-1} \operatorname{acosh}(ax)^3 / (75a^3) - 1088x^2 \sqrt{a^2x^2-1} \operatorname{acosh}(ax) / (5625a^3) + 32x \operatorname{acosh}(ax)^2 / (25a^4) + 16576x / (5625a^4) - 32\sqrt{a^2x^2-1} \operatorname{acosh}(ax)^3 / (75a^5) - 16576\sqrt{a^2x^2-1} \operatorname{acosh}(ax) / (5625a^5), \text{Ne}(a, 0)), (\pi^4 x^5 / 80, \text{True}))$

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^4,x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*acosh(a*x)^4,x)`

[Out] `int(x^4*acosh(a*x)^4, x)`

3.34 $\int x^3 \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=214

$$\frac{45x^2}{128a^2} + \frac{3x^4}{128} - \frac{45x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{32a} - \frac{45\cosh^{-1}(ax)^2}{128a^4}$$

[Out] $45/128*x^2/a^2+3/128*x^4-45/128*arccosh(a*x)^2/a^4+9/16*x^2*arccosh(a*x)^2/a^2+3/16*x^4*arccosh(a*x)^2-3/32*arccosh(a*x)^4/a^4+1/4*x^4*arccosh(a*x)^4-45/64*x*arccosh(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-3/32*x^3*arccosh(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-3/8*x*arccosh(a*x)^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-1/4*x^3*arccosh(a*x)^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.85, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5883, 5939, 5893, 30}

$$\frac{3\cosh^{-1}(ax)^4}{32a^4} - \frac{45\cosh^{-1}(ax)^2}{128a^4} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{8a^3} - \frac{45x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{64a^3} + \frac{45x^2}{128a^2} + \frac{9x^2\cosh^{-1}(ax)^2}{16a^2} + \frac{1}{4}x^4\cosh^{-1}(ax)^4 + \frac{3}{16}x^4\cosh^{-1}(ax)^2 - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{4a} - \frac{3x^3\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{32a} - \frac{3x^4}{128}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{ArcCosh}[a*x]^4, x]$

[Out] $(45*x^2)/(128*a^2) + (3*x^4)/128 - (45*x*\text{Sqrt}[-1 + a*x]*\text{Sqrt}[1 + a*x]*\text{ArcCosh}[a*x])/(64*a^3) - (3*x^3*\text{Sqrt}[-1 + a*x]*\text{Sqrt}[1 + a*x]*\text{ArcCosh}[a*x])/(32*a) - (45*\text{ArcCosh}[a*x]^2)/(128*a^4) + (9*x^2*\text{ArcCosh}[a*x]^2)/(16*a^2) + (3*x^4*\text{ArcCosh}[a*x]^2)/16 - (3*x*\text{Sqrt}[-1 + a*x]*\text{Sqrt}[1 + a*x]*\text{ArcCosh}[a*x]^3)/(8*a^3) - (x^3*\text{Sqrt}[-1 + a*x]*\text{Sqrt}[1 + a*x]*\text{ArcCosh}[a*x]^3)/(4*a) - (3*\text{ArcCosh}[a*x]^4)/(32*a^4) + (x^4*\text{ArcCosh}[a*x]^4)/4$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ /; } \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 5883

$\text{Int}[(a_. + \text{ArcCosh}[c_.](x_)]*(b_.))^{(n_.)}*((d_.)(x_))^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m + 1)}*((a + b*\text{ArcCosh}[c*x])^n/(d*(m + 1))), x] - \text{Dist}[b*c*(n/(d*(m + 1))), \text{Int}[(d*x)^{(m + 1)}*((a + b*\text{ArcCosh}[c*x])^{(n - 1)})/(\text{Sqrt}[1 + c*x]*\text{Sqrt}[-1 + c*x]), x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 5893

$\text{Int}[(a_. + \text{ArcCosh}[c_.](x_)]*(b_.))^{(n_.)}/(\text{Sqrt}[(d1_.) + (e1_.)(x_)]*\text{Sqrt}[(d2_.) + (e2_.)(x_)]), x_Symbol] \text{ :> } \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[\text{Sqrt}[1 +$

$c*x]/\text{Sqrt}[d1 + e1*x]]*\text{Simp}[\text{Sqrt}[-1 + c*x]/\text{Sqrt}[d2 + e2*x]]*(a + b*\text{ArcCosh}[c*x])^{(n + 1)}, x] /; \text{FreeQ}\{a, b, c, d1, e1, d2, e2, n\}, x\} \&\& \text{EqQ}[e1, c*d1] \&\& \text{EqQ}[e2, (-c)*d2] \&\& \text{NeQ}[n, -1]$

Rule 5939

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((f_.)*(x_.))^{(m_.)}*((d1_.) + (e1_.)*(x_.))^{(p_.)}*((d2_.) + (e2_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m - 1)}*(d1 + e1*x)^{(p + 1)}*(d2 + e2*x)^{(p + 1)}*((a + b*\text{ArcCosh}[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (\text{Dist}[f^2*((m - 1)/(c^2*(m + 2*p + 1))), \text{Int}[(f*x)^{(m - 2)}*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*\text{ArcCosh}[c*x])^n, x], x] - \text{Dist}[b*f*(n/(c*(m + 2*p + 1)))*\text{Simp}[(d1 + e1*x)^p/(1 + c*x)^p]*\text{Simp}[(d2 + e2*x)^p/(-1 + c*x)^p], \text{Int}[(f*x)^{(m - 1)}*(1 + c*x)^{(p + 1/2)}*(-1 + c*x)^{(p + 1/2)}*(a + b*\text{ArcCosh}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d1, e1, d2, e2, f, p\}, x\} \&\& \text{EqQ}[e1, c*d1] \&\& \text{EqQ}[e2, (-c)*d2] \&\& \text{GtQ}[n, 0] \&\& \text{IGtQ}[m, 1] \&\& \text{NeQ}[m + 2*p + 1, 0]$

Rubi steps

$$\begin{aligned} \int x^3 \cosh^{-1}(ax)^4 dx &= \frac{1}{4}x^4 \cosh^{-1}(ax)^4 - a \int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\ &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4 \cosh^{-1}(ax)^4 + \frac{3}{4} \int x^3 \cosh^{-1}(ax)^2 dx \\ &= \frac{3}{16}x^4 \cosh^{-1}(ax)^2 - \frac{3x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{8a^3} - \frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{4a} \\ &= -\frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{32a} + \frac{9x^2 \cosh^{-1}(ax)^2}{16a^2} + \frac{3}{16}x^4 \cosh^{-1}(ax)^2 - \frac{3x}{16a} \\ &= \frac{3x^4}{128} - \frac{45x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{32a} \\ &= \frac{45x^2}{128a^2} + \frac{3x^4}{128} - \frac{45x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{32a} \end{aligned}$$

Mathematica [A]

time = 0.08, size = 143, normalized size = 0.67

$$\frac{3a^2x^2(15+a^2x^2) - 6ax\sqrt{-1+ax}\sqrt{1+ax}(15+2a^2x^2)\cosh^{-1}(ax) + 3(-15+24a^2x^2+8a^4x^4)\cosh^{-1}(ax)^2 - 16ax\sqrt{-1+ax}\sqrt{1+ax}(3+2a^2x^2)\cosh^{-1}(ax)^3 + 4(-3+8a^4x^4)\cosh^{-1}(ax)^4}{128a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCosh[a*x]^4,x]

[Out] $(3a^2x^2(15 + a^2x^2) - 6ax\sqrt{-1 + ax}\sqrt{1 + ax}(15 + 2a^2x^2) \operatorname{ArcCosh}[ax] + 3(-15 + 24a^2x^2 + 8a^4x^4) \operatorname{ArcCosh}[ax]^2 - 16ax\sqrt{-1 + ax}\sqrt{1 + ax}(3 + 2a^2x^2) \operatorname{ArcCosh}[ax]^3 + 4(-3 + 8a^4x^4) \operatorname{ArcCosh}[ax]^4) / (128a^4)$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arccosh(a*x)^4,x)

[Out] int(x^3*arccosh(a*x)^4,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccosh(a*x)^4,x, algorithm="maxima")

[Out] $1/4x^4 \log(ax + \sqrt{ax + 1}) \sqrt{ax - 1}^4 - \int (a^3x^6 + \sqrt{ax + 1} \sqrt{ax - 1} a^2x^5 - ax^4) \log(ax + \sqrt{ax + 1}) \sqrt{ax - 1}^3 / (a^3x^3 + (a^2x^2 - 1) \sqrt{ax + 1} \sqrt{ax - 1} - ax), x$

Fricas [A]

time = 0.35, size = 176, normalized size = 0.82

$$\frac{3a^4x^4 + 4(8a^4x^4 - 3) \log(ax + \sqrt{a^2x^2 - 1})^4 - 16(2a^3x^3 + 3ax) \sqrt{a^2x^2 - 1} \log(ax + \sqrt{a^2x^2 - 1})^3 + 45a^2x^2 + 3(8a^4x^4 + 24a^2x^2 - 15) \log(ax + \sqrt{a^2x^2 - 1})^2 - 6(2a^3x^3 + 15ax) \sqrt{a^2x^2 - 1} \log(ax + \sqrt{a^2x^2 - 1})}{128a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccosh(a*x)^4,x, algorithm="fricas")

[Out] $1/128(3a^4x^4 + 4(8a^4x^4 - 3) \log(ax + \sqrt{a^2x^2 - 1})^4 - 16(2a^3x^3 + 3ax) \sqrt{a^2x^2 - 1} \log(ax + \sqrt{a^2x^2 - 1})^3 + 45a^2x^2 + 3(8a^4x^4 + 24a^2x^2 - 15) \log(ax + \sqrt{a^2x^2 - 1})^2 - 6(2a^3x^3 + 15ax) \sqrt{a^2x^2 - 1} \log(ax + \sqrt{a^2x^2 - 1})) / a^4$

Sympy [A]

time = 0.70, size = 197, normalized size = 0.92

$$\begin{cases} \frac{x^4 \operatorname{acosh}^4(ax)}{4} + \frac{3x^4 \operatorname{acosh}^3(ax)}{16} + \frac{3x^4}{128} - \frac{x^3 \sqrt{a^2x^2 - 1} \operatorname{acosh}^3(ax)}{4a} - \frac{3x^3 \sqrt{a^2x^2 - 1} \operatorname{acosh}(ax)}{32a} + \frac{9x^2 \operatorname{acosh}^2(ax)}{16a^2} + \frac{45x^2}{128a^2} - \frac{3x \sqrt{a^2x^2 - 1} \operatorname{acosh}^3(ax)}{8a^2} - \frac{45x \sqrt{a^2x^2 - 1} \operatorname{acosh}(ax)}{64a^2} - \frac{3 \operatorname{acosh}^4(ax)}{32a^4} - \frac{45 \operatorname{acosh}^2(ax)}{128a^4} & \text{for } a \neq 0 \\ \frac{x^4 x^4}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**4*acosh(a*x)**4/4 + 3*x**4*acosh(a*x)**2/16 + 3*x**4/128 - x*
*3*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(4*a) - 3*x**3*sqrt(a**2*x**2 - 1)*aco
sh(a*x)/(32*a) + 9*x**2*acosh(a*x)**2/(16*a**2) + 45*x**2/(128*a**2) - 3*x*
sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(8*a**3) - 45*x*sqrt(a**2*x**2 - 1)*acosh
(a*x)/(64*a**3) - 3*acosh(a*x)**4/(32*a**4) - 45*acosh(a*x)**2/(128*a**4),
Ne(a, 0)), (pi**4*x**4/64, True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a*x)^4,x)
```

```
[Out] int(x^3*acosh(a*x)^4, x)
```

3.35 $\int x^2 \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=182

$$\frac{160x}{27a^2} + \frac{8x^3}{81} - \frac{160\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{27a} + \frac{8x\cosh^{-1}(ax)^2}{3a^2} + \frac{4}{9}$$

[Out] $160/27*x/a^2+8/81*x^3+8/3*x*\operatorname{arccosh}(a*x)^2/a^2+4/9*x^3*\operatorname{arccosh}(a*x)^2+1/3*x^3*\operatorname{arccosh}(a*x)^4-160/27*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-8/27*x^2*\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-8/9*\operatorname{arccosh}(a*x)^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-4/9*x^2*\operatorname{arccosh}(a*x)^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a$

Rubi [A]

time = 0.57, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5883, 5939, 5915, 5879, 8, 30}

$$\frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{9a^3} - \frac{160\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{27a^3} + \frac{160x}{27a^2} + \frac{8x\cosh^{-1}(ax)^2}{3a^2} + \frac{1}{3}x^3\cosh^{-1}(ax)^4 + \frac{4}{9}x^3\cosh^{-1}(ax)^2 - \frac{4x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{9a} - \frac{8x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{27a} + \frac{8x^3}{81}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCosh[a*x]^4,x]`

[Out] $(160*x)/(27*a^2) + (8*x^3)/81 - (160*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x])/(27*a^3) - (8*x^2*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x])/(27*a) + (8*x*\operatorname{ArcCosh}[a*x]^2)/(3*a^2) + (4*x^3*\operatorname{ArcCosh}[a*x]^2)/9 - (8*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x]^3)/(9*a^3) - (4*x^2*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x]^3)/(9*a) + (x^3*\operatorname{ArcCosh}[a*x]^4)/3$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 30

`Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 5879

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

Rule 5883

`Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c`

$(n/(d*(m + 1))), \text{Int}[(d*x)^{(m + 1)}*((a + b*\text{ArcCosh}[c*x])^{(n - 1)})/(\text{Sqrt}[1 + c*x]*\text{Sqrt}[-1 + c*x]), x, x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 5915

$\text{Int}[(a_. + \text{ArcCosh}[(c_.)*(x_.)]*(b_.))^{(n_.)}*(x_.)*((d1_.) + (e1_.)*(x_.))^{(p_.)}*((d2_.) + (e2_.)*(x_.))^{(p_.)}, x_Symbol] \ :> \ \text{Simp}[(d1 + e1*x)^{(p + 1)}*(d2 + e2*x)^{(p + 1)}*((a + b*\text{ArcCosh}[c*x])^{(n/(2*e1*e2*(p + 1))})], x] - \text{Dist}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d1 + e1*x)^p/(1 + c*x)^p]*\text{Simp}[(d2 + e2*x)^p/(-1 + c*x)^p], \text{Int}[(1 + c*x)^{(p + 1/2)}*(-1 + c*x)^{(p + 1/2)}*(a + b*\text{ArcCosh}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d1, e1, d2, e2, p\}, x] \ \&\& \ \text{EqQ}[e1, c*d1] \ \&\& \ \text{EqQ}[e2, (-c)*d2] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

Rule 5939

$\text{Int}[(a_. + \text{ArcCosh}[(c_.)*(x_.)]*(b_.))^{(n_.)}*((f_.)*(x_.))^{(m_.)}*((d1_.) + (e1_.)*(x_.))^{(p_.)}*((d2_.) + (e2_.)*(x_.))^{(p_.)}, x_Symbol] \ :> \ \text{Simp}[f*(f*x)^{(m - 1)}*(d1 + e1*x)^{(p + 1)}*(d2 + e2*x)^{(p + 1)}*((a + b*\text{ArcCosh}[c*x])^{(n/(e1*e2*(m + 2*p + 1))})], x] + (\text{Dist}[f^2*((m - 1)/(c^2*(m + 2*p + 1))], \text{Int}[(f*x)^{(m - 2)}*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*\text{ArcCosh}[c*x])^n, x], x] - \text{Dist}[b*f*(n/(c*(m + 2*p + 1)))*\text{Simp}[(d1 + e1*x)^p/(1 + c*x)^p]*\text{Simp}[(d2 + e2*x)^p/(-1 + c*x)^p], \text{Int}[(f*x)^{(m - 1)}*(1 + c*x)^{(p + 1/2)}*(-1 + c*x)^{(p + 1/2)}*(a + b*\text{ArcCosh}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d1, e1, d2, e2, f, p\}, x] \ \&\& \ \text{EqQ}[e1, c*d1] \ \&\& \ \text{EqQ}[e2, (-c)*d2] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0]$

Rubi steps

$$\begin{aligned}
 \int x^2 \cosh^{-1}(ax)^4 dx &= \frac{1}{3}x^3 \cosh^{-1}(ax)^4 - \frac{1}{3}(4a) \int \frac{x^3 \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= -\frac{4x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^4 + \frac{4}{3} \int x^2 \cosh^{-1}(ax)^2 dx \\
 &= \frac{4}{9}x^3 \cosh^{-1}(ax)^2 - \frac{8\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{9a^3} - \frac{4x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{9a} \\
 &= -\frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{27a} + \frac{8x \cosh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3 \cosh^{-1}(ax)^2 - \frac{8\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{27a} \\
 &= \frac{8x^3}{81} - \frac{160\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{27a^3} - \frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{27a} \\
 &= \frac{160x}{27a^2} + \frac{8x^3}{81} - \frac{160\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{27a^3} - \frac{8x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{27a}
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 122, normalized size = 0.67

$$\frac{8ax(60 + a^2x^2) - 24\sqrt{-1+ax}\sqrt{1+ax}(20 + a^2x^2)\cosh^{-1}(ax) + 36ax(6 + a^2x^2)\cosh^{-1}(ax)^2 - 36\sqrt{-1+ax}\sqrt{1+ax}(2 + a^2x^2)\cosh^{-1}(ax)^3 + 27a^3x^3\cosh^{-1}(ax)^4}{81a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcCosh[a*x]^4,x]

[Out] (8*a*x*(60 + a^2*x^2) - 24*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(20 + a^2*x^2)*ArcCosh[a*x] + 36*a*x*(6 + a^2*x^2)*ArcCosh[a*x]^2 - 36*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^3 + 27*a^3*x^3*ArcCosh[a*x]^4)/(81*a^3)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arccosh(a*x)^4,x)**[Out]** int(x^2*arccosh(a*x)^4,x)**Maxima [A]**

time = 0.27, size = 143, normalized size = 0.79

$$\frac{1}{3}x^3\operatorname{arccosh}(ax)^4 - \frac{4}{9}a\left(\frac{\sqrt{a^2x^2-1}x^2}{a^2} + \frac{2\sqrt{a^2x^2-1}}{a^4}\right)\operatorname{arccosh}(ax)^3 - \frac{4}{81}\left(2a\left(\frac{3\left(\sqrt{a^2x^2-1}x^2 + \frac{20\sqrt{a^2x^2-1}}{a^2}\right)\operatorname{arccosh}(ax)}{a^2} - \frac{a^2x^3 + 60x}{a^4}\right) - \frac{9(a^2x^3 + 6x)\operatorname{arccosh}(ax)^2}{a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^4,x, algorithm="maxima")

[Out] 1/3*x^3*arccosh(a*x)^4 - 4/9*a*(sqrt(a^2*x^2 - 1)*x^2/a^2 + 2*sqrt(a^2*x^2 - 1)/a^4)*arccosh(a*x)^3 - 4/81*(2*a*(3*(sqrt(a^2*x^2 - 1)*x^2 + 20*sqrt(a^2*x^2 - 1)/a^2)*arccosh(a*x)/a^3 - (a^2*x^3 + 60*x)/a^4) - 9*(a^2*x^3 + 6*x)*arccosh(a*x)^2/a^3)*a

Fricas [A]

time = 0.37, size = 154, normalized size = 0.85

$$\frac{27a^3x^3\log(ax + \sqrt{a^2x^2-1})^4 + 8a^3x^3 - 36(a^2x^2 + 2)\sqrt{a^2x^2-1}\log(ax + \sqrt{a^2x^2-1})^3 + 36(a^3x^3 + 6ax)\log(ax + \sqrt{a^2x^2-1})^2 - 24(a^2x^2 + 20)\sqrt{a^2x^2-1}\log(ax + \sqrt{a^2x^2-1}) + 480ax}{81a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^4,x, algorithm="fricas")

```
[Out] 1/81*(27*a^3*x^3*log(a*x + sqrt(a^2*x^2 - 1))^4 + 8*a^3*x^3 - 36*(a^2*x^2 + 2)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 36*(a^3*x^3 + 6*a*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 24*(a^2*x^2 + 20)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1)) + 480*a*x)/a^3
```

Sympy [A]

time = 0.47, size = 165, normalized size = 0.91

$$\begin{cases} \frac{x^3 \operatorname{acosh}^4(ax)}{3} + \frac{4x^3 \operatorname{acosh}^2(ax)}{9} + \frac{8x^3}{81} - \frac{4x^2 \sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{9a} - \frac{8x^2 \sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{27a} + \frac{8x \operatorname{acosh}^2(ax)}{3a^2} + \frac{160x}{27a^2} - \frac{8 \sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{9a^3} - \frac{160 \sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{27a^3} & \text{for } a \neq 0 \\ \frac{\pi^4 x^3}{48} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**4,x)
```

```
[Out] Piecewise((x**3*acosh(a*x)**4/3 + 4*x**3*acosh(a*x)**2/9 + 8*x**3/81 - 4*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(9*a) - 8*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/(27*a) + 8*x*acosh(a*x)**2/(3*a**2) + 160*x/(27*a**2) - 8*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/(9*a**3) - 160*sqrt(a**2*x**2 - 1)*acosh(a*x)/(27*a**3), Ne(a, 0)), (pi**4*x**3/48, True))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(a*x)^4,x)
```

```
[Out] int(x^2*acosh(a*x)^4, x)
```

3.36 $\int x \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=120

$$\frac{3x^2}{4} - \frac{3x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a} - \frac{3\cosh^{-1}(ax)^2}{4a^2} + \frac{3}{2}x^2\cosh^{-1}(ax)^2 - \frac{x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a}$$

[Out] $\frac{3}{4}x^2 - \frac{3}{4}a\operatorname{arccosh}(ax)^2/a^2 + \frac{3}{2}x^2\operatorname{arccosh}(ax)^2 - \frac{1}{4}a\operatorname{arccosh}(ax)^4/a^2 + \frac{1}{2}x^2\operatorname{arccosh}(ax)^4 - \frac{3}{2}x\operatorname{arccosh}(ax)(ax-1)^{1/2}(ax+1)^{1/2}/a - \operatorname{arccosh}(ax)^3(ax-1)^{1/2}(ax+1)^{1/2}/a$

Rubi [A]

time = 0.40, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,

Rules used = {5883, 5939, 5893, 30}

$$-\frac{\cosh^{-1}(ax)^4}{4a^2} - \frac{3\cosh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^4 + \frac{3}{2}x^2\cosh^{-1}(ax)^2 - \frac{x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{2a} + \frac{3x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x*ArcCosh[a*x]^4,x]

[Out] $(3*x^2)/4 - (3*x*\sqrt{-1+ax}*\sqrt{1+ax}*\operatorname{ArcCosh}[a*x])/(2*a) - (3*\operatorname{ArcCosh}[a*x]^2)/(4*a^2) + (3*x^2*\operatorname{ArcCosh}[a*x]^2)/2 - (x*\sqrt{-1+ax}*\sqrt{1+ax}*\operatorname{ArcCosh}[a*x]^3)/a - \operatorname{ArcCosh}[a*x]^4/(4*a^2) + (x^2*\operatorname{ArcCosh}[a*x]^4)/2$

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 5883

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*((d_)*(x_)^(m_), x_Symbol] :> Simp[(d*x)^(m+1)*((a+b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a+b*ArcCosh[c*x])^(n-1)/(Sqrt[1+c*x]*Sqrt[-1+c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5893

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)/(Sqrt[(d1_) + (e1_)*(x_)])*Sqrt[(d2_) + (e2_)*(x_)], x_Symbol] :> Simp[(1/(b*c*(n+1)))*Simp[Sqrt[1+c*x]/Sqrt[d1+e1*x]]*Simp[Sqrt[-1+c*x]/Sqrt[d2+e2*x]]*(a+b*ArcCosh[c*x])^(n+1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x \cosh^{-1}(ax)^4 dx &= \frac{1}{2}x^2 \cosh^{-1}(ax)^4 - (2a) \int \frac{x^2 \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a} + \frac{1}{2}x^2 \cosh^{-1}(ax)^4 + 3 \int x \cosh^{-1}(ax)^2 dx - \dots \\
&= \frac{3}{2}x^2 \cosh^{-1}(ax)^2 - \frac{x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a} - \frac{\cosh^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^4 - \dots \\
&= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{2a} + \frac{3}{2}x^2 \cosh^{-1}(ax)^2 - \frac{x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} \\
&= \frac{3x^2}{4} - \frac{3x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{2a} - \frac{3 \cosh^{-1}(ax)^2}{4a^2} + \frac{3}{2}x^2 \cosh^{-1}(ax)^2 - \dots
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 104, normalized size = 0.87

$$\frac{3a^2x^2 - 6ax\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax) + (-3 + 6a^2x^2) \cosh^{-1}(ax)^2 - 4ax\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3 + (-1 + 2a^2x^2) \cosh^{-1}(ax)^4}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcCosh[a*x]^4,x]

[Out] (3*a^2*x^2 - 6*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + (-3 + 6*a^2*x^2)*ArcCosh[a*x]^2 - 4*a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3 + (-1 + 2*a^2*x^2)*ArcCosh[a*x]^4)/(4*a^2)

Maple [A]

time = 1.75, size = 104, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{a^2 x^2 \operatorname{arccosh}(ax)^4}{2} - ax \operatorname{arccosh}(ax)^3 \sqrt{ax-1} \sqrt{ax+1} - \frac{\operatorname{arccosh}(ax)^4}{4} + \frac{3a^2 x^2 \operatorname{arccosh}(ax)^2}{2} - \frac{3ax \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1}}{2}}{a^2}$
default	$\frac{\frac{a^2 x^2 \operatorname{arccosh}(ax)^4}{2} - ax \operatorname{arccosh}(ax)^3 \sqrt{ax-1} \sqrt{ax+1} - \frac{\operatorname{arccosh}(ax)^4}{4} + \frac{3a^2 x^2 \operatorname{arccosh}(ax)^2}{2} - \frac{3ax \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1}}{2}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccosh(a*x)^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{a^2} \left(\frac{1}{2} a^2 x^2 \operatorname{arccosh}(ax)^4 - ax \operatorname{arccosh}(ax)^3 (ax-1)^{1/2} (ax+1)^{1/2} - \frac{1}{4} \operatorname{arccosh}(ax)^4 + \frac{3}{2} a^2 x^2 \operatorname{arccosh}(ax)^2 - \frac{3}{2} ax \operatorname{arccosh}(ax) (ax-1)^{1/2} (ax+1)^{1/2} - \frac{3}{4} \operatorname{arccosh}(ax)^2 + \frac{3}{4} a^2 x^2 \right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^4,x, algorithm="maxima")`

[Out] $\frac{1}{2} x^2 \log(ax + \sqrt{ax+1} \sqrt{ax-1})^4 - \int (2(a^3 x^4 + \sqrt{ax+1} \sqrt{ax-1} a^2 x^3 - ax^2) \log(ax + \sqrt{ax+1} \sqrt{ax-1})^3 / (a^3 x^3 + (a^2 x^2 - 1) \sqrt{ax+1} \sqrt{ax-1} - ax), x)$

Fricas [A]

time = 0.36, size = 138, normalized size = 1.15

$$\frac{4 \sqrt{a^2 x^2 - 1} ax \log(ax + \sqrt{a^2 x^2 - 1})^3 - (2a^2 x^2 - 1) \log(ax + \sqrt{a^2 x^2 - 1})^4 - 3a^2 x^2 + 6 \sqrt{a^2 x^2 - 1} ax \log(ax + \sqrt{a^2 x^2 - 1}) - 3(2a^2 x^2 - 1) \log(ax + \sqrt{a^2 x^2 - 1})^2}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^4,x, algorithm="fricas")`

[Out] $-\frac{1}{4} (4 \sqrt{a^2 x^2 - 1} a x \log(ax + \sqrt{a^2 x^2 - 1})^3 - (2a^2 x^2 - 1) \log(ax + \sqrt{a^2 x^2 - 1})^4 - 3a^2 x^2 + 6 \sqrt{a^2 x^2 - 1} ax \log(ax + \sqrt{a^2 x^2 - 1}) - 3(2a^2 x^2 - 1) \log(ax + \sqrt{a^2 x^2 - 1})^2) / a^2$

Sympy [A]

time = 0.32, size = 110, normalized size = 0.92

$$\begin{cases} \frac{x^2 \operatorname{acosh}^4(ax)}{2} + \frac{3x^2 \operatorname{acosh}^2(ax)}{2} + \frac{3x^2}{4} - \frac{x \sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{a} - \frac{3x \sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{2a} - \frac{\operatorname{acosh}^4(ax)}{4a^2} - \frac{3 \operatorname{acosh}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi^4 x^2}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acosh(a*x)**4,x)`

[Out] `Piecewise((x**2*acosh(a*x)**4/2 + 3*x**2*acosh(a*x)**2/2 + 3*x**2/4 - x*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/a - 3*x*sqrt(a**2*x**2 - 1)*acosh(a*x)/(2*a) - acosh(a*x)**4/(4*a**2) - 3*acosh(a*x)**2/(4*a**2), Ne(a, 0)), (pi**4*x**2/32, True))`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^4,x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acosh(a*x)^4,x)`

[Out] `int(x*acosh(a*x)^4, x)`

3.37 $\int \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=77

$$24x - \frac{24\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{a} + 12x\cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{a} + x\cosh^{-1}(ax)^4$$

[Out] 24*x+12*x*arccosh(a*x)^2+x*arccosh(a*x)^4-24*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-4*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a

Rubi [A]

time = 0.19, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5879, 5915, 8}

$$x\cosh^{-1}(ax)^4 - \frac{4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{a} + 12x\cosh^{-1}(ax)^2 - \frac{24\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a} + 24x$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^4,x]

[Out] 24*x - (24*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/a + 12*x*ArcCosh[a*x]^2 - (4*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x]^3)/a + x*ArcCosh[a*x]^4

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.], x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n-1))/(sqrt[1 + c*x]*sqrt[-1 + c*x])], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_.*(x_)*((d1_.) + (e1_.)*(x_))^(p_.)*((d2_.) + (e2_.)*(x_))^(q_.), x_Symbol] := Simp[(d1 + e1*x)^(p+1)*(d2 + e2*x)^(q+1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p+1))), x] - Dist[b*(n/(2*c*(p+1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p+1/2)*(-1 + c*x)^(p+1/2)*(a + b*ArcCosh[c*x])^(n-1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(ax)^4 dx &= x \cosh^{-1}(ax)^4 - (4a) \int \frac{x \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a} + x \cosh^{-1}(ax)^4 + 12 \int \cosh^{-1}(ax)^2 dx \\
&= 12x \cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a} + x \cosh^{-1}(ax)^4 - (24a) \int \frac{1}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{24\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} + 12x \cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a} + x \cosh^{-1}(ax)^4 \\
&= 24x - \frac{24\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} + 12x \cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 77, normalized size = 1.00

$$24x - \frac{24\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{a} + 12x \cosh^{-1}(ax)^2 - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{a} + x \cosh^{-1}(ax)^4$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^4,x]`

```
[Out] 24*x - (24*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x])/a + 12*x*ArcCosh[a*x]^2 - (4*sqrt[-1 + a*x]*sqrt[1 + a*x]*ArcCosh[a*x]^3)/a + x*ArcCosh[a*x]^4
```

Maple [A]

time = 1.80, size = 71, normalized size = 0.92

method	result
derivativedivides	$\frac{\operatorname{arccosh}(ax)^4 ax - 4 \operatorname{arccosh}(ax)^3 \sqrt{ax-1} \sqrt{ax+1} + 12 \operatorname{arccosh}(ax)^2 ax - 24 \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1} + \operatorname{arccosh}(ax)^4}{a}$
default	$\frac{\operatorname{arccosh}(ax)^4 ax - 4 \operatorname{arccosh}(ax)^3 \sqrt{ax-1} \sqrt{ax+1} + 12 \operatorname{arccosh}(ax)^2 ax - 24 \operatorname{arccosh}(ax) \sqrt{ax-1} \sqrt{ax+1} + \operatorname{arccosh}(ax)^4}{a}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^4,x,method=_RETURNVERBOSE)`

```
[Out] 1/a*(arccosh(a*x)^4*a*x-4*arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+12*arccosh(a*x)^2*a*x-24*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+24*a*x)
```

Maxima [A]

time = 0.27, size = 73, normalized size = 0.95

$$x \operatorname{arccosh}(ax)^4 - \frac{4\sqrt{a^2x^2-1} \operatorname{arccosh}(ax)^3}{a} + 12 \left(\frac{x \operatorname{arccosh}(ax)^2}{a} + \frac{2 \left(x - \frac{\sqrt{a^2x^2-1} \operatorname{arccosh}(ax)}{a} \right)}{a} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4,x, algorithm="maxima")

[Out] $x \operatorname{arccosh}(a x)^4 - 4 \sqrt{a^2 x^2 - 1} \operatorname{arccosh}(a x)^3 / a + 12 (x \operatorname{arccosh}(a x))^2 / a + 2 (x - \sqrt{a^2 x^2 - 1}) \operatorname{arccosh}(a x) / a) / a * a$

Fricas [A]

time = 0.45, size = 112, normalized size = 1.45

$$\frac{ax \log(ax + \sqrt{a^2 x^2 - 1})^4 + 12 ax \log(ax + \sqrt{a^2 x^2 - 1})^2 - 4 \sqrt{a^2 x^2 - 1} \log(ax + \sqrt{a^2 x^2 - 1})^3 + 24 ax - 24 \sqrt{a^2 x^2 - 1} \log(ax + \sqrt{a^2 x^2 - 1})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4,x, algorithm="fricas")

[Out] $(a x \log(a x + \sqrt{a^2 x^2 - 1})^4 + 12 a x \log(a x + \sqrt{a^2 x^2 - 1})^2 - 4 \sqrt{a^2 x^2 - 1} \log(a x + \sqrt{a^2 x^2 - 1})^3 + 24 a x - 24 \sqrt{a^2 x^2 - 1} \log(a x + \sqrt{a^2 x^2 - 1})) / a$

Sympy [A]

time = 0.17, size = 70, normalized size = 0.91

$$\begin{cases} x \operatorname{acosh}^4(ax) + 12x \operatorname{acosh}^2(ax) + 24x - \frac{4\sqrt{a^2 x^2 - 1} \operatorname{acosh}^3(ax)}{a} - \frac{24\sqrt{a^2 x^2 - 1} \operatorname{acosh}(ax)}{a} & \text{for } a \neq 0 \\ \frac{\pi^4 x}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**4,x)

[Out] Piecewise((x*acosh(a*x)**4 + 12*x*acosh(a*x)**2 + 24*x - 4*sqrt(a**2*x**2 - 1)*acosh(a*x)**3/a - 24*sqrt(a**2*x**2 - 1)*acosh(a*x)/a, Ne(a, 0)), (pi**4*x/16, True))

Giac [A]

time = 0.46, size = 125, normalized size = 1.62

$$x \log(ax + \sqrt{a^2 x^2 - 1})^4 - 4 \left(\frac{\sqrt{a^2 x^2 - 1} \log(ax + \sqrt{a^2 x^2 - 1})^3}{a^2} - \frac{3 \left(x \log(ax + \sqrt{a^2 x^2 - 1})^2 + 2a \left(\frac{x}{a} - \frac{\sqrt{a^2 x^2 - 1} \log(ax + \sqrt{a^2 x^2 - 1})}{a^2} \right) \right)}{a} \right) / a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4,x, algorithm="giac")

[Out] $x \log(a x + \sqrt{a^2 x^2 - 1})^4 - 4 (\sqrt{a^2 x^2 - 1} \log(a x + \sqrt{a^2 x^2 - 1})^3 / a^2 - 3 (x \log(a x + \sqrt{a^2 x^2 - 1})^2 + 2 a (x / a - \sqrt{a^2 x^2 - 1} \log(a x + \sqrt{a^2 x^2 - 1}) / a^2)) / a) / a * a$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^4, x)`

[Out] `int(acosh(a*x)^4, x)`

3.38 $\int \frac{\cosh^{-1}(ax)^4}{x} dx$

Optimal. Leaf size=103

$$-\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log\left(1 + e^{2 \cosh^{-1}(ax)}\right) + 2 \cosh^{-1}(ax)^3 \text{PolyLog}\left(2, -e^{2 \cosh^{-1}(ax)}\right) - 3 \cosh^{-1}(ax)^2$$

```
[Out] -1/5*arccosh(a*x)^5+arccosh(a*x)^4*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+2*arccosh(a*x)^3*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3*arccosh(a*x)^2*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)+3*arccosh(a*x)*polylog(4,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)-3/2*polylog(5,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
```

Rubi [A]

time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5882, 3799, 2221, 2611, 6744, 2320, 6724}

$$2 \cosh^{-1}(ax)^3 \text{Li}_2(-e^{2 \cosh^{-1}(ax)}) - 3 \cosh^{-1}(ax)^2 \text{Li}_3(-e^{2 \cosh^{-1}(ax)}) + 3 \cosh^{-1}(ax) \text{Li}_4(-e^{2 \cosh^{-1}(ax)}) - \frac{3}{2} \text{Li}_5(-e^{2 \cosh^{-1}(ax)}) - \frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log(e^{2 \cosh^{-1}(ax)} + 1)$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^4/x, x]

```
[Out] -1/5*ArcCosh[a*x]^5 + ArcCosh[a*x]^4*Log[1 + E^(2*ArcCosh[a*x])] + 2*ArcCosh[a*x]^3*PolyLog[2, -E^(2*ArcCosh[a*x])] - 3*ArcCosh[a*x]^2*PolyLog[3, -E^(2*ArcCosh[a*x])] + 3*ArcCosh[a*x]*PolyLog[4, -E^(2*ArcCosh[a*x])] - (3*PolyLog[5, -E^(2*ArcCosh[a*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611


```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))]^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(c
+ d*x)^m*(E^(2*(-I)*e + f*fz*x))/(1 + E^(2*(-I)*e + f*fz*x))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] := Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.)
*(x_)))]^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^4}{x} dx &= \text{Subst} \left(\int x^4 \tanh(x) dx, x, \cosh^{-1}(ax) \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + 2 \text{Subst} \left(\int \frac{e^{2x} x^4}{1 + e^{2x}} dx, x, \cosh^{-1}(ax) \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) - 4 \text{Subst} \left(\int x^3 \log(1 + e^{2x}) dx, x, \cosh^{-1}(ax) \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + 2 \cosh^{-1}(ax)^3 \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + 2 \cosh^{-1}(ax)^3 \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + 2 \cosh^{-1}(ax)^3 \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + 2 \cosh^{-1}(ax)^3 \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right) \\
&= -\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) + 2 \cosh^{-1}(ax)^3 \text{Li}_2 \left(-e^{2 \cosh^{-1}(ax)} \right)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 103, normalized size = 1.00

$$\frac{1}{5} \cosh^{-1}(ax)^5 + \cosh^{-1}(ax)^4 \log \left(1 + e^{2 \cosh^{-1}(ax)} \right) - 2 \cosh^{-1}(ax)^3 \text{PolyLog} \left(2, -e^{2 \cosh^{-1}(ax)} \right) - 3 \cosh^{-1}(ax)^2 \text{PolyLog} \left(3, -e^{2 \cosh^{-1}(ax)} \right) - 3 \cosh^{-1}(ax) \text{PolyLog} \left(4, -e^{2 \cosh^{-1}(ax)} \right) - \frac{3}{2} \text{PolyLog} \left(5, -e^{2 \cosh^{-1}(ax)} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^4/x, x]`

```
[Out] ArcCosh[a*x]^5/5 + ArcCosh[a*x]^4*Log[1 + E^(-2*ArcCosh[a*x])] - 2*ArcCosh[a*x]^3*PolyLog[2, -E^(-2*ArcCosh[a*x])] - 3*ArcCosh[a*x]^2*PolyLog[3, -E^(-2*ArcCosh[a*x])] - 3*ArcCosh[a*x]*PolyLog[4, -E^(-2*ArcCosh[a*x])] - (3*PolyLog[5, -E^(-2*ArcCosh[a*x])])/2
```

Maple [A]

time = 2.01, size = 165, normalized size = 1.60

method	result
derivativedivides	$-\frac{\text{arccosh}(ax)^5}{5} + \text{arccosh}(ax)^4 \ln \left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2 \right) + 2\text{arccosh}(ax)^3 \text{poly}$
default	$-\frac{\text{arccosh}(ax)^5}{5} + \text{arccosh}(ax)^4 \ln \left(1 + (ax + \sqrt{ax-1} \sqrt{ax+1})^2 \right) + 2\text{arccosh}(ax)^3 \text{poly}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^4/x,x,method=_RETURNVERBOSE)`

[Out] $-1/5*\operatorname{arccosh}(a*x)^5+\operatorname{arccosh}(a*x)^4*\ln(1+(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)+2*\operatorname{arccosh}(a*x)^3*\operatorname{polylog}(2,-(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)-3*\operatorname{arccosh}(a*x)^2*\operatorname{polylog}(3,-(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)+3*\operatorname{arccosh}(a*x)*\operatorname{polylog}(4,-(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)-3/2*\operatorname{polylog}(5,-(a*x+(a*x-1)^{(1/2)}*(a*x+1)^{(1/2}))^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^4/x,x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)^4/x, x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^4/x,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)^4/x, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^4(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**4/x,x)`

[Out] `Integral(acosh(a*x)**4/x, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^4/x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^4/x,x)
```

```
[Out] int(acosh(a*x)^4/x, x)
```

3.39 $\int \frac{\cosh^{-1}(ax)^4}{x^2} dx$

Optimal. Leaf size=150

$$-\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -ie^{\cosh^{-1}(ax)}\right) + 12ia \cosh^{-1}(ax)^2 \operatorname{PolyLog}\left(2, ie^{\cosh^{-1}(ax)}\right) - 24ia \cosh^{-1}(ax) \operatorname{PolyLog}\left(3, -ie^{\cosh^{-1}(ax)}\right) + 24ia \cosh^{-1}(ax) \operatorname{PolyLog}\left(3, ie^{\cosh^{-1}(ax)}\right) - 24ia \operatorname{PolyLog}\left(4, -ie^{\cosh^{-1}(ax)}\right) + 24ia \operatorname{PolyLog}\left(4, ie^{\cosh^{-1}(ax)}\right)$$

```
[Out] -arccosh(a*x)^4/x+8*a*arccosh(a*x)^3*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-12*I*a*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+12*I*a*arccosh(a*x)^2*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+24*I*a*arccosh(a*x)*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-24*I*a*arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-24*I*a*polylog(4,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+24*I*a*polylog(4,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
```

Rubi [A]

time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5883, 5947, 4265, 2611, 6744, 2320, 6724}

$$8a \cosh^{-1}(ax)^3 \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \operatorname{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) + 12ia \cosh^{-1}(ax)^2 \operatorname{Li}_2\left(ie^{\cosh^{-1}(ax)}\right) + 24ia \cosh^{-1}(ax) \operatorname{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right) - 24ia \cosh^{-1}(ax) \operatorname{Li}_3\left(ie^{\cosh^{-1}(ax)}\right) - 24ia \operatorname{Li}_4\left(-ie^{\cosh^{-1}(ax)}\right) + 24ia \operatorname{Li}_4\left(ie^{\cosh^{-1}(ax)}\right) - \frac{\cosh^{-1}(ax)^4}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^4/x^2,x]

```
[Out] -(ArcCosh[a*x]^4/x) + 8*a*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]] - (12*I)*a*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (12*I)*a*ArcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]] + (24*I)*a*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]] - (24*I)*a*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]] - (24*I)*a*PolyLog[4, (-I)*E^ArcCosh[a*x]] + (24*I)*a*PolyLog[4, I*E^ArcCosh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x))))^n]/(b*c*n*Log[F]), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m-1)*PolyLog[2, (-e)*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^4}{x^2} dx &= -\frac{\cosh^{-1}(ax)^4}{x} + (4a) \int \frac{\cosh^{-1}(ax)^3}{x\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + (4a) \text{Subst}\left(\int x^3 \text{sech}(x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - (12ia) \text{Subst}\left(\int x^2 \log(1 - ie^x) dx, x, \cosh^{-1}(ax)\right) \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right) \\
&= -\frac{\cosh^{-1}(ax)^4}{x} + 8a \cosh^{-1}(ax)^3 \tan^{-1}\left(e^{\cosh^{-1}(ax)}\right) - 12ia \cosh^{-1}(ax)^2 \text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 478 vs. 2(150) = 300.
time = 0.46, size = 478, normalized size = 3.19

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^4/x^2, x]

[Out] a*(((7*I)/16)*Pi^4 + (Pi^3*ArcCosh[a*x])/2 - ((3*I)/2)*Pi^2*ArcCosh[a*x]^2 - 2*Pi*ArcCosh[a*x]^3 + I*ArcCosh[a*x]^4 - ArcCosh[a*x]^4/(a*x) + (Pi^3*Log[1 + I/E^ArcCosh[a*x]])/2 - (3*I)*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - 6*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + (4*I)*ArcCosh[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] + (3*I)*Pi^2*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] + 6*Pi*ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - (Pi^3*Log[1 + I/E^ArcCosh[a*x]])/2 - (4*I)*ArcCosh[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] + (Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]])/2 + (3*I)*(Pi - (2*I)*ArcCosh[a*x])^2*PolyLog[2, (-I)/E^ArcCosh[a*x]] - (12*I)*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]] + (3*I)*Pi^2*PolyLog[2, I/E^ArcCosh[a*x]] + 12*Pi*ArcCosh[a*x]*PolyLog[2, I/E^ArcCosh[a*x]] + 12*Pi*PolyLog[3, (-I)/E^ArcCosh[a*x]] - (24*I)*ArcCosh[a*x]*PolyLog[3, (-I)/E^ArcCosh[a*x]] + (24*I)*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]] - 12*Pi*PolyLog[3, I/E^ArcCosh[a*x]] - (24*I)*PolyLog[4, (-I)/E^ArcCosh[a*x]] - (24*I)*PolyLog[4, (-I)*E^ArcCosh[a*x]])

Maple [F]

time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^4/x^2,x)``[Out] int(arccosh(a*x)^4/x^2,x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^4/x^2,x, algorithm="maxima")`

```
[Out] -log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^4/x + integrate(4*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(a^3*x^4 - a*x^2 + (a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^4/x^2,x, algorithm="fricas")``[Out] integral(arccosh(a*x)^4/x^2, x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^4(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(acosh(a*x)**4/x**2,x)``[Out] Integral(acosh(a*x)**4/x**2, x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^4/x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^4/x^2,x)
```

```
[Out] int(acosh(a*x)^4/x^2, x)
```

3.40 $\int \frac{\cosh^{-1}(ax)^4}{x^3} dx$

Optimal. Leaf size=115

$$2a^2 \cosh^{-1}(ax)^3 + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2 \cosh^{-1}(ax)^2 \log\left(1 + e^{2\cosh^{-1}(ax)}\right)$$

[Out] $2a^2 \operatorname{arccosh}(ax)^3 - 1/2 \operatorname{arccosh}(ax)^4/x^2 - 6a^2 \operatorname{arccosh}(ax)^2 \ln(1 + (ax + (ax-1)^{(1/2)}(ax+1)^{(1/2)})^2) - 6a^2 \operatorname{arccosh}(ax) \operatorname{polylog}(2, -(ax + (ax-1)^{(1/2)}(ax+1)^{(1/2)})^2) + 3a^2 \operatorname{polylog}(3, -(ax + (ax-1)^{(1/2)}(ax+1)^{(1/2)})^2) + 2a \operatorname{arccosh}(ax)^3 (ax-1)^{(1/2)}(ax+1)^{(1/2)}/x$

Rubi [A]

time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$,

Rules used = {5883, 5918, 5882, 3799, 2221, 2611, 2320, 6724}

$$-6a^2 \cosh^{-1}(ax) \operatorname{Li}_2(-e^{2\cosh^{-1}(ax)}) + 3a^2 \operatorname{Li}_3(-e^{2\cosh^{-1}(ax)}) + 2a^2 \cosh^{-1}(ax)^3 - 6a^2 \cosh^{-1}(ax)^2 \log(e^{2\cosh^{-1}(ax)} + 1) - \frac{\cosh^{-1}(ax)^4}{2x^2} + \frac{2a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^3}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcCosh}[a*x]^4/x^3, x]$

[Out] $2a^2 \operatorname{ArcCosh}[a*x]^3 + (2a \operatorname{Sqrt}[-1 + a*x] \operatorname{Sqrt}[1 + a*x] \operatorname{ArcCosh}[a*x]^3)/x - \operatorname{ArcCosh}[a*x]^4/(2*x^2) - 6a^2 \operatorname{ArcCosh}[a*x]^2 \operatorname{Log}[1 + E^{(2*\operatorname{ArcCosh}[a*x])}] - 6a^2 \operatorname{ArcCosh}[a*x] \operatorname{PolyLog}[2, -E^{(2*\operatorname{ArcCosh}[a*x])}] + 3a^2 \operatorname{PolyLog}[3, -E^{(2*\operatorname{ArcCosh}[a*x])}]$

Rule 2221

$\operatorname{Int}[(((F_)^\wedge((g_.) * ((e_.) + (f_.) * (x_))))^\wedge(n_.) * ((c_.) + (d_.) * (x_))^\wedge(m_.) / ((a_.) + (b_.) * ((F_)^\wedge((g_.) * ((e_.) + (f_.) * (x_))))^\wedge(n_.)], x_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^\wedge m / (b*f*g*n*\operatorname{Log}[F])] * \operatorname{Log}[1 + b*((F)^\wedge(g*(e + f*x)))^\wedge n/a], x] - \operatorname{Dist}[d*(m / (b*f*g*n*\operatorname{Log}[F])), \operatorname{Int}[(c + d*x)^\wedge(m - 1) * \operatorname{Log}[1 + b*((F)^\wedge(g*(e + f*x)))^\wedge n/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \operatorname{IGTQ}[m, 0]$

Rule 2320

$\operatorname{Int}[u, x_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_.) * (v_))^\wedge(n_)^\wedge(m_) /; \operatorname{FreeQ}\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& \operatorname{!MatchQ}[u, E^\wedge((c_.) * ((a_.) + (b_.) * x)) * (F_)[v_] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

Rule 2611

$\operatorname{Int}[\operatorname{Log}[1 + (e_.) * ((F_)^\wedge((c_.) * ((a_.) + (b_.) * (x_))))^\wedge(n_.)] * ((f_.) + (g_.) * (x_))^\wedge(m_.), x_Symbol] \rightarrow \operatorname{Simp}[(-f + g*x)^\wedge m * (\operatorname{PolyLog}[2, (-e) * (F)^\wedge(c*(a +$

$(b*x))^n/(b*c*n*\text{Log}[F]), x] + \text{Dist}[g*(m/(b*c*n*\text{Log}[F])), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x)))^n}], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3799

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\text{tan}[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[(-I)*((c + d*x)^{(m+1)}/(d*(m+1))), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*(E^{(2*((-I)*e + f*fz*x))}/(1 + E^{(2*((-I)*e + f*fz*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 5882

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}/(x_), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Subst}[\text{Int}[x^n*\text{Tanh}[-a/b + x/b], x], x, a + b*\text{ArcCosh}[c*x]], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0]$

Rule 5883

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}*((d_.)*(x_)]^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcCosh}[c*x])^n/(d*(m+1))), x] - \text{Dist}[b*c*(n/(d*(m+1))), \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcCosh}[c*x])^{(n-1)})/(\text{Sqrt}[1 + c*x]*\text{Sqrt}[-1 + c*x])], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

Rule 5918

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}*((f_.)*(x_)]^{(m_.)}*((d1_.) + (e1_.)*(x_)]^{(p_.)}*((d2_.) + (e2_.)*(x_)]^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d1 + e1*x)^{(p+1)}*(d2 + e2*x)^{(p+1)}*((a + b*\text{ArcCosh}[c*x])^n/(d1*d2*f*(m+1))), x] + \text{Dist}[b*c*(n/(f*(m+1)))*\text{Simp}[(d1 + e1*x)^p/(1 + c*x)^p]*\text{Simp}[(d2 + e2*x)^p/(-1 + c*x)^p], \text{Int}[(f*x)^{(m+1)}*(1 + c*x)^{(p+1/2)}*(-1 + c*x)^{(p+1/2)}*(a + b*\text{ArcCosh}[c*x])^{(n-1)}], x], x] /; \text{FreeQ}\{a, b, c, d1, e1, d2, e2, f, m, p\}, x] \&\& \text{EqQ}[e1, c*d1] \&\& \text{EqQ}[e2, (-c)*d2] \&\& \text{GtQ}[n, 0] \&\& \text{EqQ}[m + 2*p + 3, 0] \&\& \text{NeQ}[p, -1]$

Rule 6724

$\text{Int}[\text{PolyLog}[n, (c_.)*((a_.) + (b_.)*(x_)]^{(p_.)}]/((d_.) + (e_.)*(x_))], x_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)^4}{x^3} dx &= -\frac{\cosh^{-1}(ax)^4}{2x^2} + (2a) \int \frac{\cosh^{-1}(ax)^3}{x^2 \sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - (6a^2) \int \frac{\cosh^{-1}(ax)^2}{x} dx, x \\
&= \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - (6a^2) \text{Subst} \left(\int x^2 \tanh(x) dx, x \right) \\
&= 2a^2 \cosh^{-1}(ax)^3 + \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - (12a^2) \text{Subst} \\
&= 2a^2 \cosh^{-1}(ax)^3 + \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2 \cosh^{-1}(ax) \\
&= 2a^2 \cosh^{-1}(ax)^3 + \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2 \cosh^{-1}(ax) \\
&= 2a^2 \cosh^{-1}(ax)^3 + \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2 \cosh^{-1}(ax) \\
&= 2a^2 \cosh^{-1}(ax)^3 + \frac{2a \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3}{x} - \frac{\cosh^{-1}(ax)^4}{2x^2} - 6a^2 \cosh^{-1}(ax)
\end{aligned}$$

Mathematica [A]

time = 0.77, size = 112, normalized size = 0.97

$$-\frac{\cosh^{-1}(ax)^4}{2x^2} + a^2 \left(2 \cosh^{-1}(ax)^2 \left(-\cosh^{-1}(ax) + \frac{\sqrt{-1+ax} (1+ax) \cosh^{-1}(ax)}{ax} - 3 \log(1 + e^{-2 \cosh^{-1}(ax)}) \right) + 6 \cosh^{-1}(ax) \text{PolyLog}(2, -e^{-2 \cosh^{-1}(ax)}) + 3 \text{PolyLog}(3, -e^{-2 \cosh^{-1}(ax)}) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^4/x^3,x]

[Out] $-1/2 * \text{ArcCosh}[a*x]^4/x^2 + a^2 * (2 * \text{ArcCosh}[a*x]^2 * (-\text{ArcCosh}[a*x] + (\text{Sqrt}[-(1 + a*x)/(1 + a*x)] * (1 + a*x) * \text{ArcCosh}[a*x])) / (a*x) - 3 * \text{Log}[1 + E^{-2 * \text{ArcCosh}[a*x]}]) + 6 * \text{ArcCosh}[a*x] * \text{PolyLog}[2, -E^{-2 * \text{ArcCosh}[a*x]}]) + 3 * \text{PolyLog}[3, -E^{-2 * \text{ArcCosh}[a*x]}])$

Maple [A]

time = 3.64, size = 149, normalized size = 1.30

method	result
derivativedivides	$a^2 \left(-\frac{\text{arccosh}(ax)^3 (4a^2 x^2 - 4\sqrt{ax+1} \sqrt{ax-1} ax + \text{arccosh}(ax))}{2a^2 x^2} + 4 \text{arccosh}(ax)^3 - 6 \text{arccosh}(ax) \right)$

default	$a^2 \left(-\frac{\operatorname{arccosh}(ax)^3 (4a^2x^2 - 4\sqrt{ax+1}\sqrt{ax-1}ax + \operatorname{arccosh}(ax))}{2a^2x^2} + 4\operatorname{arccosh}(ax)^3 - 6\operatorname{arccosh}(ax) \right)$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^4/x^3,x,method=_RETURNVERBOSE)`

[Out] $a^2 * (-1/2 * \operatorname{arccosh}(a*x)^3 * (4*a^2*x^2 - 4*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x + \operatorname{arccosh}(a*x)) / a^2/x^2 + 4*\operatorname{arccosh}(a*x)^3 - 6*\operatorname{arccosh}(a*x)^2 * \ln(1 + (a*x + (a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2) - 6*\operatorname{arccosh}(a*x) * \operatorname{polylog}(2, -(a*x + (a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2) + 3*\operatorname{polylog}(3, -(a*x + (a*x-1)^{(1/2)}*(a*x+1)^{(1/2)})^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^4/x^3,x, algorithm="maxima")`

[Out] $-1/2 * \log(a*x + \sqrt{a*x + 1}) * \sqrt{a*x - 1}^4 / x^2 + \operatorname{integrate}(2*(a^3*x^2 + \sqrt{a*x + 1}) * \sqrt{a*x - 1} * a^2*x - a) * \log(a*x + \sqrt{a*x + 1}) * \sqrt{a*x - 1}^3 / (a^3*x^5 - a*x^3 + (a^2*x^4 - x^2) * \sqrt{a*x + 1}) * \sqrt{a*x - 1}, x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^4/x^3,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)^4/x^3, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^4(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**4/x**3,x)`

[Out] `Integral(acosh(a*x)**4/x**3, x)`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(ax)^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^4/x^3,x)

[Out] int(acosh(a*x)^4/x^3, x)

3.41 $\int \frac{\cosh^{-1}(ax)^4}{x^4} dx$

Optimal. Leaf size=268

$$\frac{2a^2 \cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3 \cosh^{-1}(ax) \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) +$$

```
[Out] 2*a^2*arccosh(a*x)^2/x-1/3*arccosh(a*x)^4/x^3-8*a^3*arccosh(a*x)*arctan(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2))+4/3*a^3*arccosh(a*x)^3*arctan(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))+4*I*a^3*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-2
*I*a^3*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-4*I*a
^3*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+2*I*a^3*arccosh(a*x)^2*po
lylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+4*I*a^3*arccosh(a*x)*polylog(3
,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-4*I*a^3*arccosh(a*x)*polylog(3,I*(a
x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-4*I*a^3*polylog(4,-I*(a*x+(a*x-1)^(1/2)*(a
x+1)^(1/2)))+4*I*a^3*polylog(4,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))+2/3*a*a
rccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/x^2
```

Rubi [A]

time = 0.53, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5883, 5933, 5947, 4265, 2611, 6744, 2320, 6724, 2317, 2438}

$$\frac{2a^2 \cosh^{-1}(ax) \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 8a^3 \cosh^{-1}(ax) \operatorname{ArcTan}\left(e^{\cosh^{-1}(ax)}\right) - 2a^2 \cosh^{-1}(ax)^2 \operatorname{Li}_2\left(-e^{\cosh^{-1}(ax)}\right) + 2a^2 \cosh^{-1}(ax)^2 \operatorname{Li}_2\left(e^{\cosh^{-1}(ax)}\right) + 4a^2 \cosh^{-1}(ax) \operatorname{Li}_2\left(-e^{\cosh^{-1}(ax)}\right) - 4a^2 \cosh^{-1}(ax) \operatorname{Li}_2\left(e^{\cosh^{-1}(ax)}\right) + 4a^2 \operatorname{Li}_2\left(-e^{\cosh^{-1}(ax)}\right) - 4a^2 \operatorname{Li}_2\left(e^{\cosh^{-1}(ax)}\right) + 4a^2 \operatorname{Li}_2\left(e^{\cosh^{-1}(ax)}\right) + \frac{2a^2 \cosh^{-1}(ax)^2}{x} - \frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^4/x^4,x]

```
[Out] (2*a^2*ArcCosh[a*x]^2)/x + (2*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3
)/(3*x^2) - ArcCosh[a*x]^4/(3*x^3) - 8*a^3*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*
x]] + (4*a^3*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/3 + (4*I)*a^3*PolyLog[2
, (-I)*E^ArcCosh[a*x]] - (2*I)*a^3*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh
[a*x]] - (4*I)*a^3*PolyLog[2, I*E^ArcCosh[a*x]] + (2*I)*a^3*ArcCosh[a*x]^2*
PolyLog[2, I*E^ArcCosh[a*x]] + (4*I)*a^3*ArcCosh[a*x]*PolyLog[3, (-I)*E^Arc
Cosh[a*x]] - (4*I)*a^3*ArcCosh[a*x]*PolyLog[3, I*E^ArcCosh[a*x]] - (4*I)*a^
3*PolyLog[4, (-I)*E^ArcCosh[a*x]] + (4*I)*a^3*PolyLog[4, I*E^ArcCosh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
```

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m, x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
- E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n)*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n)*((f_.)*(x_)^(m_))*((d1_) + (e
1_.)*(x_)^(p_))*((d2_) + (e2_.)*(x_)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```


Rule 5947

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.))/(Sqrt[(d1_) + (e1_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] := Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && IntegerQ[m]

Rule 6724

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6744

Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.))))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^{-1}(ax)^4}{x^4} dx &= -\frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{1}{3}(4a) \int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{-1+ax}\sqrt{1+ax}} dx \\
 &= \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - (2a^2) \int \frac{\cosh^{-1}(ax)^2}{x^2} dx + \frac{1}{3}(2a^3) \int \frac{\cosh^{-1}(ax)}{x} dx \\
 &= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{1}{3}(2a^3) \operatorname{Subst}\left[\int \frac{\cosh^{-1}(u)}{u} du, u, ax\right] \\
 &= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} + \frac{4}{3}a^3\cosh^{-1}(ax) \\
 &= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax) \\
 &= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax) \\
 &= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax) \\
 &= \frac{2a^2\cosh^{-1}(ax)^2}{x} + \frac{2a\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}{3x^2} - \frac{\cosh^{-1}(ax)^4}{3x^3} - 8a^3\cosh^{-1}(ax)
 \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 595 vs. $2(268) = 536$.
time = 2.15, size = 595, normalized size = 2.22

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^4/x^4,x]

[Out] $a^3 \cdot \left(\frac{I}{2} \cdot (8 + \pi^2 - (4I)\pi \operatorname{ArcCosh}[a*x] - 4 \operatorname{ArcCosh}[a*x]^2) \cdot \operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcCosh}[a*x]}] - \frac{I}{96} \cdot (7\pi^4 + (8I)\pi^3 \operatorname{ArcCosh}[a*x] + 24\pi^2 \operatorname{ArcCosh}[a*x]^2 + \frac{(192I)\operatorname{ArcCosh}[a*x]^2}{(a*x)} - (32I)\pi \operatorname{ArcCosh}[a*x]^3 + \frac{(64I)\sqrt{(-1+a*x)/(1+a*x)} \cdot (1+a*x) \operatorname{ArcCosh}[a*x]^3}{a^2 x^2} - 16 \operatorname{ArcCosh}[a*x]^4 - \frac{(32I)\operatorname{ArcCosh}[a*x]^4}{a^3 x^3} - 384 \operatorname{ArcCosh}[a*x] \cdot \operatorname{Log}[1 - I/E^{\operatorname{ArcCosh}[a*x]}] + (8I)\pi^3 \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] + 384 \operatorname{ArcCosh}[a*x] \cdot \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] + 48\pi^2 \operatorname{ArcCosh}[a*x] \cdot \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] - (96I)\pi \operatorname{ArcCosh}[a*x]^2 \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] - 64 \operatorname{ArcCosh}[a*x]^3 \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] - 48\pi^2 \operatorname{ArcCosh}[a*x] \cdot \operatorname{Log}[1 - I/E^{\operatorname{ArcCosh}[a*x]}] + (96I)\pi \operatorname{ArcCosh}[a*x]^2 \operatorname{Log}[1 - I/E^{\operatorname{ArcCosh}[a*x]}] - (8I)\pi^3 \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] + 64 \operatorname{ArcCosh}[a*x]^3 \operatorname{Log}[1 + I/E^{\operatorname{ArcCosh}[a*x]}] + (8I)\pi^3 \operatorname{Log}[\operatorname{Tan}[(\pi + (2I)\operatorname{ArcCosh}[a*x])/4]] + 384 \operatorname{PolyLog}[2, I/E^{\operatorname{ArcCosh}[a*x]}] + 192 \operatorname{ArcCosh}[a*x]^2 \operatorname{PolyLog}[2, (-I)E^{\operatorname{ArcCosh}[a*x]}] - 48\pi^2 \operatorname{PolyLog}[2, I/E^{\operatorname{ArcCosh}[a*x]}] + (192I)\pi \operatorname{ArcCosh}[a*x] \operatorname{PolyLog}[2, I/E^{\operatorname{ArcCosh}[a*x]}] + (192I)\pi \operatorname{PolyLog}[3, (-I)/E^{\operatorname{ArcCosh}[a*x]}] + 384 \operatorname{ArcCosh}[a*x] \operatorname{PolyLog}[3, (-I)/E^{\operatorname{ArcCosh}[a*x]}] - 384 \operatorname{ArcCosh}[a*x] \operatorname{PolyLog}[3, (-I)E^{\operatorname{ArcCosh}[a*x]}] - (192I)\pi \operatorname{PolyLog}[3, I/E^{\operatorname{ArcCosh}[a*x]}] + 384 \operatorname{PolyLog}[4, (-I)/E^{\operatorname{ArcCosh}[a*x]}] + 384 \operatorname{PolyLog}[4, (-I)E^{\operatorname{ArcCosh}[a*x]}] \right)$

Maple [F]

time = 3.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^4}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^4/x^4,x)

[Out] int(arccosh(a*x)^4/x^4,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4/x^4,x, algorithm="maxima")

[Out] $-1/3 \log(ax + \sqrt{ax + 1}) \sqrt{ax - 1}^4/x^3 + \int (4/3(a^3x^2 + \sqrt{ax + 1}) \sqrt{ax - 1} a^2x - a) \log(ax + \sqrt{ax + 1}) \sqrt{ax - 1}^3 / (a^3x^6 - ax^4 + (a^2x^5 - x^3) \sqrt{ax + 1}) \sqrt{ax - 1}, x$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4/x^4,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^4/x^4, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^4(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**4/x**4,x)

[Out] Integral(acosh(a*x)**4/x**4, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^4/x^4,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^4/x^4, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(ax)^4}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^4/x^4,x)

[Out] int(acosh(a*x)^4/x^4, x)

$$3.42 \quad \int \frac{x^6}{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=55

$$\frac{5\text{Shi}(\cosh^{-1}(ax))}{64a^7} + \frac{9\text{Shi}(3\cosh^{-1}(ax))}{64a^7} + \frac{5\text{Shi}(5\cosh^{-1}(ax))}{64a^7} + \frac{\text{Shi}(7\cosh^{-1}(ax))}{64a^7}$$

[Out] 5/64*Shi(arccosh(a*x))/a^7+9/64*Shi(3*arccosh(a*x))/a^7+5/64*Shi(5*arccosh(a*x))/a^7+1/64*Shi(7*arccosh(a*x))/a^7

Rubi [A]

time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5887, 5556, 3379}

$$\frac{5\text{Shi}(\cosh^{-1}(ax))}{64a^7} + \frac{9\text{Shi}(3\cosh^{-1}(ax))}{64a^7} + \frac{5\text{Shi}(5\cosh^{-1}(ax))}{64a^7} + \frac{\text{Shi}(7\cosh^{-1}(ax))}{64a^7}$$

Antiderivative was successfully verified.

[In] Int[x^6/ArcCosh[a*x],x]

[Out] (5*SinhIntegral[ArcCosh[a*x]])/(64*a^7) + (9*SinhIntegral[3*ArcCosh[a*x]])/(64*a^7) + (5*SinhIntegral[5*ArcCosh[a*x]])/(64*a^7) + SinhIntegral[7*ArcCosh[a*x]]/(64*a^7)

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^6(x) \sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^7} \\
&= \frac{\text{Subst}\left(\int \left(\frac{5 \sinh(x)}{64x} + \frac{9 \sinh(3x)}{64x} + \frac{5 \sinh(5x)}{64x} + \frac{\sinh(7x)}{64x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^7} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(7x)}{x} dx, x, \cosh^{-1}(ax)\right)}{64a^7} + \frac{5 \text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{64a^7} + \frac{5 \text{Subst}\left(\int \frac{\sinh(3x)}{x} dx, x, \cosh^{-1}(ax)\right)}{64a^7} \\
&= \frac{5 \text{Shi}(\cosh^{-1}(ax))}{64a^7} + \frac{9 \text{Shi}(3 \cosh^{-1}(ax))}{64a^7} + \frac{5 \text{Shi}(5 \cosh^{-1}(ax))}{64a^7} + \frac{\text{Shi}(7 \cosh^{-1}(ax))}{64a^7}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 40, normalized size = 0.73

$$\frac{5 \text{Shi}(\cosh^{-1}(ax)) + 9 \text{Shi}(3 \cosh^{-1}(ax)) + 5 \text{Shi}(5 \cosh^{-1}(ax)) + \text{Shi}(7 \cosh^{-1}(ax))}{64a^7}$$

Antiderivative was successfully verified.

`[In] Integrate[x^6/ArcCosh[a*x],x]`

```
[Out] (5*SinhIntegral[ArcCosh[a*x]] + 9*SinhIntegral[3*ArcCosh[a*x]] + 5*SinhIntegral[5*ArcCosh[a*x]] + SinhIntegral[7*ArcCosh[a*x]])/(64*a^7)
```

Maple [A]

time = 2.40, size = 40, normalized size = 0.73

method	result
derivativedivides	$\frac{5 \text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{64} + \frac{9 \text{hyperbolicSineIntegral}(3 \text{arccosh}(ax))}{64} + \frac{5 \text{hyperbolicSineIntegral}(5 \text{arccosh}(ax))}{64} + \frac{\text{hyperbolicSineIntegral}(7 \text{arccosh}(ax))}{64}$
default	$\frac{5 \text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{64} + \frac{9 \text{hyperbolicSineIntegral}(3 \text{arccosh}(ax))}{64} + \frac{5 \text{hyperbolicSineIntegral}(5 \text{arccosh}(ax))}{64} + \frac{\text{hyperbolicSineIntegral}(7 \text{arccosh}(ax))}{64}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^6/arccosh(a*x),x,method=_RETURNVERBOSE)`

```
[Out] 1/a^7*(5/64*Shi(arccosh(a*x))+9/64*Shi(3*arccosh(a*x))+5/64*Shi(5*arccosh(a*x))+1/64*Shi(7*arccosh(a*x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arccosh(a*x),x, algorithm="maxima")

[Out] integrate(x^6/arccosh(a*x), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arccosh(a*x),x, algorithm="fricas")

[Out] integral(x^6/arccosh(a*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/acosh(a*x),x)

[Out] Integral(x**6/acosh(a*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arccosh(a*x),x, algorithm="giac")

[Out] integrate(x^6/arccosh(a*x), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/acosh(a*x),x)

[Out] int(x^6/acosh(a*x), x)

3.43 $\int \frac{x^5}{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}(2 \cosh^{-1}(ax))}{32a^6} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{8a^6} + \frac{\text{Shi}(6 \cosh^{-1}(ax))}{32a^6}$$

[Out] 5/32*Shi(2*arccosh(a*x))/a^6+1/8*Shi(4*arccosh(a*x))/a^6+1/32*Shi(6*arccosh(a*x))/a^6

Rubi [A]

time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5887, 5556, 3379}

$$\frac{5\text{Shi}(2 \cosh^{-1}(ax))}{32a^6} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{8a^6} + \frac{\text{Shi}(6 \cosh^{-1}(ax))}{32a^6}$$

Antiderivative was successfully verified.

[In] Int[x^5/ArcCosh[a*x], x]

[Out] (5*SinhIntegral[2*ArcCosh[a*x]])/(32*a^6) + SinhIntegral[4*ArcCosh[a*x]]/(8*a^6) + SinhIntegral[6*ArcCosh[a*x]]/(32*a^6)

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^5(x) \sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^6} \\
&= \frac{\text{Subst}\left(\int \left(\frac{5 \sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^6} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x} dx, x, \cosh^{-1}(ax)\right)}{32a^6} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} dx, x, \cosh^{-1}(ax)\right)}{8a^6} + \frac{5 \text{Subst}\left(\int \frac{\sinh(2x)}{x} dx, x, \cosh^{-1}(ax)\right)}{32a^6} \\
&= \frac{5 \text{Shi}(2 \cosh^{-1}(ax))}{32a^6} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{8a^6} + \frac{\text{Shi}(6 \cosh^{-1}(ax))}{32a^6}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 33, normalized size = 0.77

$$\frac{5 \text{Shi}(2 \cosh^{-1}(ax)) + 4 \text{Shi}(4 \cosh^{-1}(ax)) + \text{Shi}(6 \cosh^{-1}(ax))}{32a^6}$$

Antiderivative was successfully verified.

`[In] Integrate[x^5/ArcCosh[a*x], x]``[Out] (5*SinhIntegral[2*ArcCosh[a*x]] + 4*SinhIntegral[4*ArcCosh[a*x]] + SinhIntegral[6*ArcCosh[a*x]])/(32*a^6)`**Maple [A]**

time = 2.28, size = 33, normalized size = 0.77

method	result	size
derivativedivides	$\frac{5 \text{hyperbolicSineIntegral}(2 \text{arccosh}(ax))}{32} + \frac{\text{hyperbolicSineIntegral}(4 \text{arccosh}(ax))}{8} + \frac{\text{hyperbolicSineIntegral}(6 \text{arccosh}(ax))}{32}$	33
default	$\frac{5 \text{hyperbolicSineIntegral}(2 \text{arccosh}(ax))}{32} + \frac{\text{hyperbolicSineIntegral}(4 \text{arccosh}(ax))}{8} + \frac{\text{hyperbolicSineIntegral}(6 \text{arccosh}(ax))}{32}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^5/arccosh(a*x), x, method=_RETURNVERBOSE)``[Out] 1/a^6*(5/32*Shi(2*arccosh(a*x))+1/8*Shi(4*arccosh(a*x))+1/32*Shi(6*arccosh(a*x)))`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/arccosh(a*x), x)
```

Fricas [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/arccosh(a*x), x)
```

Sympy [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^5}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/acosh(a*x),x)
```

```
[Out] Integral(x**5/acosh(a*x), x)
```

Giac [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arccosh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

```
time = 0.00, size = -1, normalized size = -0.02
```

$$\int \frac{x^5}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/acosh(a*x),x)
```

```
[Out] int(x^5/acosh(a*x), x)
```

$$3.44 \quad \int \frac{x^4}{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=41

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{8a^5} + \frac{3\text{Shi}(3 \cosh^{-1}(ax))}{16a^5} + \frac{\text{Shi}(5 \cosh^{-1}(ax))}{16a^5}$$

[Out] 1/8*Shi(arccosh(a*x))/a^5+3/16*Shi(3*arccosh(a*x))/a^5+1/16*Shi(5*arccosh(a*x))/a^5

Rubi [A]

time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5887, 5556, 3379}

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{8a^5} + \frac{3\text{Shi}(3 \cosh^{-1}(ax))}{16a^5} + \frac{\text{Shi}(5 \cosh^{-1}(ax))}{16a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCosh[a*x], x]

[Out] SinhIntegral[ArcCosh[a*x]]/(8*a^5) + (3*SinhIntegral[3*ArcCosh[a*x]])/(16*a^5) + SinhIntegral[5*ArcCosh[a*x]]/(16*a^5)

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^(n)*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^4(x) \sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x} + \frac{3 \sinh(3x)}{16x} + \frac{\sinh(5x)}{16x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{x} dx, x, \cosh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{3 \text{Subst}\left(\int \frac{\sinh(3x)}{x} dx, x, \cosh^{-1}(ax)\right)}{16a^5} \\
&= \frac{\text{Shi}(\cosh^{-1}(ax))}{8a^5} + \frac{3 \text{Shi}(3 \cosh^{-1}(ax))}{16a^5} + \frac{\text{Shi}(5 \cosh^{-1}(ax))}{16a^5}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 31, normalized size = 0.76

$$\frac{2 \text{Shi}(\cosh^{-1}(ax)) + 3 \text{Shi}(3 \cosh^{-1}(ax)) + \text{Shi}(5 \cosh^{-1}(ax))}{16a^5}$$

Antiderivative was successfully verified.

`[In] Integrate[x^4/ArcCosh[a*x], x]``[Out] (2*SinhIntegral[ArcCosh[a*x]] + 3*SinhIntegral[3*ArcCosh[a*x]] + SinhIntegral[5*ArcCosh[a*x]])/(16*a^5)`**Maple [A]**

time = 2.48, size = 31, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{8} + \frac{3 \text{hyperbolicSineIntegral}(3 \text{arccosh}(ax))}{16a^5} + \frac{\text{hyperbolicSineIntegral}(5 \text{arccosh}(ax))}{16}$	31
default	$\frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{8} + \frac{3 \text{hyperbolicSineIntegral}(3 \text{arccosh}(ax))}{16a^5} + \frac{\text{hyperbolicSineIntegral}(5 \text{arccosh}(ax))}{16}$	31

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^4/arccosh(a*x), x, method=_RETURNVERBOSE)``[Out] 1/a^5*(1/8*Shi(arccosh(a*x))+3/16*Shi(3*arccosh(a*x))+1/16*Shi(5*arccosh(a*x)))`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arccosh(a*x),x, algorithm="maxima")`

[Out] `integrate(x^4/arccosh(a*x), x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arccosh(a*x),x, algorithm="fricas")`

[Out] `integral(x^4/arccosh(a*x), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/acosh(a*x),x)`

[Out] `Integral(x**4/acosh(a*x), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arccosh(a*x),x, algorithm="giac")`

[Out] `integrate(x^4/arccosh(a*x), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/acosh(a*x),x)`

[Out] `int(x^4/acosh(a*x), x)`

$$3.45 \quad \int \frac{x^3}{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=29

$$\frac{\text{Shi}(2 \cosh^{-1}(ax))}{4a^4} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{8a^4}$$

[Out] 1/4*Shi(2*arccosh(a*x))/a^4+1/8*Shi(4*arccosh(a*x))/a^4

Rubi [A]

time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5887, 5556, 3379}

$$\frac{\text{Shi}(2 \cosh^{-1}(ax))}{4a^4} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{8a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCosh[a*x], x]

[Out] SinhIntegral[2*ArcCosh[a*x]]/(4*a^4) + SinhIntegral[4*ArcCosh[a*x]]/(8*a^4)

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^3(x) \sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} dx, x, \cosh^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x} dx, x, \cosh^{-1}(ax)\right)}{4a^4} \\
&= \frac{\text{Shi}(2 \cosh^{-1}(ax))}{4a^4} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{8a^4}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 24, normalized size = 0.83

$$\frac{2\text{Shi}(2 \cosh^{-1}(ax)) + \text{Shi}(4 \cosh^{-1}(ax))}{8a^4}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3/ArcCosh[a*x], x]``[Out] (2*SinhIntegral[2*ArcCosh[a*x]] + SinhIntegral[4*ArcCosh[a*x]])/(8*a^4)`**Maple [A]**

time = 2.12, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\text{hyperbolicSineIntegral}(2 \operatorname{arccosh}(ax))}{4} + \frac{\text{hyperbolicSineIntegral}(4 \operatorname{arccosh}(ax))}{8}$ a^4	24
default	$\frac{\text{hyperbolicSineIntegral}(2 \operatorname{arccosh}(ax))}{4} + \frac{\text{hyperbolicSineIntegral}(4 \operatorname{arccosh}(ax))}{8}$ a^4	24

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3/arccosh(a*x), x, method=_RETURNVERBOSE)``[Out] 1/a^4*(1/4*Shi(2*arccosh(a*x))+1/8*Shi(4*arccosh(a*x)))`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^3/arccosh(a*x), x, algorithm="maxima")`

[Out] integrate(x^3/arccosh(a*x), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x),x, algorithm="fricas")

[Out] integral(x^3/arccosh(a*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/acosh(a*x),x)

[Out] Integral(x**3/acosh(a*x), x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acosh(a*x),x)

[Out] int(x^3/acosh(a*x), x)

3.46 $\int \frac{x^2}{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{4a^3} + \frac{\text{Shi}(3 \cosh^{-1}(ax))}{4a^3}$$

[Out] 1/4*Shi(arccosh(a*x))/a^3+1/4*Shi(3*arccosh(a*x))/a^3

Rubi [A]

time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5887, 5556, 3379}

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{4a^3} + \frac{\text{Shi}(3 \cosh^{-1}(ax))}{4a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcCosh[a*x],x]

[Out] SinhIntegral[ArcCosh[a*x]]/(4*a^3) + SinhIntegral[3*ArcCosh[a*x]]/(4*a^3)

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^m, x_Symbol] :> Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x} + \frac{\sinh(3x)}{4x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x} dx, x, \cosh^{-1}(ax)\right)}{4a^3} \\
&= \frac{\text{Shi}(\cosh^{-1}(ax))}{4a^3} + \frac{\text{Shi}(3 \cosh^{-1}(ax))}{4a^3}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 20, normalized size = 0.74

$$\frac{\text{Shi}(\cosh^{-1}(ax)) + \text{Shi}(3 \cosh^{-1}(ax))}{4a^3}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/ArcCosh[a*x], x]``[Out] (SinhIntegral[ArcCosh[a*x]] + SinhIntegral[3*ArcCosh[a*x]])/(4*a^3)`**Maple [A]**

time = 1.70, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{4} + \frac{\text{hyperbolicSineIntegral}(3 \text{arccosh}(ax))}{4a^3}$	22
default	$\frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{4} + \frac{\text{hyperbolicSineIntegral}(3 \text{arccosh}(ax))}{4a^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/arccosh(a*x), x, method=_RETURNVERBOSE)``[Out] 1/a^3*(1/4*Shi(arccosh(a*x))+1/4*Shi(3*arccosh(a*x)))`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/arccosh(a*x), x, algorithm="maxima")`

[Out] integrate(x^2/arccosh(a*x), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x),x, algorithm="fricas")

[Out] integral(x^2/arccosh(a*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/acosh(a*x),x)

[Out] Integral(x**2/acosh(a*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x),x, algorithm="giac")

[Out] integrate(x^2/arccosh(a*x), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/acosh(a*x),x)

[Out] int(x^2/acosh(a*x), x)

$$3.47 \quad \int \frac{x}{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=14

$$\frac{\text{Shi}(2 \cosh^{-1}(ax))}{2a^2}$$

[Out] 1/2*Shi(2*arccosh(a*x))/a^2

Rubi [A]

time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5887, 5556, 12, 3379}

$$\frac{\text{Shi}(2 \cosh^{-1}(ax))}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCosh[a*x],x]

[Out] SinhIntegral[2*ArcCosh[a*x]]/(2*a^2)

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x} dx, x, \cosh^{-1}(ax)\right)}{2a^2} \\
&= \frac{\text{Shi}(2 \cosh^{-1}(ax))}{2a^2}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 14, normalized size = 1.00

$$\frac{\text{Shi}(2 \cosh^{-1}(ax))}{2a^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x/ArcCosh[a*x], x]``[Out] SinhIntegral[2*ArcCosh[a*x]]/(2*a^2)`**Maple [A]**

time = 2.26, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\text{hyperbolicSineIntegral}(2 \operatorname{arccosh}(ax))}{2a^2}$	13
default	$\frac{\text{hyperbolicSineIntegral}(2 \operatorname{arccosh}(ax))}{2a^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/arccosh(a*x), x, method=_RETURNVERBOSE)``[Out] 1/2*Shi(2*arccosh(a*x))/a^2`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/arccosh(a*x), x, algorithm="maxima")`

[Out] integrate(x/arccosh(a*x), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x),x, algorithm="fricas")

[Out] integral(x/arccosh(a*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(a*x),x)

[Out] Integral(x/acosh(a*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x),x, algorithm="giac")

[Out] integrate(x/arccosh(a*x), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x),x)

[Out] int(x/acosh(a*x), x)

$$3.48 \quad \int \frac{1}{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=9

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{a}$$

[Out] Shi(arccosh(a*x))/a

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5881, 3379}

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^(-1),x]

[Out] SinhIntegral[ArcCosh[a*x]]/a

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_, x_Symbol] :> Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\cosh^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a} \\ &= \frac{\text{Shi}(\cosh^{-1}(ax))}{a} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 9, normalized size = 1.00

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^(-1),x]

[Out] SinhIntegral[ArcCosh[a*x]]/a

Maple [A]

time = 1.66, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{a}$	10
default	$\frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{a}$	10

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccosh(a*x),x,method=_RETURNVERBOSE)

[Out] Shi(arccosh(a*x))/a

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x),x, algorithm="maxima")

[Out] integrate(1/arccosh(a*x), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x),x, algorithm="fricas")

[Out] integral(1/arccosh(a*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acosh(a*x),x)

[Out] Integral(1/acosh(a*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x),x, algorithm="giac")

[Out] integrate(1/arccosh(a*x), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/acosh(a*x),x)

[Out] int(1/acosh(a*x), x)

$$3.49 \quad \int \frac{1}{x \cosh^{-1}(ax)} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]), x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)} dx = \int \frac{1}{x \cosh^{-1}(ax)} dx$$

Mathematica [A]

time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]), x]

[Out] Integrate[1/(x*ArcCosh[a*x]), x]

Maple [A]

time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/arccosh(a*x),x)`

[Out] `int(1/x/arccosh(a*x),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x),x, algorithm="maxima")`

[Out] `integrate(1/(x*arccosh(a*x)), x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x),x, algorithm="fricas")`

[Out] `integral(1/(x*arccosh(a*x)), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/acosh(a*x),x)`

[Out] `Integral(1/(x*acosh(a*x)), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x),x, algorithm="giac")`

[Out] `integrate(1/(x*arccosh(a*x)), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*acosh(a*x)),x)
```

```
[Out] int(1/(x*acosh(a*x)), x)
```

$$3.50 \quad \int \frac{1}{x^2 \cosh^{-1}(ax)} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \cosh^{-1}(ax)}, x\right)$$

[Out] Unintegrable(1/x^2/arccosh(a*x), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*ArcCosh[a*x]), x]

[Out] Defer[Int][1/(x^2*ArcCosh[a*x]), x]

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)} dx = \int \frac{1}{x^2 \cosh^{-1}(ax)} dx$$

Mathematica [A]

time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*ArcCosh[a*x]), x]

[Out] Integrate[1/(x^2*ArcCosh[a*x]), x]

Maple [A]

time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \text{arccosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/arccosh(a*x),x)`

[Out] `int(1/x^2/arccosh(a*x),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccosh(a*x),x, algorithm="maxima")`

[Out] `integrate(1/(x^2*arccosh(a*x)), x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccosh(a*x),x, algorithm="fricas")`

[Out] `integral(1/(x^2*arccosh(a*x)), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/acosh(a*x),x)`

[Out] `Integral(1/(x**2*acosh(a*x)), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccosh(a*x),x, algorithm="giac")`

[Out] `integrate(1/(x^2*arccosh(a*x)), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acosh(a*x)),x)
```

```
[Out] int(1/(x^2*acosh(a*x)), x)
```

3.51 $\int \frac{x^4}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=73

$$-\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{8a^5} + \frac{9\text{Chi}(3\cosh^{-1}(ax))}{16a^5} + \frac{5\text{Chi}(5\cosh^{-1}(ax))}{16a^5}$$

[Out] 1/8*Chi(arccosh(a*x))/a^5+9/16*Chi(3*arccosh(a*x))/a^5+5/16*Chi(5*arccosh(a*x))/a^5-x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5885, 3382}

$$\frac{\text{Chi}(\cosh^{-1}(ax))}{8a^5} + \frac{9\text{Chi}(3\cosh^{-1}(ax))}{16a^5} + \frac{5\text{Chi}(5\cosh^{-1}(ax))}{16a^5} - \frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCosh[a*x]^2,x]

[Out] -((x^4*sqrt[-1 + a*x]*sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCosh[a*x]]/(8*a^5) + (9*CoshIntegral[3*ArcCosh[a*x]])/(16*a^5) + (5*CoshIntegral[5*ArcCosh[a*x]])/(16*a^5)

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x^m*sqrt[1 + c*x]*sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^2} dx = -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8x} - \frac{9 \cosh(3x)}{16x} - \frac{5 \cosh(5x)}{16x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{5 \text{Subst}\left(\int \frac{\cosh(5x)}{x} dx, x, \cosh^{-1}(ax)\right)}{16a^5}$$

$$= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{8a^5} + \frac{9 \text{Chi}(3 \cosh^{-1}(ax))}{16a^5} + \frac{5 \text{Chi}(5 \cosh^{-1}(ax))}{16a^5}$$

Mathematica [A]

time = 0.16, size = 101, normalized size = 1.38

$$\frac{-16a^4 x^4 \sqrt{\frac{-1+ax}{1+ax}} - 16a^5 x^5 \sqrt{\frac{-1+ax}{1+ax}} + 2 \cosh^{-1}(ax) \text{Chi}(\cosh^{-1}(ax)) + 9 \cosh^{-1}(ax) \text{Chi}(3 \cosh^{-1}(ax)) + 5 \cosh^{-1}(ax) \text{Chi}(5 \cosh^{-1}(ax))}{16a^5 \cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/ArcCosh[a*x]^2,x]

[Out] $(-16a^4 x^4 \sqrt{(-1+ax)/(1+ax)} - 16a^5 x^5 \sqrt{(-1+ax)/(1+ax)}) + 2 \text{ArcCosh}[a*x] \text{CoshIntegral}[\text{ArcCosh}[a*x]] + 9 \text{ArcCosh}[a*x] \text{CoshIntegral}[3 \text{ArcCosh}[a*x]] + 5 \text{ArcCosh}[a*x] \text{CoshIntegral}[5 \text{ArcCosh}[a*x]] / (16a^5 \text{ArcCosh}[a*x])$

Maple [A]

time = 2.39, size = 83, normalized size = 1.14

method	result
derivativedivides	$-\frac{\sqrt{ax-1} \sqrt{ax+1}}{8 \text{arccosh}(ax)} + \frac{\text{hyperbolicCosineIntegral}(\text{arccosh}(ax))}{8} - \frac{3 \sinh(3 \text{arccosh}(ax))}{16 \text{arccosh}(ax)} + \frac{9 \text{hyperbolicCosineIntegral}(3 \text{arccosh}(ax))}{16 a^5}$
default	$-\frac{\sqrt{ax-1} \sqrt{ax+1}}{8 \text{arccosh}(ax)} + \frac{\text{hyperbolicCosineIntegral}(\text{arccosh}(ax))}{8} - \frac{3 \sinh(3 \text{arccosh}(ax))}{16 \text{arccosh}(ax)} + \frac{9 \text{hyperbolicCosineIntegral}(3 \text{arccosh}(ax))}{16 a^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccosh(a*x)^2,x,method=_RETURNVERBOSE)

[Out] $1/a^5 * (-1/8 / \text{arccosh}(a*x) * (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)} + 1/8 * \text{Chi}(\text{arccosh}(a*x)) - 3/16 / \text{arccosh}(a*x) * \sinh(3 * \text{arccosh}(a*x)) + 9/16 * \text{Chi}(3 * \text{arccosh}(a*x)) - 1/16 / \text{arccosh}(a*x) * \sinh(5 * \text{arccosh}(a*x)) + 5/16 * \text{Chi}(5 * \text{arccosh}(a*x)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^2,x, algorithm="maxima")
[Out] -(a^3*x^7 - a*x^5 + (a^2*x^6 - x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2
+ sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
1))) + integrate((5*a^5*x^8 - 10*a^3*x^6 + 5*a*x^4 + (5*a^3*x^6 - 3*a*x^4)
*(a*x + 1)*(a*x - 1) + (10*a^4*x^7 - 13*a^2*x^5 + 4*x^3)*sqrt(a*x + 1)*sqrt
(a*x - 1))/((a^5*x^4 + (a*x + 1)*(a*x - 1)*a^3*x^2 - 2*a^3*x^2 + 2*(a^4*x^3
- a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x
- 1))), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^2,x, algorithm="fricas")
[Out] integral(x^4/arccosh(a*x)^2, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)**2,x)
[Out] Integral(x**4/acosh(a*x)**2, x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^4/arccosh(a*x)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/acosh(a*x)^2,x)
[Out] int(x^4/acosh(a*x)^2, x)
```

$$3.52 \quad \int \frac{x^3}{\cosh^{-1}(ax)^2} dx$$

Optimal. Leaf size=61

$$-\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}(2\cosh^{-1}(ax))}{2a^4} + \frac{\text{Chi}(4\cosh^{-1}(ax))}{2a^4}$$

[Out] 1/2*Chi(2*arccosh(a*x))/a^4+1/2*Chi(4*arccosh(a*x))/a^4-x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)

Rubi [A]

time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5885, 3382}

$$\frac{\text{Chi}(2\cosh^{-1}(ax))}{2a^4} + \frac{\text{Chi}(4\cosh^{-1}(ax))}{2a^4} - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCosh[a*x]^2,x]

[Out] -((x^3*Sqrt[-1+a*x]*Sqrt[1+a*x])/(a*ArcCosh[a*x])) + CoshIntegral[2*ArcCosh[a*x]]/(2*a^4) + CoshIntegral[4*ArcCosh[a*x]]/(2*a^4)

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n*(x_)^m, x_Symbol] :> Simp[x^m*Sqrt[1+c*x]*Sqrt[-1+c*x]*((a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1))), x] + Dist[1/(b^2*c^(m+1)*(n+1)), Subst[Int[ExpandTrigReduce[x^(n+1), Cosh[-a/b+x/b]^(m-1)*(m-(m+1)*Cosh[-a/b+x/b]^2), x], x], x, a+b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\cosh^{-1}(ax)^2} dx &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2x} - \frac{\cosh(4x)}{2x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
&= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} dx, x, \cosh^{-1}(ax)\right)}{2a^4} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x} dx, x, \cosh^{-1}(ax)\right)}{2a^4} \\
&= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Chi}(2 \cosh^{-1}(ax))}{2a^4} + \frac{\text{Chi}(4 \cosh^{-1}(ax))}{2a^4}
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 58, normalized size = 0.95

$$-\frac{2a^3 x^3 \sqrt{\frac{-1+ax}{1+ax}} (1+ax)}{\cosh^{-1}(ax)} + \frac{\text{Chi}(2 \cosh^{-1}(ax)) + \text{Chi}(4 \cosh^{-1}(ax))}{2a^4}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[x^3/ArcCosh[a*x]^2,x]`

```
[Out] ((-2*a^3*x^3*sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))/ArcCosh[a*x] + CoshIntegral[2*ArcCosh[a*x]] + CoshIntegral[4*ArcCosh[a*x]])/(2*a^4)
```

Maple [A]

time = 2.08, size = 54, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{\sinh(2 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(2 \operatorname{arccosh}(ax))}{2} - \frac{\sinh(4 \operatorname{arccosh}(ax))}{8 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(4 \operatorname{arccosh}(ax))}{2}}{a^4}$
default	$\frac{-\frac{\sinh(2 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(2 \operatorname{arccosh}(ax))}{2} - \frac{\sinh(4 \operatorname{arccosh}(ax))}{8 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(4 \operatorname{arccosh}(ax))}{2}}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3/arccosh(a*x)^2,x,method=_RETURNVERBOSE)`

```
[Out] 1/a^4*(-1/4/arccosh(a*x)*sinh(2*arccosh(a*x))+1/2*Chi(2*arccosh(a*x))-1/8/a*rccosh(a*x)*sinh(4*arccosh(a*x))+1/2*Chi(4*arccosh(a*x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^2,x, algorithm="maxima")

[Out] $-(a^3x^6 - ax^4 + (a^2x^5 - x^3)\sqrt{ax + 1}\sqrt{ax - 1})/((a^3x^2 + \sqrt{ax + 1}\sqrt{ax - 1}a^2x - a)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})) + \int (4a^5x^7 - 8a^3x^5 + 4ax^3 + 2(2a^3x^5 - ax^3)(ax + 1)(ax - 1) + (8a^4x^6 - 10a^2x^4 + 3x^2)\sqrt{ax + 1}\sqrt{ax - 1})/((a^5x^4 + (ax + 1)(ax - 1)a^3x^2 - 2a^3x^2 + 2(a^4x^3 - a^2x)\sqrt{ax + 1}\sqrt{ax - 1} + a)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})) dx$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^2,x, algorithm="fricas")

[Out] integral(x^3/arccosh(a*x)^2, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/acosh(a*x)**2,x)

[Out] Integral(x**3/acosh(a*x)**2, x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/acosh(a*x)^2,x)
```

```
[Out] int(x^3/acosh(a*x)^2, x)
```

3.53 $\int \frac{x^2}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=59

$$-\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{4a^3} + \frac{3\text{Chi}(3 \cosh^{-1}(ax))}{4a^3}$$

[Out] $1/4*\text{Chi}(\text{arccosh}(a*x))/a^3+3/4*\text{Chi}(3*\text{arccosh}(a*x))/a^3-x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\text{arccosh}(a*x)$

Rubi [A]

time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5885, 3382}

$$\frac{\text{Chi}(\cosh^{-1}(ax))}{4a^3} + \frac{3\text{Chi}(3 \cosh^{-1}(ax))}{4a^3} - \frac{x^2 \sqrt{ax-1} \sqrt{ax+1}}{a \cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/\text{ArcCosh}[a*x]^2, x]$

[Out] $-((x^2*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(a*\text{ArcCosh}[a*x])) + \text{CoshIntegral}[\text{ArcCosh}[a*x]]/(4*a^3) + (3*\text{CoshIntegral}[3*\text{ArcCosh}[a*x]])/(4*a^3)$

Rule 3382

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[c*f*(fz/d) + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

Rule 5885

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^m*\text{Sqrt}[1+c*x]*\text{Sqrt}[-1+c*x]*((a+b*\text{ArcCosh}[c*x])^{(n+1)})/(b*c*(n+1)), x] + \text{Dist}[1/(b^2*c^{(m+1)}*(n+1)), \text{Subst}[\text{Int}[\text{ExpandTrigReduce}[x^{(n+1)}, \text{Cosh}[-a/b+x/b]^{(m-1)}*(m-(m+1)*\text{Cosh}[-a/b+x/b]^2), x], x], x, a+b*\text{ArcCosh}[c*x]], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{GeQ}[n, -2] \&\& \text{LtQ}[n, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\cosh^{-1}(ax)^2} dx &= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} - \frac{3 \cosh(3x)}{4x}\right) dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{3 \text{Subst}\left(\int \frac{\cosh(3x)}{x} dx, x, \cosh^{-1}(ax)\right)}{4a^3} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{4a^3} + \frac{3 \text{Chi}(3 \cosh^{-1}(ax))}{4a^3}
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 58, normalized size = 0.98

$$-\frac{4a^2 x^2 \sqrt{\frac{-1+ax}{1+ax}}}{\cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax)) + 3 \text{Chi}(3 \cosh^{-1}(ax))}{4a^3}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[x^2/ArcCosh[a*x]^2,x]`

```
[Out] ((-4*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))/ArcCosh[a*x] + CoshIntegral[ArcCosh[a*x]] + 3*CoshIntegral[3*ArcCosh[a*x]])/(4*a^3)
```

Maple [A]

time = 2.49, size = 59, normalized size = 1.00

method	result
derivativedivides	$-\frac{\sqrt{ax-1} \sqrt{ax+1}}{4 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(\operatorname{arccosh}(ax))}{4} - \frac{\sinh(3 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \frac{3 \operatorname{hyperbolicCosineIntegral}(3 \operatorname{arccosh}(ax))}{4}$
default	$-\frac{\sqrt{ax-1} \sqrt{ax+1}}{4 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(\operatorname{arccosh}(ax))}{4} - \frac{\sinh(3 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \frac{3 \operatorname{hyperbolicCosineIntegral}(3 \operatorname{arccosh}(ax))}{4}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/arccosh(a*x)^2,x,method=_RETURNVERBOSE)`

```
[Out] 1/a^3*(-1/4/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+1/4*Chi(arccosh(a*x))-1/4/arccosh(a*x)*sinh(3*arccosh(a*x))+3/4*Chi(3*arccosh(a*x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^2,x, algorithm="maxima")

[Out] $-(a^3x^5 - ax^3 + (a^2x^4 - x^2)\sqrt{ax + 1}\sqrt{ax - 1})/((a^3x^2 + \sqrt{ax + 1}\sqrt{ax - 1})a^2x - a)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1}) + \int (3a^5x^6 - 6a^3x^4 + (3a^3x^4 - ax^2)(ax + 1)(ax - 1) + 3ax^2 + (6a^4x^5 - 7a^2x^3 + 2x)\sqrt{ax + 1}\sqrt{ax - 1})/((a^5x^4 + (ax + 1)(ax - 1)a^3x^2 - 2a^3x^2 + 2(a^4x^3 - a^2x)\sqrt{ax + 1}\sqrt{ax - 1} + a)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})) dx$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^2,x, algorithm="fricas")

[Out] integral(x^2/arccosh(a*x)^2, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/acosh(a*x)**2,x)

[Out] Integral(x**2/acosh(a*x)**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^2,x, algorithm="giac")

[Out] integrate(x^2/arccosh(a*x)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/acosh(a*x)^2,x)

[Out] int(x^2/acosh(a*x)^2, x)

3.54 $\int \frac{x}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=42

$$-\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}(2\cosh^{-1}(ax))}{a^2}$$

[Out] Chi(2*arccosh(a*x))/a^2-x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5885, 3382}

$$\frac{\text{Chi}(2\cosh^{-1}(ax))}{a^2} - \frac{x\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCosh[a*x]^2,x]

[Out] -((x*sqrt[-1 + a*x]*sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[2*ArcCosh[a*x]]/a^2

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^(n_)*(x_)^(m_.), x_Symbol] := Simp[x^m*sqrt[1 + c*x]*sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1))*Cosh[-a/b + x/b]^2], x], x], x, a + b*ArcCosh[c*x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x}{\cosh^{-1}(ax)^2} dx &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\ &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}(2\cosh^{-1}(ax))}{a^2} \end{aligned}$$

Mathematica [A]

time = 0.18, size = 44, normalized size = 1.05

$$\frac{-\frac{ax\sqrt{\frac{-1+ax}{1+ax}}(1+ax)}{\cosh^{-1}(ax)} + \text{Chi}(2\cosh^{-1}(ax))}{a^2}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[x/ArcCosh[a*x]^2,x]``[Out] (-((a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))/ArcCosh[a*x]) + CoshIntegral[2*ArcCosh[a*x]])/a^2`**Maple [A]**

time = 2.45, size = 28, normalized size = 0.67

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\operatorname{arccosh}(ax))}{2\operatorname{arccosh}(ax)} + \operatorname{hyperbolicCosineIntegral}(2\operatorname{arccosh}(ax))}{a^2}$	28
default	$\frac{-\frac{\sinh(2\operatorname{arccosh}(ax))}{2\operatorname{arccosh}(ax)} + \operatorname{hyperbolicCosineIntegral}(2\operatorname{arccosh}(ax))}{a^2}$	28

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/arccosh(a*x)^2,x,method=_RETURNVERBOSE)``[Out] 1/a^2*(-1/2/arccosh(a*x)*sinh(2*arccosh(a*x))+Chi(2*arccosh(a*x)))`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/arccosh(a*x)^2,x, algorithm="maxima")`

```
[Out] -(a^3*x^4 - a*x^2 + (a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2 +
sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
))) + integrate((2*a^5*x^5 + 2*(a*x + 1)*(a*x - 1)*a^3*x^3 - 4*a^3*x^3 + (4
*a^4*x^4 - 4*a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a*x - 1) + 2*a*x)/((a^5*x^4 +
(a*x + 1)*(a*x - 1)*a^3*x^2 - 2*a^3*x^2 + 2*(a^4*x^3 - a^2*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^2,x, algorithm="fricas")

[Out] integral(x/arccosh(a*x)^2, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(a*x)**2,x)

[Out] Integral(x/acosh(a*x)**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^2,x, algorithm="giac")

[Out] integrate(x/arccosh(a*x)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x)^2,x)

[Out] int(x/acosh(a*x)^2, x)

$$3.55 \quad \int \frac{1}{\cosh^{-1}(ax)^2} dx$$

Optimal. Leaf size=39

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{a\cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{a}$$

[Out] Chi(arccosh(a*x))/a-(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)

Rubi [A]

time = 0.12, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5880, 5953, 3382}

$$\frac{\text{Chi}(\cosh^{-1}(ax))}{a} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^(-2), x]

[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x])) + CoshIntegral[ArcCosh[a*x]]/a

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 5880

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_), x_Symbol] :> Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^2} dx &= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + a \int \frac{x}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)} dx \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a} \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{a \cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{a}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 60, normalized size = 1.54

$$\frac{1 - ax + \sqrt{\frac{-1 + ax}{1 + ax}} \cosh^{-1}(ax) \text{Chi}(\cosh^{-1}(ax))}{a \sqrt{\frac{-1 + ax}{1 + ax}} \cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[ArcCosh[a*x]^(-2), x]`

```
[Out] (1 - a*x + Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]])/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x])
```

Maple [A]

time = 1.82, size = 33, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{ax-1} \sqrt{ax+1}}{\text{arccosh}(ax)} + \frac{\text{hyperbolicCosineIntegral}(\text{arccosh}(ax))}{a}$	33
default	$-\frac{\sqrt{ax-1} \sqrt{ax+1}}{\text{arccosh}(ax)} + \frac{\text{hyperbolicCosineIntegral}(\text{arccosh}(ax))}{a}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/arccosh(a*x)^2, x, method=_RETURNVERBOSE)`

```
[Out] 1/a*(-1/arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)+Chi(arccosh(a*x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x)^2,x, algorithm="maxima")

[Out] $-(a^3x^3 + (a^2x^2 - 1)\sqrt{ax + 1}\sqrt{ax - 1} - ax)/((a^3x^2 + \sqrt{ax + 1}\sqrt{ax - 1})a^2x - a)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1}) + \int (a^4x^4 - 2a^2x^2 + (a^2x^2 + 1)(ax + 1)(ax - 1) + (2a^3x^3 - ax)\sqrt{ax + 1}\sqrt{ax - 1} + 1)/((a^4x^4 + (ax + 1)(ax - 1)a^2x^2 - 2a^2x^2 + 2(a^3x^3 - ax)\sqrt{ax + 1}\sqrt{ax - 1} + 1)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})) dx$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x)^2,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^(-2), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acosh(a*x)**2,x)

[Out] Integral(acosh(a*x)**(-2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x)^2,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^(-2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/acosh(a*x)^2,x)

[Out] int(1/acosh(a*x)^2, x)

$$3.56 \quad \int \frac{1}{x \cosh^{-1}(ax)^2} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)^2}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^2,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^2),x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^2), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^2} dx = \int \frac{1}{x \cosh^{-1}(ax)^2} dx$$

Mathematica [A]

time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^2),x]

[Out] Integrate[1/(x*ArcCosh[a*x]^2), x]

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccosh(a*x)^2,x)

[Out] int(1/x/arccosh(a*x)^2,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^2,x, algorithm="maxima")

[Out] $-(a^3x^3 + (a^2x^2 - 1)\sqrt{ax + 1}\sqrt{ax - 1} - ax)/((a^3x^3 + \sqrt{ax + 1}\sqrt{ax - 1}a^2x^2 - ax)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})) + \int \frac{(2(ax + 1)(ax - 1)ax + (2a^2x^2 - 1)\sqrt{ax + 1}\sqrt{ax - 1})}{(a^5x^6 + (ax + 1)(ax - 1)a^3x^4 - 2a^3x^4 + ax^2 + 2(a^4x^5 - a^2x^3)\sqrt{ax + 1}\sqrt{ax - 1})\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})} dx$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^2,x, algorithm="fricas")

[Out] integral(1/(x*arccosh(a*x)^2), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/acosh(a*x)**2,x)

[Out] Integral(1/(x*acosh(a*x)**2), x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^2,x, algorithm="giac")

[Out] integrate(1/(x*arccosh(a*x)^2), x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*acosh(a*x)^2),x)

[Out] int(1/(x*acosh(a*x)^2), x)

$$3.57 \quad \int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \cosh^{-1}(ax)^2}, x\right)$$

[Out] Unintegrable(1/x^2/arccosh(a*x)^2,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*ArcCosh[a*x]^2), x]

[Out] Defer[Int][1/(x^2*ArcCosh[a*x]^2), x]

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx = \int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Mathematica [A]

time = 5.88, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*ArcCosh[a*x]^2), x]

[Out] Integrate[1/(x^2*ArcCosh[a*x]^2), x]

Maple [A]

time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/arccosh(a*x)^2,x)`

[Out] `int(1/x^2/arccosh(a*x)^2,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccosh(a*x)^2,x, algorithm="maxima")`

[Out]
$$-(a^3x^3 + (a^2x^2 - 1)\sqrt{ax + 1}\sqrt{ax - 1} - ax)/((a^3x^4 + \sqrt{ax + 1}\sqrt{ax - 1})a^2x^3 - ax^2)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1}) - \int (a^5x^5 - 2a^3x^3 + (a^3x^3 - 3ax)(ax + 1)(ax - 1) + (2a^4x^4 - 5a^2x^2 + 2)\sqrt{ax + 1}\sqrt{ax - 1} + ax)/((a^5x^7 + (ax + 1)(ax - 1)a^3x^5 - 2a^3x^5 + ax^3 + 2(a^4x^6 - a^2x^4)\sqrt{ax + 1}\sqrt{ax - 1})\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})), x)$$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccosh(a*x)^2,x, algorithm="fricas")`

[Out] `integral(1/(x^2*arccosh(a*x)^2), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/acosh(a*x)**2,x)`

[Out] `Integral(1/(x**2*acosh(a*x)**2), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccosh(a*x)^2), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acosh(a*x)^2),x)
```

```
[Out] int(1/(x^2*acosh(a*x)^2), x)
```

$$3.58 \quad \int \frac{x^4}{\cosh^{-1}(ax)^3} dx$$

Optimal. Leaf size=102

$$-\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} + \frac{\text{Shi}(\cosh^{-1}(ax))}{16a^5} + \frac{27\text{Shi}(3 \cosh^{-1}(ax))}{32a^5} + \frac{25\text{Shi}(5 \cosh^{-1}(ax))}{32a^5}$$

[Out] $2x^3/a^2/\text{arccosh}(ax) - 5/2x^5/\text{arccosh}(ax) + 1/16*\text{Shi}(\text{arccosh}(ax))/a^5 + 27/32*\text{Shi}(3*\text{arccosh}(ax))/a^5 + 25/32*\text{Shi}(5*\text{arccosh}(ax))/a^5 - 1/2*x^4*(ax-1)^(1/2)*(ax+1)^(1/2)/a/\text{arccosh}(ax)^2$

Rubi [A]

time = 0.43, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5886, 5951, 5887, 5556, 3379}

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{16a^5} + \frac{27\text{Shi}(3 \cosh^{-1}(ax))}{32a^5} + \frac{25\text{Shi}(5 \cosh^{-1}(ax))}{32a^5} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} - \frac{x^4 \sqrt{ax-1} \sqrt{ax+1}}{2a \cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCosh[a*x]^3,x]

[Out] $-1/2*(x^4*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(a*\text{ArcCosh}[a*x]^2) + (2*x^3)/(a^2*\text{ArcCosh}[a*x]) - (5*x^5)/(2*\text{ArcCosh}[a*x]) + \text{SinhIntegral}[\text{ArcCosh}[a*x]]/(16*a^5) + (27*\text{SinhIntegral}[3*\text{ArcCosh}[a*x]])/(32*a^5) + (25*\text{SinhIntegral}[5*\text{ArcCosh}[a*x]])/(32*a^5)$

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I

```
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\cosh^{-1}(ax)^3} dx &= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} - \frac{2 \int \frac{x^3}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2} dx}{a} + \frac{1}{2} (5a) \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} + \frac{25}{2} \int \frac{x^4}{\cosh^{-1}(ax)} dx - \frac{6}{a^5} \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} - \frac{6 \operatorname{Subst}\left(\int \frac{\cosh^2(x) \sinh(x)}{x} dx, x, \frac{1}{\sqrt{-1+ax}}\right)}{a^5} \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} - \frac{6 \operatorname{Subst}\left(\int \left(\frac{\sinh(x)}{4x} + \frac{\sinh(x)}{4x}\right) dx, x, \frac{1}{\sqrt{-1+ax}}\right)}{a^5} \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} + \frac{25 \operatorname{Subst}\left(\int \frac{\sinh(5x)}{x} dx, x, \frac{1}{\sqrt{-1+ax}}\right)}{32a^5} \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{2x^3}{a^2 \cosh^{-1}(ax)} - \frac{5x^5}{2 \cosh^{-1}(ax)} + \frac{\operatorname{Shi}(\cosh^{-1}(ax))}{16a^5} + \frac{27 \operatorname{Shi}(3 \cosh^{-1}(ax))}{32a^5}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 107, normalized size = 1.05

$$\frac{-16a^4 x^4 \sqrt{-1+ax} \sqrt{1+ax} + 64a^3 x^3 \cosh^{-1}(ax) - 80a^5 x^5 \cosh^{-1}(ax) + 2 \cosh^{-1}(ax)^2 \operatorname{Shi}(\cosh^{-1}(ax)) + 27 \cosh^{-1}(ax)^2 \operatorname{Shi}(3 \cosh^{-1}(ax)) + 25 \cosh^{-1}(ax)^2 \operatorname{Shi}(5 \cosh^{-1}(ax))}{32a^5 \cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcCosh[a*x]^3,x]

[Out]
$$\frac{(-16a^4x^4\sqrt{-1+ax}\sqrt{1+ax} + 64a^3x^3\text{ArcCosh}[ax] - 80a^5x^5\text{ArcCosh}[ax] + 2\text{ArcCosh}[ax]^2\text{SinhIntegral}[\text{ArcCosh}[ax]] + 27\text{ArcCosh}[ax]^2\text{SinhIntegral}[3\text{ArcCosh}[ax]] + 25\text{ArcCosh}[ax]^2\text{SinhIntegral}[5\text{ArcCosh}[ax]])}{(32a^5\text{ArcCosh}[ax]^2)}$$

Maple [A]

time = 2.32, size = 123, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{16\text{arccosh}(ax)^2} - \frac{ax}{16\text{arccosh}(ax)} + \frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{16} - \frac{3\sinh(3\text{arccosh}(ax))}{32\text{arccosh}(ax)^2} - \frac{9\cosh(3\text{arccosh}(ax))}{32\text{arccosh}(ax)}}{a}$
default	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{16\text{arccosh}(ax)^2} - \frac{ax}{16\text{arccosh}(ax)} + \frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{16} - \frac{3\sinh(3\text{arccosh}(ax))}{32\text{arccosh}(ax)^2} - \frac{9\cosh(3\text{arccosh}(ax))}{32\text{arccosh}(ax)}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccosh(a*x)^3,x,method=_RETURNVERBOSE)

[Out]
$$\frac{1}{a^5} \left(-\frac{1}{16} \text{arccosh}(ax)^2 (ax-1)^{1/2} (ax+1)^{1/2} - \frac{1}{16} \text{arccosh}(ax) ax + \frac{1}{16} \text{Shi}(\text{arccosh}(ax)) - \frac{3}{32} \text{arccosh}(ax)^2 \sinh(3\text{arccosh}(ax)) - \frac{9}{32} \text{arccosh}(ax) \cosh(3\text{arccosh}(ax)) + \frac{27}{32} \text{Shi}(3\text{arccosh}(ax)) - \frac{1}{32} \text{arccosh}(ax)^2 \sinh(5\text{arccosh}(ax)) - \frac{5}{32} \text{arccosh}(ax) \cosh(5\text{arccosh}(ax)) + \frac{25}{32} \text{Shi}(5\text{arccosh}(ax)) \right)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^3,x, algorithm="maxima")

[Out]
$$\frac{-1/2(a^8x^{11} - 3a^6x^9 + 3a^4x^7 - a^2x^5 + (a^5x^8 - a^3x^6)(ax+1)^{3/2}(ax-1)^{3/2} + (3a^6x^9 - 5a^4x^7 + 2a^2x^5)(ax+1)(ax-1) + (3a^7x^{10} - 7a^5x^8 + 5a^3x^6 - ax^4)\sqrt{ax+1}\sqrt{ax-1} + (5a^8x^{11} - 15a^6x^9 + 15a^4x^7 - 5a^2x^5 + (5a^5x^8 - 8a^3x^6 + 3ax^4)(ax+1)^{3/2}(ax-1)^{3/2} + (15a^6x^9 - 31a^4x^7 + 20a^2x^5 - 4x^3)(ax+1)(ax-1) + (15a^7x^{10} - 38a^5x^8 + 32a^3x^6 - 9ax^4)\sqrt{ax+1}\sqrt{ax-1})\log(ax+\sqrt{ax+1})\sqrt{ax-1})}{(a^8x^6 + (ax+1)^{3/2}(ax-1)^{3/2}a^5x^3 - 3a^6x^4 + 3a^4x^2 + 3(a^6x^4 - a^4x^2)(ax+1)(ax-1) + 3(a^7x^5 - 2a^5x^3 + a^3x)\sqrt{ax+1}\sqrt{ax-1} - a^2)\log(ax+\sqrt{ax+1})\sqrt{ax-1}}$$

$a*x + 1)*\sqrt{a*x - 1))^2) + \text{integrate}(1/2*(25*a^{10}*x^{12} - 100*a^8*x^{10} + 150*a^6*x^8 - 100*a^4*x^6 + 25*a^2*x^4 + (25*a^6*x^8 - 24*a^4*x^6 + 3*a^2*x^4)*(a*x + 1)^2*(a*x - 1)^2 + (100*a^7*x^9 - 172*a^5*x^7 + 87*a^3*x^5 - 12*a*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 3*(50*a^8*x^{10} - 124*a^6*x^8 + 105*a^4*x^6 - 35*a^2*x^4 + 4*x^2)*(a*x + 1)*(a*x - 1) + (100*a^9*x^{11} - 324*a^7*x^9 + 381*a^5*x^7 - 193*a^3*x^5 + 36*a*x^3)*\sqrt{a*x + 1}*\sqrt{a*x - 1})/(a^{10}*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^6*x^4 - 4*a^8*x^6 + 6*a^6*x^4 - 4*a^4*x^2 + 4*(a^7*x^5 - a^5*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 6*(a^8*x^6 - 2*a^6*x^4 + a^4*x^2)*(a*x + 1)*(a*x - 1) + 4*(a^9*x^7 - 3*a^7*x^5 + 3*a^5*x^3 - a^3*x)*\sqrt{a*x + 1}*\sqrt{a*x - 1} + a^2)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})), x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arccosh(a*x)^3,x, algorithm="fricas")`

[Out] `integral(x^4/arccosh(a*x)^3, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/acosh(a*x)**3,x)`

[Out] `Integral(x**4/acosh(a*x)**3, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arccosh(a*x)^3,x, algorithm="giac")`

[Out] `integrate(x^4/arccosh(a*x)^3, x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/acosh(a*x)^3,x)
```

```
[Out] int(x^4/acosh(a*x)^3, x)
```

$$3.59 \quad \int \frac{x^3}{\cosh^{-1}(ax)^3} dx$$

Optimal. Leaf size=87

$$-\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + \frac{\operatorname{Shi}(2\cosh^{-1}(ax))}{2a^4} + \frac{\operatorname{Shi}(4\cosh^{-1}(ax))}{a^4}$$

[Out] $3/2*x^2/a^2/\operatorname{arccosh}(a*x) - 2*x^4/\operatorname{arccosh}(a*x) + 1/2*\operatorname{Shi}(2*\operatorname{arccosh}(a*x))/a^4 + \operatorname{Shi}(4*\operatorname{arccosh}(a*x))/a^4 - 1/2*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^2$

Rubi [A]

time = 0.39, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5886, 5951, 5887, 5556, 3379, 12}

$$\frac{\operatorname{Shi}(2\cosh^{-1}(ax))}{2a^4} + \frac{\operatorname{Shi}(4\cosh^{-1}(ax))}{a^4} + \frac{3x^2}{2a^2\cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCosh[a*x]^3, x]

[Out] $-1/2*(x^3*\sqrt{-1+a*x}*\sqrt{1+a*x})/(a*\operatorname{ArcCosh}[a*x]^2) + (3*x^2)/(2*a^2*\operatorname{ArcCosh}[a*x]) - (2*x^4)/\operatorname{ArcCosh}[a*x] + \operatorname{SinhIntegral}[2*\operatorname{ArcCosh}[a*x]]/(2*a^4) + \operatorname{SinhIntegral}[4*\operatorname{ArcCosh}[a*x]]/a^4$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^{(m_.)}, x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1))

)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/(Sqrt[(d1_.) + (e1_.)*(x_.)]*Sqrt[(d2_.) + (e2_.)*(x_.)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\cosh^{-1}(ax)^3} dx &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} - \frac{3 \int \frac{x^2}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2} dx}{2a} + (2a) \int \frac{1}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)} dx \\
 &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2 \cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + 8 \int \frac{x^3}{\cosh^{-1}(ax)} dx - \frac{3}{a^4} \int \frac{1}{\cosh^{-1}(ax)} dx \\
 &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2 \cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} - \frac{3 \text{Subst}\left(\int \frac{\cosh(x) \sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2 \cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} - \frac{3 \text{Subst}\left(\int \frac{\sinh(2x)}{2x} dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2 \cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{3x^2}{2a^2 \cosh^{-1}(ax)} - \frac{2x^4}{\cosh^{-1}(ax)} + \frac{\text{Shi}(2 \cosh^{-1}(ax))}{2a^4} + \frac{\text{Shi}(4 \cosh^{-1}(ax))}{a^4}
 \end{aligned}$$

Mathematica [A]

time = 0.13, size = 75, normalized size = 0.86

$$\frac{a^2 x^2 \left(ax \sqrt{-1 + ax} \sqrt{1 + ax} + (-3 + 4a^2 x^2) \cosh^{-1}(ax) \right)}{\cosh^{-1}(ax)^2} + \frac{\text{Shi}(2 \cosh^{-1}(ax)) + 2\text{Shi}(4 \cosh^{-1}(ax))}{2a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCosh[a*x]^3,x]

[Out] (-((a^2*x^2*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] + (-3 + 4*a^2*x^2)*ArcCosh[a*x]))/ArcCosh[a*x]^2) + SinhIntegral[2*ArcCosh[a*x]] + 2*SinhIntegral[4*ArcCosh[a*x]])/(2*a^4)

Maple [A]

time = 2.08, size = 82, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{\sinh(2 \operatorname{arccosh}(ax))}{8 \operatorname{arccosh}(ax)^2} - \frac{\cosh(2 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicSineIntegral}(2 \operatorname{arccosh}(ax))}{2} - \frac{\sinh(4 \operatorname{arccosh}(ax))}{16 \operatorname{arccosh}(ax)^2} - \frac{\cosh(4 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \operatorname{hyperbolicSineIntegral}(4 \operatorname{arccosh}(ax))}{a^4}$
default	$\frac{-\frac{\sinh(2 \operatorname{arccosh}(ax))}{8 \operatorname{arccosh}(ax)^2} - \frac{\cosh(2 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicSineIntegral}(2 \operatorname{arccosh}(ax))}{2} - \frac{\sinh(4 \operatorname{arccosh}(ax))}{16 \operatorname{arccosh}(ax)^2} - \frac{\cosh(4 \operatorname{arccosh}(ax))}{4 \operatorname{arccosh}(ax)} + \operatorname{hyperbolicSineIntegral}(4 \operatorname{arccosh}(ax))}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arccosh(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/a^4*(-1/8/arccosh(a*x)^2*sinh(2*arccosh(a*x))-1/4/arccosh(a*x)*cosh(2*arccosh(a*x))+1/2*Shi(2*arccosh(a*x))-1/16/arccosh(a*x)^2*sinh(4*arccosh(a*x))-1/4/arccosh(a*x)*cosh(4*arccosh(a*x))+Shi(4*arccosh(a*x)))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^3,x, algorithm="maxima")

[Out] -1/2*(a^8*x^10 - 3*a^6*x^8 + 3*a^4*x^6 - a^2*x^4 + (a^5*x^7 - a^3*x^5)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (3*a^6*x^8 - 5*a^4*x^6 + 2*a^2*x^4)*(a*x + 1)*(a*x - 1) + (3*a^7*x^9 - 7*a^5*x^7 + 5*a^3*x^5 - a*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1) + (4*a^8*x^10 - 12*a^6*x^8 + 12*a^4*x^6 - 4*a^2*x^4 + 2*(2*a^5*x^7 - 3*a^3*x^5 + a*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 3*(4*a^6*x^8 - 8*a^4*x^6 + 5*a^2*x^4 - x^2)*(a*x + 1)*(a*x - 1) + (12*a^7*x^9 - 30*a^5*x^7 + 25*a^3*x^5 - 7*a*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/((a^8*x^6 + (a*x + 1)^(3/2)*(a*x - 1)^(3/2)*a^5*x^3 - 3*a^6

```
*x^4 + 3*a^4*x^2 + 3*(a^6*x^4 - a^4*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^5 -
  2*a^5*x^3 + a^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a^2*log(a*x + sqrt(a*x +
  1)*sqrt(a*x - 1))^2) + integrate(1/2*(16*a^10*x^11 - 64*a^8*x^9 + 96*a^6*x
^7 - 64*a^4*x^5 + 4*(4*a^6*x^7 - 3*a^4*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 16*a^
2*x^3 + (64*a^7*x^8 - 100*a^5*x^6 + 42*a^3*x^4 - 3*a*x^2)*(a*x + 1)^(3/2)*(
a*x - 1)^(3/2) + 6*(16*a^8*x^9 - 38*a^6*x^7 + 30*a^4*x^5 - 9*a^2*x^3 + x)*(
a*x + 1)*(a*x - 1) + (64*a^9*x^10 - 204*a^7*x^8 + 234*a^5*x^6 - 115*a^3*x^4
+ 21*a*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^10*x^8 + (a*x + 1)^2*(a*x - 1
)^2*a^6*x^4 - 4*a^8*x^6 + 6*a^6*x^4 - 4*a^4*x^2 + 4*(a^7*x^5 - a^5*x^3)*(a*
x + 1)^(3/2)*(a*x - 1)^(3/2) + 6*(a^8*x^6 - 2*a^6*x^4 + a^4*x^2)*(a*x + 1)*
(a*x - 1) + 4*(a^9*x^7 - 3*a^7*x^5 + 3*a^5*x^3 - a^3*x)*sqrt(a*x + 1)*sqrt(
a*x - 1) + a^2)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccosh(a*x)^3, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**3,x)
```

```
[Out] Integral(x**3/acosh(a*x)**3, x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acosh(a*x)^3,x)

[Out] int(x^3/acosh(a*x)^3, x)

3.60 $\int \frac{x^2}{\cosh^{-1}(ax)^3} dx$

Optimal. Leaf size=85

$$-\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} + \frac{\text{Shi}(\cosh^{-1}(ax))}{8a^3} + \frac{9\text{Shi}(3\cosh^{-1}(ax))}{8a^3}$$

[Out] $x/a^2/\text{arccosh}(a*x) - 3/2*x^3/\text{arccosh}(a*x) + 1/8*\text{Shi}(\text{arccosh}(a*x))/a^3 + 9/8*\text{Shi}(3*\text{arccosh}(a*x))/a^3 - 1/2*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\text{arccosh}(a*x)^2$

Rubi [A]

time = 0.33, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5886, 5951, 5887, 5556, 3379, 5881}

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{8a^3} + \frac{9\text{Shi}(3\cosh^{-1}(ax))}{8a^3} + \frac{x}{a^2\cosh^{-1}(ax)} - \frac{3x^3}{2\cosh^{-1}(ax)} - \frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] `Int[x^2/ArcCosh[a*x]^3,x]`

[Out] $-1/2*(x^2*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(a*\text{ArcCosh}[a*x]^2) + x/(a^2*\text{ArcCosh}[a*x]) - (3*x^3)/(2*\text{ArcCosh}[a*x]) + \text{SinhIntegral}[\text{ArcCosh}[a*x]]/(8*a^3) + (9*\text{SinhIntegral}[3*\text{ArcCosh}[a*x]])/(8*a^3)$

Rule 3379

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]`

Rule 5556

`Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

Rule 5881

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]`

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\cosh^{-1}(ax)^3} dx &= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} - \frac{\int \frac{x}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^2} dx}{a} + \frac{1}{2}(3a) \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{x}{a^2 \cosh^{-1}(ax)} - \frac{3x^3}{2 \cosh^{-1}(ax)} + \frac{9}{2} \int \frac{x^2}{\cosh^{-1}(ax)} dx - \frac{1}{2} \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{x}{a^2 \cosh^{-1}(ax)} - \frac{3x^3}{2 \cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{x}{a^2 \cosh^{-1}(ax)} - \frac{3x^3}{2 \cosh^{-1}(ax)} - \frac{\text{Shi}(\cosh^{-1}(ax))}{a^3} + \frac{9 \text{Subst}\left(\int \frac{1}{\sqrt{-1+ax}} dx, x, \cosh^{-1}(ax)\right)}{2a^3} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{x}{a^2 \cosh^{-1}(ax)} - \frac{3x^3}{2 \cosh^{-1}(ax)} - \frac{\text{Shi}(\cosh^{-1}(ax))}{a^3} + \frac{9 \text{Subst}\left(\int \frac{1}{\sqrt{-1+ax}} dx, x, \cosh^{-1}(ax)\right)}{2a^3} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{2a \cosh^{-1}(ax)^2} + \frac{x}{a^2 \cosh^{-1}(ax)} - \frac{3x^3}{2 \cosh^{-1}(ax)} + \frac{\text{Shi}(\cosh^{-1}(ax))}{8a^3} + \frac{9 \text{Shi}(\cosh^{-1}(ax))}{8a^3}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 69, normalized size = 0.81

$$\frac{-\frac{4ax\left(ax\sqrt{-1+ax}\sqrt{1+ax}+(-2+3a^2x^2)\cosh^{-1}(ax)\right)}{\cosh^{-1}(ax)^2} + \text{Shi}(\cosh^{-1}(ax)) + 9\text{Shi}(3\cosh^{-1}(ax))}{8a^3}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/ArcCosh[a*x]^3,x]`

```
[Out] ((-4*a*x*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x] + (-2 + 3*a^2*x^2)*ArcCosh[a*x])
)/ArcCosh[a*x]^2 + SinhIntegral[ArcCosh[a*x]] + 9*SinhIntegral[3*ArcCosh[a*
x]])/(8*a^3)
```

Maple [A]

time = 1.85, size = 84, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{8\text{arccosh}(ax)^2} - \frac{ax}{8\text{arccosh}(ax)} + \frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{8} - \frac{\sinh(3\text{arccosh}(ax))}{8\text{arccosh}(ax)^2} - \frac{3\cosh(3\text{arccosh}(ax))}{8\text{arccosh}(ax)}}{a^3}$
default	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{8\text{arccosh}(ax)^2} - \frac{ax}{8\text{arccosh}(ax)} + \frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{8} - \frac{\sinh(3\text{arccosh}(ax))}{8\text{arccosh}(ax)^2} - \frac{3\cosh(3\text{arccosh}(ax))}{8\text{arccosh}(ax)}}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/arccosh(a*x)^3,x,method=_RETURNVERBOSE)`

```
[Out] 1/a^3*(-1/8/arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/8/arccosh(a*x)*a*x
+1/8*Shi(arccosh(a*x))-1/8/arccosh(a*x)^2*sinh(3*arccosh(a*x))-3/8/arccosh(
a*x)*cosh(3*arccosh(a*x))+9/8*Shi(3*arccosh(a*x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/arccosh(a*x)^3,x, algorithm="maxima")`

```
[Out] -1/2*(a^8*x^9 - 3*a^6*x^7 + 3*a^4*x^5 - a^2*x^3 + (a^5*x^6 - a^3*x^4)*(a*x
+ 1)^(3/2)*(a*x - 1)^(3/2) + (3*a^6*x^7 - 5*a^4*x^5 + 2*a^2*x^3)*(a*x + 1)*
(a*x - 1) + (3*a^7*x^8 - 7*a^5*x^6 + 5*a^3*x^4 - a*x^2)*sqrt(a*x + 1)*sqrt(
a*x - 1) + (3*a^8*x^9 - 9*a^6*x^7 + 9*a^4*x^5 - 3*a^2*x^3 + (3*a^5*x^6 - 4*
a^3*x^4 + a*x^2)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (9*a^6*x^7 - 17*a^4*x^5
+ 10*a^2*x^3 - 2*x)*(a*x + 1)*(a*x - 1) + (9*a^7*x^8 - 22*a^5*x^6 + 18*a^3*
```

$$x^4 - 5ax^2) \sqrt{ax+1} \sqrt{ax-1}) \log(ax + \sqrt{ax+1} \sqrt{ax-1})) / ((a^8x^6 + (ax+1)^{3/2}(ax-1)^{3/2}a^5x^3 - 3a^6x^4 + 3a^4x^2 + 3(a^6x^4 - a^4x^2)(ax+1)(ax-1) + 3(a^7x^5 - 2a^5x^3 + a^3x) \sqrt{ax+1} \sqrt{ax-1} - a^2) \log(ax + \sqrt{ax+1} \sqrt{ax-1}))^2) + \text{integrate}(1/2(9a^{10}x^{10} - 36a^8x^8 + 54a^6x^6 - 36a^4x^4 + (9a^6x^6 - 4a^4x^4 - a^2x^2)(ax+1)^2(ax-1)^2 + (36a^7x^7 - 48a^5x^5 + 13a^3x^3 + 2ax)(ax+1)^{3/2}(ax-1)^{3/2} + 9a^2x^2 + (54a^8x^8 - 120a^6x^6 + 83a^4x^4 - 19a^2x^2 + 2)(ax+1)(ax-1) + (36a^9x^9 - 112a^7x^7 + 123a^5x^5 - 57a^3x^3 + 10ax) \sqrt{ax+1} \sqrt{ax-1})) / ((a^{10}x^8 + (ax+1)^2(ax-1)^2a^6x^4 - 4a^8x^6 + 6a^6x^4 - 4a^4x^2 + 4(a^7x^5 - a^5x^3)(ax+1)^{3/2}(ax-1)^{3/2} + 6(a^8x^6 - 2a^6x^4 + a^4x^2)(ax+1)(ax-1) + 4(a^9x^7 - 3a^7x^5 + 3a^5x^3 - a^3x) \sqrt{ax+1} \sqrt{ax-1} + a^2) \log(ax + \sqrt{ax+1} \sqrt{ax-1})), x)$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(ax)^3,x, algorithm="fricas")

[Out] integral(x^2/arccosh(ax)^3, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/acosh(ax)**3,x)

[Out] Integral(x**2/acosh(ax)**3, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(ax)^3,x, algorithm="giac")

[Out] integrate(x^2/arccosh(ax)^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/acosh(a*x)^3,x)

[Out] int(x^2/acosh(a*x)^3, x)

3.61 $\int \frac{x}{\cosh^{-1}(ax)^3} dx$

Optimal. Leaf size=68

$$-\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{\text{Shi}(2\cosh^{-1}(ax))}{a^2}$$

[Out] $1/2/a^2/\text{arccosh}(a*x)-x^2/\text{arccosh}(a*x)+\text{Shi}(2*\text{arccosh}(a*x))/a^2-1/2*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\text{arccosh}(a*x)^2$

Rubi [A]

time = 0.26, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5886, 5951, 5887, 5556, 12, 3379, 5893}

$$\frac{\text{Shi}(2\cosh^{-1}(ax))}{a^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} - \frac{x\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCosh[a*x]^3,x]

[Out] $-1/2*(x*\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(a*\text{ArcCosh}[a*x]^2) + 1/(2*a^2*\text{ArcCosh}[a*x]) - x^2/\text{ArcCosh}[a*x] + \text{SinhIntegral}[2*\text{ArcCosh}[a*x]]/a^2$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.)^(n_.)*(x_))^(m_.), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1))

```

)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

```

Rule 5887

```

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

```

Rule 5893

```

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/(Sqrt[(d1_.) + (e1_.)*(x_)]*Sq
rt[(d2_.) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

```

Rule 5951

```

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_.) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cosh^{-1}(ax)^3} dx &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2} dx}{2a} + a \int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}} dx \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + 2 \int \frac{x}{\cosh^{-1}(ax)} dx \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2a^2\cosh^{-1}(ax)} - \frac{x^2}{\cosh^{-1}(ax)} + \frac{\text{Shi}(2\cosh^{-1}(ax))}{a^2}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 67, normalized size = 0.99

$$-\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1-2a^2x^2}{2a^2\cosh^{-1}(ax)} + \frac{\text{Shi}(2\cosh^{-1}(ax))}{a^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x/ArcCosh[a*x]^3,x]`

```
[Out] -1/2*(x*sqrt[-1 + a*x]*sqrt[1 + a*x])/(a*ArcCosh[a*x]^2) + (1 - 2*a^2*x^2)/
(2*a^2*ArcCosh[a*x]) + SinhIntegral[2*ArcCosh[a*x]]/a^2
```

Maple [A]

time = 2.29, size = 43, normalized size = 0.63

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\operatorname{arccosh}(ax))}{4\operatorname{arccosh}(ax)^2} - \frac{\cosh(2\operatorname{arccosh}(ax))}{2\operatorname{arccosh}(ax)} + \operatorname{hyperbolicSineIntegral}(2\operatorname{arccosh}(ax))}{a^2}$	43
default	$\frac{-\frac{\sinh(2\operatorname{arccosh}(ax))}{4\operatorname{arccosh}(ax)^2} - \frac{\cosh(2\operatorname{arccosh}(ax))}{2\operatorname{arccosh}(ax)} + \operatorname{hyperbolicSineIntegral}(2\operatorname{arccosh}(ax))}{a^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/arccosh(a*x)^3,x,method=_RETURNVERBOSE)`

[Out] $1/a^2*(-1/4/\operatorname{arccosh}(ax)^2*\sinh(2*\operatorname{arccosh}(ax))-1/2/\operatorname{arccosh}(ax)*\cosh(2*\operatorname{arccosh}(ax))+\operatorname{Shi}(2*\operatorname{arccosh}(ax)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/arccosh(ax)^3,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/2*(a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - a^3*x^3)*(a*x + 1)^{(3/2)} \\ & *(a*x - 1)^{(3/2)} - a^2*x^2 + (3*a^6*x^6 - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)* \\ & (a*x - 1) + (3*a^7*x^7 - 7*a^5*x^5 + 5*a^3*x^3 - a*x)*\sqrt{a*x + 1}*\sqrt{a*x \\ & - 1} + (2*a^8*x^8 - 6*a^6*x^6 + 6*a^4*x^4 + 2*(a^5*x^5 - a^3*x^3)*(a*x + \\ & 1)^{(3/2)}*(a*x - 1)^{(3/2)} - 2*a^2*x^2 + (6*a^6*x^6 - 10*a^4*x^4 + 5*a^2*x^2 \\ & - 1)*(a*x + 1)*(a*x - 1) + (6*a^7*x^7 - 14*a^5*x^5 + 11*a^3*x^3 - 3*a*x)*\sqrt{ \\ & a*x + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))/((a^8*x^8 \\ & + (a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)}*a^5*x^3 - 3*a^6*x^4 + 3*a^4*x^2 + 3*(a^6*x^4 \\ & - a^4*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^5 - 2*a^5*x^3 + a^3*x)*\sqrt{ \\ & a*x + 1}*\sqrt{a*x - 1} - a^2)*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))^2) + \\ & \operatorname{integrate}(1/2*(4*a^9*x^9 + 4*(a*x + 1)^2*(a*x - 1)^2*a^5*x^5 - 16*a^7*x^7 + \\ & 24*a^5*x^5 - 16*a^3*x^3 + (16*a^6*x^6 - 16*a^4*x^4 + 3)*(a*x + 1)^{(3/2)}*(a \\ & *x - 1)^{(3/2)} + 24*(a^7*x^7 - 2*a^5*x^5 + a^3*x^3)*(a*x + 1)*(a*x - 1) + (1 \\ & 6*a^8*x^8 - 48*a^6*x^6 + 48*a^4*x^4 - 19*a^2*x^2 + 3)*\sqrt{a*x + 1}*\sqrt{a*x \\ & - 1} + 4*a*x)/((a^9*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^5*x^4 - 4*a^7*x^6 + 6 \\ & *a^5*x^4 - 4*a^3*x^2 + 4*(a^6*x^5 - a^4*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} \\ &) + 6*(a^7*x^6 - 2*a^5*x^4 + a^3*x^2)*(a*x + 1)*(a*x - 1) + 4*(a^8*x^7 - 3* \\ & a^6*x^5 + 3*a^4*x^3 - a^2*x)*\sqrt{a*x + 1}*\sqrt{a*x - 1} + a)*\log(a*x + \sqrt{ \\ & a*x + 1}*\sqrt{a*x - 1})), x) \end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/arccosh(ax)^3,x, algorithm="fricas")`

[Out] `integral(x/arccosh(ax)^3, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(a*x)**3,x)

[Out] Integral(x/acosh(a*x)**3, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^3,x, algorithm="giac")

[Out] integrate(x/arccosh(a*x)^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x)^3,x)

[Out] int(x/acosh(a*x)^3, x)

$$3.62 \quad \int \frac{1}{\cosh^{-1}(ax)^3} dx$$

Optimal. Leaf size=55

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{x}{2\cosh^{-1}(ax)} + \frac{\text{Shi}(\cosh^{-1}(ax))}{2a}$$

[Out] $-1/2*x/\text{arccosh}(a*x)+1/2*\text{Shi}(\text{arccosh}(a*x))/a-1/2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\text{arccosh}(a*x)^2$

Rubi [A]

time = 0.13, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5880, 5951, 5881, 3379}

$$\frac{\text{Shi}(\cosh^{-1}(ax))}{2a} - \frac{x}{2\cosh^{-1}(ax)} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^(-3), x]

[Out] $-1/2*(\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(a*\text{ArcCosh}[a*x]^2) - x/(2*\text{ArcCosh}[a*x]) + \text{SinhIntegral}[\text{ArcCosh}[a*x]]/(2*a)$

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 5880

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n, x_Symbol] :> Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n, x_Symbol] :> Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n)*((f_.)*(x_))^m, x_Symbol] :> Simp[(f*x)^m*((a

```

+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^3} dx &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} + \frac{1}{2}a \int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2} dx \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{x}{2\cosh^{-1}(ax)} + \frac{1}{2} \int \frac{1}{\cosh^{-1}(ax)} dx \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{x}{2\cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{2a} \\
&= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{x}{2\cosh^{-1}(ax)} + \frac{\text{Shi}(\cosh^{-1}(ax))}{2a}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 55, normalized size = 1.00

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)^2} - \frac{x}{2\cosh^{-1}(ax)} + \frac{\text{Shi}(\cosh^{-1}(ax))}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^(-3), x]

[Out] -1/2*(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x]^2) - x/(2*ArcCosh[a*x]) + SinhIntegral[ArcCosh[a*x]]/(2*a)

Maple [A]

time = 2.50, size = 45, normalized size = 0.82

method	result	size
derivativedivides	$ -\frac{\sqrt{ax-1}\sqrt{ax+1}}{2\text{arccosh}(ax)^2} - \frac{ax}{2\text{arccosh}(ax)} + \frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{2} $	45
default	$ -\frac{\sqrt{ax-1}\sqrt{ax+1}}{2\text{arccosh}(ax)^2} - \frac{ax}{2\text{arccosh}(ax)} + \frac{\text{hyperbolicSineIntegral}(\text{arccosh}(ax))}{2} $	45

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/arccosh(a*x)^3,x,method=_RETURNVERBOSE)`

[Out] $1/a*(-1/2/\operatorname{arccosh}(a*x)^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}-1/2/\operatorname{arccosh}(a*x)*a*x+1/2*\operatorname{Shi}(\operatorname{arccosh}(a*x)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^3,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/2*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - a^2*x^2)*(a*x + 1)^{(3/2)} \\ & *(a*x - 1)^{(3/2)} + (3*a^5*x^5 - 5*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (3 \\ & *a^6*x^6 - 7*a^4*x^4 + 5*a^2*x^2 - 1)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1) - a*x + (\\ & a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - 1)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} \\ & + 3*(a^5*x^5 - a^3*x^3)*(a*x + 1)*(a*x - 1) + (3*a^6*x^6 - 6*a^4*x^4 + \\ & 4*a^2*x^2 - 1)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1) - a*x)*\log(a*x + \operatorname{sqrt}(a*x + 1)* \\ & \operatorname{sqrt}(a*x - 1)))/((a^7*x^6 + (a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)}*a^4*x^3 - 3*a^5 \\ & *x^4 + 3*a^3*x^2 + 3*(a^5*x^4 - a^3*x^2)*(a*x + 1)*(a*x - 1) + 3*(a^6*x^5 - \\ & 2*a^4*x^3 + a^2*x)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1) - a)*\log(a*x + \operatorname{sqrt}(a*x + 1) \\ &)*\operatorname{sqrt}(a*x - 1))^2) + \operatorname{integrate}(1/2*(a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 + (a^4 \\ & *x^4 + 3)*(a*x + 1)^2*(a*x - 1)^2 + (4*a^5*x^5 - 4*a^3*x^3 + 3*a*x)*(a*x + \\ & 1)^{(3/2)}*(a*x - 1)^{(3/2)} - 4*a^2*x^2 + 3*(2*a^6*x^6 - 4*a^4*x^4 + a^2*x^2 + \\ & 1)*(a*x + 1)*(a*x - 1) + (4*a^7*x^7 - 12*a^5*x^5 + 9*a^3*x^3 - a*x)*\operatorname{sqrt}(a \\ & *x + 1)*\operatorname{sqrt}(a*x - 1) + 1)/((a^8*x^8 + (a*x + 1)^2*(a*x - 1)^2*a^4*x^4 - 4* \\ & a^6*x^6 + 6*a^4*x^4 + 4*(a^5*x^5 - a^3*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} \\ & - 4*a^2*x^2 + 6*(a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*(a*x + 1)*(a*x - 1) + 4*(a \\ & ^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 - a*x)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1) + 1)*\log(\\ & a*x + \operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))), x) \end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^3,x, algorithm="fricas")`

[Out] `integral(arccosh(a*x)^(-3), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acosh(a*x)**3,x)

[Out] Integral(acosh(a*x)**(-3), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccosh(a*x)^3,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^(-3), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/acosh(a*x)^3,x)

[Out] int(1/acosh(a*x)^3, x)

$$3.63 \quad \int \frac{1}{x \cosh^{-1}(ax)^3} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)^3}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^3,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^3),x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^3), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^3} dx = \int \frac{1}{x \cosh^{-1}(ax)^3} dx$$

Mathematica [A]

time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^3),x]

[Out] Integrate[1/(x*ArcCosh[a*x]^3), x]

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccosh(a*x)^3,x)

[Out] int(1/x/arccosh(a*x)^3,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*(a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - a^3*x^3)*(a*x + 1)^{(3/2)} \\ & *(a*x - 1)^{(3/2)} - a^2*x^2 + (3*a^6*x^6 - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)* \\ & (a*x - 1) + (3*a^7*x^7 - 7*a^5*x^5 + 5*a^3*x^3 - a*x)*\sqrt{a*x + 1}*\sqrt{a*x \\ & - 1} + (2*(a^3*x^3 - a*x)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + (4*a^4*x^4 - \\ & 5*a^2*x^2 + 1)*(a*x + 1)*(a*x - 1) + (2*a^5*x^5 - 3*a^3*x^3 + a*x)*\sqrt{a*x \\ & + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))/((a^8*x^8 + (a \\ & *x + 1)^{(3/2)}*(a*x - 1)^{(3/2)}*a^5*x^5 - 3*a^6*x^6 + 3*a^4*x^4 - a^2*x^2 + 3 \\ & *(a^6*x^6 - a^4*x^4)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^7 - 2*a^5*x^5 + a^3*x^3 \\ &)*\sqrt{a*x + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})^2) - \\ & \text{integrate}(1/2*(4*(a^4*x^4 - 2*a^2*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (12*a^5*x^5 \\ & - 22*a^3*x^3 + 7*a*x)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(6*a^6*x^6 - 10 \\ & *a^4*x^4 + 5*a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + (4*a^7*x^7 - 6*a^5*x^5 + 3* \\ & a^3*x^3 - a*x)*\sqrt{a*x + 1}*\sqrt{a*x - 1}))/((a^{10}*x^{11} + (a*x + 1)^2*(a*x \\ & - 1)^2*a^6*x^7 - 4*a^8*x^9 + 6*a^6*x^7 - 4*a^4*x^5 + a^2*x^3 + 4*(a^7*x^8 - \\ & a^5*x^6)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 6*(a^8*x^9 - 2*a^6*x^7 + a^4*x^5) \\ & *(a*x + 1)*(a*x - 1) + 4*(a^9*x^{10} - 3*a^7*x^8 + 3*a^5*x^6 - a^3*x^4)*\sqrt{a*x \\ & + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})), x) \end{aligned}$$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^3,x, algorithm="fricas")

[Out] integral(1/(x*arccosh(a*x)^3), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/acosh(a*x)**3,x)

[Out] Integral(1/(x*acosh(a*x)**3), x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^3,x, algorithm="giac")

[Out] integrate(1/(x*arccosh(a*x)^3), x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*acosh(a*x)^3),x)

[Out] int(1/(x*acosh(a*x)^3), x)

$$3.64 \quad \int \frac{1}{x^2 \cosh^{-1}(ax)^3} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \cosh^{-1}(ax)^3}, x\right)$$

[Out] Unintegrable(1/x^2/arccosh(a*x)^3,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*ArcCosh[a*x]^3), x]

[Out] Defer[Int][1/(x^2*ArcCosh[a*x]^3), x]

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} dx = \int \frac{1}{x^2 \cosh^{-1}(ax)^3} dx$$

Mathematica [A]

time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*ArcCosh[a*x]^3), x]

[Out] Integrate[1/(x^2*ArcCosh[a*x]^3), x]

Maple [A]

time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arccosh(a*x)^3,x)

[Out] int(1/x^2/arccosh(a*x)^3,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccosh(a*x)^3,x, algorithm="maxima")

[Out]
$$-1/2*(a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - a^3*x^3)*(a*x + 1)^{(3/2)} * (a*x - 1)^{(3/2)} - a^2*x^2 + (3*a^6*x^6 - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*(a*x - 1) + (3*a^7*x^7 - 7*a^5*x^5 + 5*a^3*x^3 - a*x)*\sqrt{a*x + 1}*\sqrt{a*x - 1} - (a^8*x^8 - 3*a^6*x^6 + 3*a^4*x^4 + (a^5*x^5 - 4*a^3*x^3 + 3*a*x)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} - a^2*x^2 + (3*a^6*x^6 - 11*a^4*x^4 + 10*a^2*x^2 - 2)*(a*x + 1)*(a*x - 1) + (3*a^7*x^7 - 10*a^5*x^5 + 10*a^3*x^3 - 3*a*x)*\sqrt{a*x + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))/((a^8*x^9 + (a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)}*a^5*x^6 - 3*a^6*x^7 + 3*a^4*x^5 - a^2*x^3 + 3*(a^6*x^7 - a^4*x^5)*(a*x + 1)*(a*x - 1) + 3*(a^7*x^8 - 2*a^5*x^6 + a^3*x^4)*\sqrt{a*x + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))^2 + \int (1/2*(a^{10}*x^{10} - 4*a^8*x^8 + 6*a^6*x^6 - 4*a^4*x^4 + (a^6*x^6 - 12*a^4*x^4 + 15*a^2*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (4*a^7*x^7 - 40*a^5*x^5 + 57*a^3*x^3 - 18*a*x)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + a^2*x^2 + 3*(2*a^8*x^8 - 16*a^6*x^6 + 25*a^4*x^4 - 13*a^2*x^2 + 2)*(a*x + 1)*(a*x - 1) + (4*a^9*x^9 - 24*a^7*x^7 + 39*a^5*x^5 - 25*a^3*x^3 + 6*a*x)*\sqrt{a*x + 1}*\sqrt{a*x - 1}))/((a^{10}*x^{12} + (a*x + 1)^2*(a*x - 1)^2*a^6*x^8 - 4*a^8*x^{10} + 6*a^6*x^8 - 4*a^4*x^6 + a^2*x^4 + 4*(a^7*x^9 - a^5*x^7)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 6*(a^8*x^{10} - 2*a^6*x^8 + a^4*x^6)*(a*x + 1)*(a*x - 1) + 4*(a^9*x^{11} - 3*a^7*x^9 + 3*a^5*x^7 - a^3*x^5)*\sqrt{a*x + 1}*\sqrt{a*x - 1}))*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1})), x)$$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccosh(a*x)^3,x, algorithm="fricas")

[Out] integral(1/(x^2*arccosh(a*x)^3), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/acosh(a*x)**3,x)

[Out] Integral(1/(x**2*acosh(a*x)**3), x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccosh(a*x)^3,x, algorithm="giac")

[Out] integrate(1/(x^2*arccosh(a*x)^3), x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*acosh(a*x)^3),x)

[Out] int(1/(x^2*acosh(a*x)^3), x)

3.65 $\int \frac{x^4}{\cosh^{-1}(ax)^4} dx$

Optimal. Leaf size=170

$$-\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2\cosh^{-1}(ax)^2} - \frac{5x^5}{6\cosh^{-1}(ax)^2} + \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{25x^4\sqrt{-1+ax}\sqrt{1+ax}}{6a\cosh^{-1}(ax)}$$

[Out] $2/3*x^3/a^2/\operatorname{arccosh}(a*x)^2 - 5/6*x^5/\operatorname{arccosh}(a*x)^2 + 1/48*\operatorname{Chi}(\operatorname{arccosh}(a*x))/a^5 + 27/32*\operatorname{Chi}(3*\operatorname{arccosh}(a*x))/a^5 + 125/96*\operatorname{Chi}(5*\operatorname{arccosh}(a*x))/a^5 - 1/3*x^4*(a*x - 1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^3 + 2*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3/\operatorname{arccosh}(a*x) - 25/6*x^4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)$

Rubi [A]

time = 0.41, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5886, 5951, 5885, 3382}

$$\frac{\operatorname{Chi}(\cosh^{-1}(ax))}{48a^5} + \frac{27\operatorname{Chi}(3\cosh^{-1}(ax))}{32a^5} + \frac{125\operatorname{Chi}(5\cosh^{-1}(ax))}{96a^5} + \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{a^3\cosh^{-1}(ax)} + \frac{2x^3}{3a^2\cosh^{-1}(ax)^2} - \frac{5x^5}{6\cosh^{-1}(ax)^2} - \frac{25x^4\sqrt{ax-1}\sqrt{ax+1}}{6a\cosh^{-1}(ax)} - \frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4/\operatorname{ArcCosh}[a*x]^4, x]$

[Out] $-1/3*(x^4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a*\operatorname{ArcCosh}[a*x]^3) + (2*x^3)/(3*a^2*\operatorname{ArcCosh}[a*x]^2) - (5*x^5)/(6*\operatorname{ArcCosh}[a*x]^2) + (2*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a^3*\operatorname{ArcCosh}[a*x]) - (25*x^4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(6*a*\operatorname{ArcCosh}[a*x]) + \operatorname{CoshIntegral}[\operatorname{ArcCosh}[a*x]]/(48*a^5) + (27*\operatorname{CoshIntegral}[3*\operatorname{ArcCosh}[a*x]])/(32*a^5) + (125*\operatorname{CoshIntegral}[5*\operatorname{ArcCosh}[a*x]])/(96*a^5)$

Rule 3382

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[c*f*(fz/d) + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

Rule 5885

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^m*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[-1+c*x]*((a+b*\operatorname{ArcCosh}[c*x])^{(n+1)})/(b*c*(n+1)), x] + \operatorname{Dist}[1/(b^2*c^{(m+1)}*(n+1)), \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandTrigReduce}[x^{(n+1)}, \operatorname{Cosh}[-a/b+x/b]^{(m-1)}*(m-(m+1)*\operatorname{Cosh}[-a/b+x/b]^2), x], x], x, a+b*\operatorname{ArcCosh}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{GeQ}[n, -2] \&\& \operatorname{LtQ}[n, -1]$

Rule 5886

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^m*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[-1+c*x]*((a+b*\operatorname{ArcCosh}[c*x])^{(n+1)})/(b*c*(n+1))$

```
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/(Sqrt[(d1_) + (e1_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^4}{\cosh^{-1}(ax)^4} dx = -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{4 \int \frac{x^3}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3} dx}{3a} + \frac{1}{3}(5a) \int \frac{1}{\sqrt{-1+ax}}$$

$$= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2 \cosh^{-1}(ax)^2} - \frac{5x^5}{6 \cosh^{-1}(ax)^2} + \frac{25}{6} \int \frac{x^4}{\cosh^{-1}(ax)^2} dx$$

$$= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2 \cosh^{-1}(ax)^2} - \frac{5x^5}{6 \cosh^{-1}(ax)^2} + \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a^3 \cosh^{-1}(ax)}$$

$$= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2 \cosh^{-1}(ax)^2} - \frac{5x^5}{6 \cosh^{-1}(ax)^2} + \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a^3 \cosh^{-1}(ax)}$$

$$= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{2x^3}{3a^2 \cosh^{-1}(ax)^2} - \frac{5x^5}{6 \cosh^{-1}(ax)^2} + \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax}}{a^3 \cosh^{-1}(ax)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 356 vs. 2(170) = 340.
 time = 0.28, size = 356, normalized size = 2.09

```
\sqrt{-1+ax} \left( \frac{32a^4 \sqrt{1+ax}}{1+ax} - 32a^4 \sqrt{\frac{1+ax}{-1+ax}} + 64a^3 \sqrt{-1+ax} \sqrt{\frac{1+ax}{-1+ax}} \sqrt{-1+ax} \sqrt{-1+ax} \cosh^{-1}(ax) - 80a^2 \sqrt{-1+ax} \sqrt{\frac{1+ax}{-1+ax}} \sqrt{-1+ax} \cosh^{-1}(ax) - 192a^2 \sqrt{\frac{1+ax}{-1+ax}} \cosh^{-1}(ax)^2 + 592a^2 \sqrt{\frac{1+ax}{-1+ax}} \cosh^{-1}(ax)^3 - 400a^2 \sqrt{\frac{1+ax}{-1+ax}} \cosh^{-1}(ax)^4 + 2(-1+ax) \cosh^{-1}(ax)^2 \text{Chi}(\cosh^{-1}(ax)) + 8(-1+ax) \cosh^{-1}(ax)^3 \text{Chi}(\cosh^{-1}(ax)) - 12 \cosh^{-1}(ax)^2 \text{Chi}(\cosh^{-1}(ax)) + 12 \cosh^{-1}(ax)^3 \text{Chi}(\cosh^{-1}(ax)) \right) \text{Shi}(\frac{1}{2} \text{arctanh}(\frac{1+ax}{-1+ax})) \sqrt{-1+ax} \sqrt{-1+ax} \cosh^{-1}(ax)
```

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcCosh[a*x]^4, x]
```

```
[Out] (Sqrt[-1 + a*x]*(32*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)] - 32*a^6*x^6*Sqrt[(-1 + a*x)/(1 + a*x)] + 64*a^3*x^3*Sqrt[-1 + a*x]*Sqrt[(-1 + a*x)/(1 + a*x)]*
```

$$\frac{\sqrt{1+ax} \operatorname{ArcCosh}[ax] - 80a^5x^5\sqrt{-1+ax}\sqrt{(-1+ax)/(1+ax)}}{(1+ax)} + \frac{\sqrt{1+ax} \operatorname{ArcCosh}[ax] - 192a^2x^2\sqrt{(-1+ax)/(1+ax)}}{(1+ax)} + \frac{\operatorname{ArcCosh}[ax]^2 + 592a^4x^4\sqrt{(-1+ax)/(1+ax)} \operatorname{ArcCosh}[ax]^2 - 400a^6x^6\sqrt{(-1+ax)/(1+ax)} \operatorname{ArcCosh}[ax]^2 + 2(-1+ax) \operatorname{ArcCosh}[ax]^3 \operatorname{CoshIntegral}[\operatorname{ArcCosh}[ax]] + 81(-1+ax) \operatorname{ArcCosh}[ax]^3 \operatorname{CoshIntegral}[3 \operatorname{ArcCosh}[ax]] - 125 \operatorname{ArcCosh}[ax]^3 \operatorname{CoshIntegral}[5 \operatorname{ArcCosh}[ax]] + 125ax \operatorname{ArcCosh}[ax]^3 \operatorname{CoshIntegral}[5 \operatorname{ArcCosh}[ax]]}{(96a^5((-1+ax)/(1+ax))^{3/2}(1+ax)^{3/2} \operatorname{ArcCosh}[ax]^3)}$$

Maple [A]

time = 2.33, size = 175, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{24\operatorname{arccosh}(ax)^3} - \frac{ax}{48\operatorname{arccosh}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{48\operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(\operatorname{arccosh}(ax))}{48} - \frac{\sinh(3\operatorname{arccosh}(ax))}{16\operatorname{arccosh}(ax)^3}}{\dots}$
default	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{24\operatorname{arccosh}(ax)^3} - \frac{ax}{48\operatorname{arccosh}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{48\operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(\operatorname{arccosh}(ax))}{48} - \frac{\sinh(3\operatorname{arccosh}(ax))}{16\operatorname{arccosh}(ax)^3}}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/arccosh(a*x)^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{a^5} \left(-\frac{1}{24} \operatorname{arccosh}(ax)^3 (ax-1)^{1/2} (ax+1)^{1/2} - \frac{1}{48} \operatorname{arccosh}(ax)^2 ax - \frac{1}{48} \operatorname{arccosh}(ax) (ax-1)^{1/2} (ax+1)^{1/2} + \frac{1}{48} \operatorname{Chi}(\operatorname{arccosh}(ax)) - \frac{1}{16} \operatorname{arccosh}(ax)^3 \sinh(3 \operatorname{arccosh}(ax)) - \frac{3}{32} \operatorname{arccosh}(ax)^2 \cosh(3 \operatorname{arccosh}(ax)) - \frac{9}{32} \operatorname{arccosh}(ax) \sinh(3 \operatorname{arccosh}(ax)) + \frac{27}{32} \operatorname{Chi}(3 \operatorname{arccosh}(ax)) - \frac{1}{48} \operatorname{arccosh}(ax)^3 \sinh(5 \operatorname{arccosh}(ax)) - \frac{5}{96} \operatorname{arccosh}(ax)^2 \cosh(5 \operatorname{arccosh}(ax)) - \frac{25}{96} \operatorname{arccosh}(ax) \sinh(5 \operatorname{arccosh}(ax)) + \frac{125}{96} \operatorname{Chi}(5 \operatorname{arccosh}(ax)) \right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arccosh(a*x)^4,x, algorithm="maxima")`

[Out]
$$-\frac{1}{6} (2a^{13}x^{15} - 10a^{11}x^{13} + 20a^9x^{11} - 20a^7x^9 + 10a^5x^7 - 2a^3x^5 + 2(a^8x^{10} - a^6x^8)(ax+1)^{5/2}(ax-1)^{5/2} + 2(5a^9x^{11} - 9a^7x^9 + 4a^5x^7)(ax+1)^2(ax-1)^2 + 4(5a^{10}x^{12} - 13a^8x^{10} + 11a^6x^8 - 3a^4x^6)(ax+1)^{3/2}(ax-1)^{3/2} + 4(5a^{11}x^{13} - 17a^9x^{11} + 21a^7x^9 - 11a^5x^7 + 2a^3x^5)(ax+1)(ax-1) + (25a^{13}x^{15} - 125a^{11}x^{13} + 250a^9x^{11} - 250a^7x^9 + 125a^5x^7 - 25a^3x^5 + (25a^8x^{10} - 49a^6x^8 + 27a^4x^6 - 3a^2x^4)(ax+1)^{5/2}(ax-1)^{5/2} + (125a^9x^{11} - 321a^7x^9 + 286a^5x^7 - 102a^3x^5 + 12ax^3)(ax+1)^2(ax-1)^2 + (250a^{10}x^{12} - 79$$

$$\begin{aligned}
& 4a^8x^{10} + 946a^6x^8 - 519a^4x^6 + 129a^2x^4 - 12x^2)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + 2(125a^{11}x^{13} - 473a^9x^{11} + 696a^7x^9 - 497a^5x^7 + 173a^3x^5 - 24ax^3)(ax + 1)(ax - 1) + (125a^{12}x^{14} - 549a^{10}x^{12} + 955a^8x^{10} - 824a^6x^8 + 354a^4x^6 - 61a^2x^4)\sqrt{ax + 1}\sqrt{ax - 1})\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})^2 + 2(5a^{12}x^{14} - 21a^{10}x^{12} + 34a^8x^{10} - 26a^6x^8 + 9a^4x^6 - a^2x^4)\sqrt{ax + 1}\sqrt{ax - 1} + (5a^{13}x^{15} - 25a^{11}x^{13} + 50a^9x^{11} - 50a^7x^9 + 25a^5x^7 - 5a^3x^5 + (5a^8x^{10} - 8a^6x^8 + 3a^4x^6)(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + (25a^9x^{11} - 57a^7x^9 + 42a^5x^7 - 10a^3x^5)(ax + 1)^2(ax - 1)^2 + (50a^{10}x^{12} - 148a^8x^{10} + 158a^6x^8 - 71a^4x^6 + 11a^2x^4)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + 2(25a^{11}x^{13} - 91a^9x^{11} + 126a^7x^9 - 81a^5x^7 + 23a^3x^5 - 2ax^3)(ax + 1)(ax - 1) + (25a^{12}x^{14} - 108a^{10}x^{12} + 183a^8x^{10} - 151a^6x^8 + 60a^4x^6 - 9a^2x^4)\sqrt{ax + 1}\sqrt{ax - 1})\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})))/((a^{13}x^{10} - 5a^{11}x^8 + (ax + 1)^{(5/2)}(ax - 1)^{(5/2)})a^8x^5 + 10a^9x^6 - 10a^7x^4 + 5a^5x^2 + 5(a^9x^6 - a^7x^4)(ax + 1)^2(ax - 1)^2 + 10(a^{10}x^7 - 2a^8x^5 + a^6x^3)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + 10(a^{11}x^8 - 3a^9x^6 + 3a^7x^4 - a^5x^2)(ax + 1)(ax - 1) - a^3 + 5(a^{12}x^9 - 4a^{10}x^7 + 6a^8x^5 - 4a^6x^3 + a^4x)\sqrt{ax + 1}\sqrt{ax - 1})\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})^3 + \text{integrate}(1/6(125a^{15}x^{16} - 750a^{13}x^{14} + 1875a^{11}x^{12} - 2500a^9x^{10} + 1875a^7x^8 - 750a^5x^6 + (125a^9x^{10} - 147a^7x^8 + 27a^5x^6 + 3a^3x^4)(ax + 1)^3(ax - 1)^3 + 125a^3x^4 + (750a^{10}x^{11} - 1485a^8x^9 + 901a^6x^7 - 147a^4x^5 - 12a^2x^3)(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + (1875a^{11}x^{12} - 5220a^9x^{10} + 5209a^7x^8 - 2185a^5x^6 + 321a^3x^4)(ax + 1)^2(ax - 1)^2 + (2500a^{12}x^{13} - 8970a^{10}x^{11} + 12366a^8x^9 - 8143a^6x^7 + 2583a^4x^5 - 360a^2x^3 + 24x)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + (1875a^{13}x^{14} - 8235a^{11}x^{12} + 14449a^9x^{10} - 12834a^7x^8 + 6030a^5x^6 - 1429a^3x^4 + 144ax^2)(ax + 1)(ax - 1) + (750a^{14}x^{15} - 3897a^{12}x^{13} + 8293a^{10}x^{11} - 9226a^8x^9 + 5655a^6x^7 - 1819a^4x^5 + 244a^2x^3)\sqrt{ax + 1}\sqrt{ax - 1})/(a^{15}x^{12} - 6a^{13}x^{10} + (ax + 1)^3(ax - 1)^3a^9x^6 + 15a^{11}x^8 - 20a^9x^6 + 15a^7x^4 - 6a^5x^2 + 6(a^{10}x^7 - a^8x^5)(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + 15(a^{11}x^8 - 2a^9x^6 + a^7x^4)(ax + 1)^2(ax - 1)^2 + 20(a^{12}x^9 - 3a^{10}x^7 + 3a^8x^5 - a^6x^3)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + 15(a^{13}x^{10} - 4a^{11}x^8 + 6a^9x^6 - 4a^7x^4 + a^5x^2)(ax + 1)(ax - 1) + a^3 + 6(a^{14}x^{11} - 5a^{12}x^9 + 10a^{10}x^7 - 10a^8x^5 + 5a^6x^3 - a^4x)\sqrt{ax + 1}\sqrt{ax - 1})\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})), x)
\end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^4,x, algorithm="fricas")

[Out] integral(x^4/arccosh(a*x)^4, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/acosh(a*x)**4,x)

[Out] Integral(x**4/acosh(a*x)**4, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^4,x, algorithm="giac")

[Out] integrate(x^4/arccosh(a*x)^4, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/acosh(a*x)^4,x)

[Out] int(x^4/acosh(a*x)^4, x)

3.66 $\int \frac{x^3}{\cosh^{-1}(ax)^4} dx$

Optimal. Leaf size=155

$$-\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x^2}{2a^2\cosh^{-1}(ax)^2} - \frac{2x^4}{3\cosh^{-1}(ax)^2} + \frac{x\sqrt{-1+ax}\sqrt{1+ax}}{a^3\cosh^{-1}(ax)} - \frac{8x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)}$$

[Out] $1/2*x^2/a^2/\operatorname{arccosh}(a*x)^2 - 2/3*x^4/\operatorname{arccosh}(a*x)^2 + 1/3*\operatorname{Chi}(2*\operatorname{arccosh}(a*x))/a^4 + 4/3*\operatorname{Chi}(4*\operatorname{arccosh}(a*x))/a^4 - 1/3*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^3 + x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3/\operatorname{arccosh}(a*x) - 8/3*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)$

Rubi [A]

time = 0.39, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5886, 5951, 5885, 3382}

$$\frac{\operatorname{Chi}(2\cosh^{-1}(ax))}{3a^4} + \frac{4\operatorname{Chi}(4\cosh^{-1}(ax))}{3a^4} + \frac{x\sqrt{ax-1}\sqrt{ax+1}}{a^3\cosh^{-1}(ax)} + \frac{x^2}{2a^2\cosh^{-1}(ax)^2} - \frac{2x^4}{3\cosh^{-1}(ax)^2} - \frac{8x^3\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)} - \frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/\operatorname{ArcCosh}[a*x]^4, x]$

[Out] $-1/3*(x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a*\operatorname{ArcCosh}[a*x]^3) + x^2/(2*a^2*\operatorname{ArcCosh}[a*x]^2) - (2*x^4)/(3*\operatorname{ArcCosh}[a*x]^2) + (x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a^3*\operatorname{ArcCosh}[a*x]) - (8*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a*\operatorname{ArcCosh}[a*x]) + \operatorname{CoshIntegral}[2*\operatorname{ArcCosh}[a*x]]/(3*a^4) + (4*\operatorname{CoshIntegral}[4*\operatorname{ArcCosh}[a*x]])/(3*a^4)$

Rule 3382

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[c*f*(fz/d) + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

Rule 5885

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}(c_.*(x_))*(b_.)]^{(n_)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^m*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[-1+c*x]*((a+b*\operatorname{ArcCosh}[c*x])^{(n+1)})/(b*c*(n+1)), x] + \operatorname{Dist}[1/(b^2*c^{(m+1)}*(n+1)), \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandTrigReduce}[x^{(n+1)}, \operatorname{Cosh}[-a/b+x/b]^{(m-1)}*(m-(m+1)*\operatorname{Cosh}[-a/b+x/b]^2), x], x], x, a+b*\operatorname{ArcCosh}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{GeQ}[n, -2] \&\& \operatorname{LtQ}[n, -1]$

Rule 5886


```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*(m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_)^(m_)))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\cosh^{-1}(ax)^4} dx &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{\int \frac{x^2}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3} dx}{a} + \frac{1}{3}(4a) \int \frac{1}{\sqrt{-1+ax}} dx \\ &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x^2}{2a^2 \cosh^{-1}(ax)^2} - \frac{2x^4}{3 \cosh^{-1}(ax)^2} + \frac{8}{3} \int \frac{x^3}{\cosh^{-1}(ax)^2} dx \\ &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x^2}{2a^2 \cosh^{-1}(ax)^2} - \frac{2x^4}{3 \cosh^{-1}(ax)^2} + \frac{x \sqrt{-1+ax} \sqrt{1+ax}}{a^3 \cosh^{-1}(ax)} \\ &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x^2}{2a^2 \cosh^{-1}(ax)^2} - \frac{2x^4}{3 \cosh^{-1}(ax)^2} + \frac{x \sqrt{-1+ax} \sqrt{1+ax}}{a^3 \cosh^{-1}(ax)} \\ &= -\frac{x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x^2}{2a^2 \cosh^{-1}(ax)^2} - \frac{2x^4}{3 \cosh^{-1}(ax)^2} + \frac{x \sqrt{-1+ax} \sqrt{1+ax}}{a^3 \cosh^{-1}(ax)} \end{aligned}$$

Mathematica [A]

time = 0.28, size = 188, normalized size = 1.21

$$\frac{\sqrt{-1+ax} \left(ax \sqrt{\frac{-1+ax}{1+ax}} \left(2a^2x^2 - 2a^4x^4 - ax\sqrt{-1+ax} \sqrt{1+ax} (-3 + 4a^2x^2) \cosh^{-1}(ax) - 2(3 - 11a^2x^2 + 8a^4x^4) \cosh^{-1}(ax)^2 \right) + 2(-1+ax) \cosh^{-1}(ax)^3 \text{Chi}(2 \cosh^{-1}(ax)) + 8(-1+ax) \cosh^{-1}(ax)^3 \text{Chi}(4 \cosh^{-1}(ax)) \right)}{6a^4 \left(\frac{-1+ax}{1+ax} \right)^{3/2} (1+ax)^{3/2} \cosh^{-1}(ax)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/ArcCosh[a*x]^4, x]

```
[Out] (Sqrt[-1 + a*x]*(a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*(2*a^2*x^2 - 2*a^4*x^4 - a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-3 + 4*a^2*x^2)*ArcCosh[a*x] - 2*(3 - 11*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x]^2) + 2*(-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral[2*ArcCosh[a*x]] + 8*(-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral[4*ArcCosh[a*x]]))/(6*a^4*((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^3)
```

Maple [A]

time = 2.05, size = 114, normalized size = 0.74

method	result
derivativedivides	$\frac{\frac{\sinh(2 \operatorname{arccosh}(ax))}{12 \operatorname{arccosh}(ax)^3} - \frac{\cosh(2 \operatorname{arccosh}(ax))}{12 \operatorname{arccosh}(ax)^2} - \frac{\sinh(2 \operatorname{arccosh}(ax))}{6 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(2 \operatorname{arccosh}(ax))}{3} - \frac{\sinh(4 \operatorname{arccosh}(ax))}{24 \operatorname{arccosh}(ax)^3} - \frac{\cosh(4 \operatorname{arccosh}(ax))}{24 \operatorname{arccosh}(ax)^2}}{a^4}$
default	$\frac{\frac{\sinh(2 \operatorname{arccosh}(ax))}{12 \operatorname{arccosh}(ax)^3} - \frac{\cosh(2 \operatorname{arccosh}(ax))}{12 \operatorname{arccosh}(ax)^2} - \frac{\sinh(2 \operatorname{arccosh}(ax))}{6 \operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(2 \operatorname{arccosh}(ax))}{3} - \frac{\sinh(4 \operatorname{arccosh}(ax))}{24 \operatorname{arccosh}(ax)^3} - \frac{\cosh(4 \operatorname{arccosh}(ax))}{24 \operatorname{arccosh}(ax)^2}}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/12/arccosh(a*x)^3*sinh(2*arccosh(a*x))-1/12/arccosh(a*x)^2*cosh(2*arccosh(a*x))-1/6/arccosh(a*x)*sinh(2*arccosh(a*x))+1/3*Chi(2*arccosh(a*x))-1/24/arccosh(a*x)^3*sinh(4*arccosh(a*x))-1/12/arccosh(a*x)^2*cosh(4*arccosh(a*x))-1/3/arccosh(a*x)*sinh(4*arccosh(a*x))+4/3*Chi(4*arccosh(a*x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^14 - 10*a^11*x^12 + 20*a^9*x^10 - 20*a^7*x^8 + 10*a^5*x^6 - 2*a^3*x^4 + 2*(a^8*x^9 - a^6*x^7)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 2*(5*a^9*x^10 - 9*a^7*x^8 + 4*a^5*x^6)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^10*x^11 - 13*a^8*x^9 + 11*a^6*x^7 - 3*a^4*x^5)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + 4*(5*a^11*x^12 - 17*a^9*x^10 + 21*a^7*x^8 - 11*a^5*x^6 + 2*a^3*x^4)*(a*x + 1)*(a*x - 1) + (16*a^13*x^14 - 80*a^11*x^12 + 160*a^9*x^10 - 160*a^7*x^8 + 80*a^5*x^6 - 16*a^3*x^4 + 4*(4*a^8*x^9 - 7*a^6*x^7 + 3*a^4*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (80*a^9*x^10 - 192*a^7*x^8 + 154*a^5*x^6 - 45*a^3*x^4 + 3*a*x^2)*(a*x + 1)^2*(a*x - 1)^2 + (160*a^10*x^11 - 488*a^8*x^9 + 550*a^6*x^7 - 279*a^4*x^5 + 63*a^2*x^3 - 6*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) + (160*a^11*x^12 - 592*a^9*x^10 + 846*a^7*x^8 - 583*a^5*x^6 + 196*a^3*x^4 - 27*a*x^2)*(a*x + 1)*(a*x - 1) + (80*a^12*x^13 - 348*a^10*x^11 + 598*a^8*x^9 - 509*a^6*x^7 + 216*a^4*x^5 - 37*a^2*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 + 2*(5*a^12*x^13 - 21*a^10*x^11 + 34*a^8*x^9 - 26*a^6*x^7 + 9*a^4*x^5 - a^2*x^3)*sqrt(a*x + 1)*sqrt(a*x - 1) + (4*a^1
```

$$\begin{aligned}
& 3x^{14} - 20a^{11}x^{12} + 40a^9x^{10} - 40a^7x^8 + 20a^5x^6 - 4a^3x^4 + \\
& 2(2a^8x^9 - 3a^6x^7 + a^4x^5)(ax + 1)^{5/2}(ax - 1)^{5/2} + (20a^9x^{10} - 44a^7x^8 + 31a^5x^6 - 7a^3x^4)(ax + 1)^2(ax - 1)^2 + (\\
& 40a^{10}x^{11} - 116a^8x^9 + 121a^6x^7 - 53a^4x^5 + 8a^2x^3)(ax + 1)^{3/2}(ax - 1)^{3/2} + (40a^{11}x^{12} - 144a^9x^{10} + 197a^7x^8 - 125a^5x^6 + 35a^3x^4 - 3ax^2)(ax + 1)(ax - 1) + (20a^{12}x^{13} - 86a^{10}x^{11} + 145a^8x^9 - 119a^6x^7 + 47a^4x^5 - 7a^2x^3)\sqrt{ax + 1} \\
& \sqrt{ax - 1})\log(ax + \sqrt{ax + 1})\sqrt{ax - 1})/((a^{13}x^{10} - 5a^{11}x^8 + (ax + 1)^{5/2}(ax - 1)^{5/2})a^8x^5 + 10a^9x^6 - 10a^7x^4 + 5a^5x^2 + 5(a^9x^6 - a^7x^4)(ax + 1)^2(ax - 1)^2 + 10(a^{10}x^7 - 2a^8x^5 + a^6x^3)(ax + 1)^{3/2}(ax - 1)^{3/2} + 10(a^{11}x^8 - 3a^9x^6 + 3a^7x^4 - a^5x^2)(ax + 1)(ax - 1) - a^3 + 5(a^{12}x^9 - 4a^{10}x^7 + 6a^8x^5 - 4a^6x^3 + a^4x)\sqrt{ax + 1})\sqrt{ax - 1})\log(ax + \sqrt{ax + 1})\sqrt{ax - 1})^3) + \text{integrate}(1/6(64a^{15}x^{15} - 384a^{13}x^{13} + 960a^{11}x^{11} - 1280a^9x^9 + 960a^7x^7 - 384a^5x^5 + 8(8a^9x^9 - 7a^7x^7)(ax + 1)^3(ax - 1)^3 + (384a^{10}x^{10} - 664a^8x^8 + 308a^6x^6 - 12a^4x^4 - 9a^2x^2)(ax + 1)^{5/2}(ax - 1)^{5/2} + 64a^3x^3 + 2(480a^{11}x^{11} - 1240a^9x^9 + 1096a^7x^7 - 360a^5x^5 + 15a^3x^3 + 9ax)(ax + 1)^2(ax - 1)^2 + 2(640a^{12}x^{12} - 2200a^{10}x^{10} + 2844a^8x^8 - 1684a^6x^6 + 433a^4x^4 - 36a^2x^2 + 3)(ax + 1)^{3/2}(ax - 1)^{3/2} + 2(480a^{13}x^{13} - 2060a^{11}x^{11} + 3496a^9x^9 - 2952a^7x^7 + 1283a^5x^5 - 274a^3x^3 + 27ax)(ax + 1)(ax - 1) + (384a^{14}x^{14} - 1976a^{12}x^{12} + 4148a^{10}x^{10} - 4524a^8x^8 + 2699a^6x^6 - 842a^4x^4 + 111a^2x^2)\sqrt{ax + 1})\sqrt{ax - 1})/((a^{15}x^{12} - 6a^{13}x^{10} + (ax + 1)^3(ax - 1)^3a^9x^6 + 15a^{11}x^8 - 20a^9x^6 + 15a^7x^4 - 6a^5x^2 + 6(a^{10}x^7 - a^8x^5)(ax + 1)^{5/2}(ax - 1)^{5/2} + 15(a^{11}x^8 - 2a^9x^6 + a^7x^4)(ax + 1)^2(ax - 1)^2 + 20(a^{12}x^9 - 3a^{10}x^7 + 3a^8x^5 - a^6x^3)(ax + 1)^{3/2}(ax - 1)^{3/2} + 15(a^{13}x^{10} - 4a^{11}x^8 + 6a^9x^6 - 4a^7x^4 + a^5x^2)(ax + 1)(ax - 1) + a^3 + 6(a^{14}x^{11} - 5a^{12}x^9 + 10a^{10}x^7 - 10a^8x^5 + 5a^6x^3 - a^4x)\sqrt{ax + 1})\sqrt{ax - 1})\log(ax + \sqrt{ax + 1})\sqrt{ax - 1})), x)
\end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccosh(a*x)^4,x, algorithm="fricas")`

[Out] `integral(x^3/arccosh(a*x)^4, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/acosh(a*x)**4,x)

[Out] Integral(x**3/acosh(a*x)**4, x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acosh(a*x)^4,x)

[Out] int(x^3/acosh(a*x)^4, x)

3.67 $\int \frac{x^2}{\cosh^{-1}(ax)^4} dx$

Optimal. Leaf size=153

$$-\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{x}{3a^2\cosh^{-1}(ax)^2} - \frac{x^3}{2\cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\cosh^{-1}(ax)} - \frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}}{2a\cosh^{-1}(ax)}$$

[Out] $1/3*x/a^2/\operatorname{arccosh}(a*x)^2 - 1/2*x^3/\operatorname{arccosh}(a*x)^2 + 1/24*\operatorname{Chi}(\operatorname{arccosh}(a*x))/a^3 + 9/8*\operatorname{Chi}(3*\operatorname{arccosh}(a*x))/a^3 - 1/3*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^3 + 1/3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3/\operatorname{arccosh}(a*x) - 3/2*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)$

Rubi [A]

time = 0.44, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5886, 5951, 5885, 3382, 5880, 5953}

$$\frac{\operatorname{Chi}(\cosh^{-1}(ax))}{24a^3} + \frac{9\operatorname{Chi}(3\cosh^{-1}(ax))}{8a^3} + \frac{\sqrt{ax-1}\sqrt{ax+1}}{3a^3\cosh^{-1}(ax)} + \frac{x}{3a^2\cosh^{-1}(ax)^2} - \frac{x^3}{2\cosh^{-1}(ax)^2} - \frac{3x^2\sqrt{ax-1}\sqrt{ax+1}}{2a\cosh^{-1}(ax)} - \frac{x^2\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/\operatorname{ArcCosh}[a*x]^4, x]$

[Out] $-1/3*(x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a*\operatorname{ArcCosh}[a*x]^3) + x/(3*a^2*\operatorname{ArcCosh}[a*x]^2) - x^3/(2*\operatorname{ArcCosh}[a*x]^2) + (\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a^3*\operatorname{ArcCosh}[a*x]) - (3*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(2*a*\operatorname{ArcCosh}[a*x]) + \operatorname{CoshIntegral}[\operatorname{ArcCosh}[a*x]]/(24*a^3) + (9*\operatorname{CoshIntegral}[3*\operatorname{ArcCosh}[a*x]])/(8*a^3)$

Rule 3382

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[c*f*(fz/d) + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

Rule 5880

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[-1 + c*x]*((a + b*\operatorname{ArcCosh}[c*x])^{(n+1)})/(b*c*(n+1)), x] - \operatorname{Dist}[c/(b*(n+1)), \operatorname{Int}[x*((a + b*\operatorname{ArcCosh}[c*x])^{(n+1)})/(\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[-1 + c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{LtQ}[n, -1]$

Rule 5885

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_)]*(b_.)]^{(n_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^m*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[-1 + c*x]*((a + b*\operatorname{ArcCosh}[c*x])^{(n+1)})/(b*c*(n+1))$

)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\cosh^{-1}(ax)^4} dx &= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{2 \int \frac{x}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3} dx}{3a} + a \int \frac{1}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x}{3a^2 \cosh^{-1}(ax)^2} - \frac{x^3}{2 \cosh^{-1}(ax)^2} + \frac{3}{2} \int \frac{x^2}{\cosh^{-1}(ax)^2} dx \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x}{3a^2 \cosh^{-1}(ax)^2} - \frac{x^3}{2 \cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a^3 \cosh^{-1}(ax)} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x}{3a^2 \cosh^{-1}(ax)^2} - \frac{x^3}{2 \cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a^3 \cosh^{-1}(ax)} \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{x}{3a^2 \cosh^{-1}(ax)^2} - \frac{x^3}{2 \cosh^{-1}(ax)^2} + \frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a^3 \cosh^{-1}(ax)}
\end{aligned}$$

Mathematica [A]

time = 0.28, size = 183, normalized size = 1.20

$$\frac{\sqrt{-1+ax} \left(-4 \sqrt{\frac{-1+ax}{1+ax}} (2a^2x^2(-1+a^2x^2) + ax\sqrt{-1+ax}\sqrt{1+ax}(-2+3a^2x^2)\cosh^{-1}(ax) + (2-11a^2x^2+9a^4x^4)\cosh^{-1}(ax)^2) + (-1+ax)\cosh^{-1}(ax)^3\text{Chi}(\cosh^{-1}(ax)) + 27(-1+ax)\cosh^{-1}(ax)^3\text{Chi}(3\cosh^{-1}(ax)) \right)}{24a^3 \left(\frac{-1+ax}{1+ax} \right)^{3/2} (1+ax)^{3/2} \cosh^{-1}(ax)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/ArcCosh[a*x]^4,x]

[Out] (Sqrt[-1 + a*x]*(-4*Sqrt[(-1 + a*x)/(1 + a*x)]*(2*a^2*x^2*(-1 + a^2*x^2) + a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-2 + 3*a^2*x^2)*ArcCosh[a*x] + (2 - 11*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x]^2) + (-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral[ArcCosh[a*x]] + 27*(-1 + a*x)*ArcCosh[a*x]^3*CoshIntegral[3*ArcCosh[a*x]])/(24*a^3*((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^3)

Maple [A]

time = 1.82, size = 121, normalized size = 0.79

method	result
derivativedivides	$ -\frac{\sqrt{ax-1} \sqrt{ax+1}}{12\text{arccosh}(ax)^3} - \frac{ax}{24\text{arccosh}(ax)^2} - \frac{\sqrt{ax-1} \sqrt{ax+1}}{24 \text{arccosh}(ax)} + \frac{\text{hyperbolicCosineIntegral}(\text{arccosh}(ax))}{24} - \frac{\sinh(3 \text{arccosh}(ax))}{12a^3} $
default	$ -\frac{\sqrt{ax-1} \sqrt{ax+1}}{12\text{arccosh}(ax)^3} - \frac{ax}{24\text{arccosh}(ax)^2} - \frac{\sqrt{ax-1} \sqrt{ax+1}}{24 \text{arccosh}(ax)} + \frac{\text{hyperbolicCosineIntegral}(\text{arccosh}(ax))}{24} - \frac{\sinh(3 \text{arccosh}(ax))}{12a^3} $

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccosh(a*x)^4,x,method=_RETURNVERBOSE)

[Out] $1/a^3*(-1/12/\operatorname{arccosh}(a*x)^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}-1/24/\operatorname{arccosh}(a*x)^2*a*x-1/24/\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}+1/24*\operatorname{Chi}(\operatorname{arccosh}(a*x))-1/12/\operatorname{arccosh}(a*x)^3*\sinh(3*\operatorname{arccosh}(a*x))-1/8/\operatorname{arccosh}(a*x)^2*\cosh(3*\operatorname{arccosh}(a*x))-3/8/\operatorname{arccosh}(a*x)*\sinh(3*\operatorname{arccosh}(a*x))+9/8*\operatorname{Chi}(3*\operatorname{arccosh}(a*x)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/arccosh(a*x)^4,x, algorithm="maxima")`

[Out] $-1/6*(2*a^{13}*x^{13} - 10*a^{11}*x^{11} + 20*a^9*x^9 - 20*a^7*x^7 + 10*a^5*x^5 + 2*(a^8*x^8 - a^6*x^6)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} - 2*a^3*x^3 + 2*(5*a^9*x^9 - 9*a^7*x^7 + 4*a^5*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^{10}*x^{10} - 13*a^8*x^8 + 11*a^6*x^6 - 3*a^4*x^4)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 4*(5*a^{11}*x^{11} - 17*a^9*x^9 + 21*a^7*x^7 - 11*a^5*x^5 + 2*a^3*x^3)*(a*x + 1)*(a*x - 1) + (9*a^{13}*x^{13} - 45*a^{11}*x^{11} + 90*a^9*x^9 - 90*a^7*x^7 + 45*a^5*x^5 + (9*a^8*x^8 - 13*a^6*x^6 + 3*a^4*x^4 + a^2*x^2)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} - 9*a^3*x^3 + (45*a^9*x^9 - 97*a^7*x^7 + 64*a^5*x^5 - 10*a^3*x^3 - 2*a*x)*(a*x + 1)^2*(a*x - 1)^2 + (90*a^{10}*x^{10} - 258*a^8*x^8 + 264*a^6*x^6 - 113*a^4*x^4 + 19*a^2*x^2 - 2)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(45*a^{11}*x^{11} - 161*a^9*x^9 + 219*a^7*x^7 - 141*a^5*x^5 + 44*a^3*x^3 - 6*a*x)*(a*x + 1)*(a*x - 1) + (45*a^{12}*x^{12} - 193*a^{10}*x^{10} + 325*a^8*x^8 - 270*a^6*x^6 + 112*a^4*x^4 - 19*a^2*x^2)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))*\log(a*x + \operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))^2 + 2*(5*a^{12}*x^{12} - 21*a^{10}*x^{10} + 34*a^8*x^8 - 26*a^6*x^6 + 9*a^4*x^4 - a^2*x^2)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1) + (3*a^{13}*x^{13} - 15*a^{11}*x^{11} + 30*a^9*x^9 - 30*a^7*x^7 + 15*a^5*x^5 + (3*a^8*x^8 - 4*a^6*x^6 + a^4*x^4)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} - 3*a^3*x^3 + (15*a^9*x^9 - 31*a^7*x^7 + 20*a^5*x^5 - 4*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + (30*a^{10}*x^{10} - 84*a^8*x^8 + 84*a^6*x^6 - 35*a^4*x^4 + 5*a^2*x^2)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(15*a^{11}*x^{11} - 53*a^9*x^9 + 71*a^7*x^7 - 44*a^5*x^5 + 12*a^3*x^3 - a*x)*(a*x + 1)*(a*x - 1) + (15*a^{12}*x^{12} - 64*a^{10}*x^{10} + 107*a^8*x^8 - 87*a^6*x^6 + 34*a^4*x^4 - 5*a^2*x^2)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))*\log(a*x + \operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1)))/((a^{13}*x^{10} - 5*a^{11}*x^8 + (a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)}*a^8*x^5 + 10*a^9*x^6 - 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 - a^7*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 10*(a^{10}*x^7 - 2*a^8*x^5 + a^6*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 10*(a^{11}*x^8 - 3*a^9*x^6 + 3*a^7*x^4 - a^5*x^2)*(a*x + 1)*(a*x - 1) - a^3 + 5*(a^{12}*x^9 - 4*a^{10}*x^7 + 6*a^8*x^5 - 4*a^6*x^3 + a^4*x)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))*\log(a*x + \operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))^3) + \operatorname{integrate}(1/6*(27*a^{14}*x^{14} - 162*a^{12}*x^{12} + 405*a^{10}*x^{10} - 540*a^8*x^8 + 405*a^6*x^6 - 162*a^4*x^4 + (27*a^8*x^8 - 13*a^6*x^6 - 3*a^4*x^4 - 3*a^2*x^2)*(a*x + 1)^3*(a*x - 1)^3 + (162*a^9*x^9 - 227*a^7*x^7 + 63*a^5*x^5 + 3*a^3*x^3 + 6*a*x)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + (405*a^{10}*x$

$$\begin{aligned} & ^{10} - 940a^8x^8 + 687a^6x^6 - 143a^4x^4 - 21a^2x^2 + 12)(ax + 1)^2 \\ & 2(ax - 1)^2 + (540a^{11}x^{11} - 1750a^9x^9 + 2058a^7x^7 - 1017a^5x^5 \\ & + 145a^3x^3 + 24ax)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + 27a^2x^2 + (40 \\ & 5a^{12}x^{12} - 1685a^{10}x^{10} + 2727a^8x^8 - 2118a^6x^6 + 782a^4x^4 - \\ & 123a^2x^2 + 12)(ax + 1)(ax - 1) + (162a^{13}x^{13} - 823a^{11}x^{11} + 16 \\ & 95a^9x^9 - 1790a^7x^7 + 1015a^5x^5 - 297a^3x^3 + 38ax)\sqrt{ax + 1} \\ & \sqrt{ax - 1}) / ((a^{14}x^{12} - 6a^{12}x^{10} + (ax + 1)^3(ax - 1)^3a^8x^6 \\ & + 15a^{10}x^8 - 20a^8x^6 + 15a^6x^4 + 6(a^9x^7 - a^7x^5)(ax + 1)^{(5/2)} \\ & (ax - 1)^{(5/2)} - 6a^4x^2 + 15(a^{10}x^8 - 2a^8x^6 + a^6x^4)(ax + 1)^2 \\ & (ax - 1)^2 + 20(a^{11}x^9 - 3a^9x^7 + 3a^7x^5 - a^5x^3)(ax + 1)^{(3/2)} \\ & (ax - 1)^{(3/2)} + 15(a^{12}x^{10} - 4a^{10}x^8 + 6a^8x^6 - 4a^6x^4 + a^4x^2) \\ & (ax + 1)(ax - 1) + 6(a^{13}x^{11} - 5a^{11}x^9 + 10a^9x^7 - 10a^7x^5 + 5a^5x^3 \\ & - a^3x)\sqrt{ax + 1}\sqrt{ax - 1} + a^2) \log(ax + \sqrt{ax + 1}\sqrt{ax - 1})), x \end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(ax)^4,x, algorithm="fricas")

[Out] integral(x^2/arccosh(ax)^4, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/acosh(ax)**4,x)

[Out] Integral(x**2/acosh(ax)**4, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(ax)^4,x, algorithm="giac")

[Out] integrate(x^2/arccosh(ax)^4, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/acosh(a*x)^4,x)

[Out] int(x^2/acosh(a*x)^4, x)

3.68 $\int \frac{x}{\cosh^{-1}(ax)^4} dx$

Optimal. Leaf size=105

$$-\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)} + \frac{2\text{Chi}(2\cosh^{-1}(ax))}{3a^2}$$

[Out] 1/6/a^2/arccosh(a*x)^2-1/3*x^2/arccosh(a*x)^2+2/3*Chi(2*arccosh(a*x))/a^2-1/3*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^3-2/3*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)

Rubi [A]

time = 0.25, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5886, 5951, 5885, 3382, 5893}

$$\frac{2\text{Chi}(2\cosh^{-1}(ax))}{3a^2} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} - \frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)} - \frac{x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCosh[a*x]^4,x]

[Out] -1/3*(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*ArcCosh[a*x]^3) + 1/(6*a^2*ArcCosh[a*x]^2) - x^2/(3*ArcCosh[a*x]^2) - (2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*ArcCosh[a*x]) + (2*CoshIntegral[2*ArcCosh[a*x]])/(3*a^2)

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*(m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x])

```
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]),
x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/(Sqrt[(d1_.) + (e1_.)*(x_.)]*Sqr
rt[(d2_.) + (e2_.)*(x_.)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_.)]*Sqrt[(d2_.) + (e2_.)*(x_.)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cosh^{-1}(ax)^4} dx &= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} - \frac{\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3} dx}{3a} + \frac{1}{3}(2a) \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} + \frac{2}{3} \int \frac{x}{\cosh^{-1}(ax)^2} dx \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)} \\
&= -\frac{x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} + \frac{1}{6a^2\cosh^{-1}(ax)^2} - \frac{x^2}{3\cosh^{-1}(ax)^2} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 131, normalized size = 1.25

$$\frac{2ax - 2a^3x^3 - \sqrt{-1+ax}\sqrt{1+ax} \frac{(-1+2a^2x^2)\cosh^{-1}(ax) + (4ax-4a^3x^3)\cosh^{-1}(ax)^2}{\cosh^{-1}(ax)^3} + 4\sqrt{\frac{-1+ax}{1+ax}}(1+ax)\text{Chi}(2\cosh^{-1}(ax))}{6a^2\sqrt{-1+ax}\sqrt{1+ax}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/ArcCosh[a*x]^4,x]

[Out] $((2ax - 2a^3x^3 - \sqrt{-1 + ax})\sqrt{1 + ax}(-1 + 2a^2x^2)\text{ArcCosh}[ax] + (4ax - 4a^3x^3)\text{ArcCosh}[ax]^2)/\text{ArcCosh}[ax]^3 + 4\sqrt{(-1 + ax)/(1 + ax)}(1 + ax)\text{CoshIntegral}[2\text{ArcCosh}[ax]])/(6a^2\sqrt{-1 + ax})\sqrt{1 + ax}$

Maple [A]

time = 2.28, size = 60, normalized size = 0.57

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2 \operatorname{arccosh}(ax))}{6 \operatorname{arccosh}(ax)^3} - \frac{\cosh(2 \operatorname{arccosh}(ax))}{6 \operatorname{arccosh}(ax)^2} - \frac{\sinh(2 \operatorname{arccosh}(ax))}{3 \operatorname{arccosh}(ax)} + \frac{2 \operatorname{hyperbolicCosineIntegral}(2 \operatorname{arccosh}(ax))}{3}}{a^2}$	60
default	$\frac{-\frac{\sinh(2 \operatorname{arccosh}(ax))}{6 \operatorname{arccosh}(ax)^3} - \frac{\cosh(2 \operatorname{arccosh}(ax))}{6 \operatorname{arccosh}(ax)^2} - \frac{\sinh(2 \operatorname{arccosh}(ax))}{3 \operatorname{arccosh}(ax)} + \frac{2 \operatorname{hyperbolicCosineIntegral}(2 \operatorname{arccosh}(ax))}{3}}{a^2}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccosh(a*x)^4,x,method=_RETURNVERBOSE)

[Out] $1/a^2*(-1/6/\operatorname{arccosh}(ax)^3*\sinh(2*\operatorname{arccosh}(ax))-1/6/\operatorname{arccosh}(ax)^2*\cosh(2*a*\operatorname{rccosh}(ax))-1/3/\operatorname{arccosh}(ax)*\sinh(2*\operatorname{arccosh}(ax))+2/3*\Chi(2*\operatorname{arccosh}(ax)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^4,x, algorithm="maxima")

[Out] $-1/6*(2a^{12}x^{12} - 10a^{10}x^{10} + 20a^8x^8 - 20a^6x^6 + 10a^4x^4 + 2*(a^7x^7 - a^5x^5)*(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + 2*(5a^8x^8 - 9a^6x^6 + 4a^4x^4)*(ax + 1)^2(ax - 1)^2 + 4*(5a^9x^9 - 13a^7x^7 + 11a^5x^5 - 3a^3x^3)*(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} - 2a^2x^2 + 4*(5a^{10}x^{10} - 17a^8x^8 + 21a^6x^6 - 11a^4x^4 + 2a^2x^2)*(ax + 1)*(ax - 1) + (4a^{12}x^{12} - 20a^{10}x^{10} + 40a^8x^8 - 40a^6x^6 + 20a^4x^4 + 4*(a^7x^7 - a^5x^5)*(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + (20a^8x^8 - 36a^6x^6 + 16a^4x^4 + 3a^2x^2 - 3)*(ax + 1)^2(ax - 1)^2 + (40a^9x^9 - 104a^7x^7 + 88a^5x^5 - 21a^3x^3 - 3ax)*(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} - 4a^2x^2 + (40a^{10}x^{10} - 136a^8x^8 + 168a^6x^6 - 91a^4x^4 + 22a^2x^2 - 3)*(ax + 1)*(ax - 1) + (20a^{11}x^{11} - 84a^9x^9 + 136a^7x^7 - 107a^5x^5 + 42a^3x^3 - 7ax)*\sqrt{ax + 1}\sqrt{ax - 1})*\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})^2 + 2*(5a^{11}x^{11} - 21a^9x^9 + 34a^7x^7 - 26a^5x^5 + 9a^3x^3 - ax)*\sqrt{ax + 1}\sqrt{ax - 1} + (2a^{12}x^{12} - 10a^{10}x^{10} + 20a^8x^8 - 20a^6x^6 + 10a^4x^4 + 2*(a^7x^7 - a^5x^5)*(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + (10a^8x^8 - 18a^6x^6 + 9a^4x^4$

$$\begin{aligned} &^4 - a^2x^2)(ax + 1)^2(ax - 1)^2 + (20a^9x^9 - 52a^7x^7 + 47a^5x^5 \\ &- 17a^3x^3 + 2ax)(ax + 1)^{3/2}(ax - 1)^{3/2} - 2a^2x^2 + (20a^{10}x^{10} - 68a^8x^8 + 87a^6x^6 - 51a^4x^4 + 13a^2x^2 - 1)(ax + 1) \\ &)(ax - 1) + (10a^{11}x^{11} - 42a^9x^9 + 69a^7x^7 - 55a^5x^5 + 21a^3x^3 - 3ax) \sqrt{ax + 1} \sqrt{ax - 1} \log(ax + \sqrt{ax + 1} \sqrt{ax - 1})) / ((a^{12}x^{10} - 5a^{10}x^8 + (ax + 1)^{5/2}(ax - 1)^{5/2}a^7x^5 \\ &+ 10a^8x^6 - 10a^6x^4 + 5a^4x^2 + 5(a^8x^6 - a^6x^4)(ax + 1)^2(a \\ &ax - 1)^2 + 10(a^9x^7 - 2a^7x^5 + a^5x^3)(ax + 1)^{3/2}(ax - 1)^{3/2} + 10(a^{10}x^8 - 3a^8x^6 + 3a^6x^4 - a^4x^2)(ax + 1)(ax - 1) \\ &+ 5(a^{11}x^9 - 4a^9x^7 + 6a^7x^5 - 4a^5x^3 + a^3x) \sqrt{ax + 1} \sqrt{ax - 1} - a^2) \log(ax + \sqrt{ax + 1} \sqrt{ax - 1})^3 + \text{integrate}(1/ \\ &6(8a^{13}x^{13} - 48a^{11}x^{11} + 8(ax + 1)^3(ax - 1)^3a^7x^7 + 120a^9x^9 - 160a^7x^7 + 120a^5x^5 + (48a^8x^8 - 48a^6x^6 + 4a^4x^4 - 1 \\ &2a^2x^2 + 15)(ax + 1)^{5/2}(ax - 1)^{5/2} - 48a^3x^3 + 8(15a^9x^9 - 30a^7x^7 + 17a^5x^5 - 5a^3x^3 + 3ax)(ax + 1)^2(ax - 1)^2 + \\ &2(80a^{10}x^{10} - 240a^8x^8 + 252a^6x^6 - 104a^4x^4 + 3a^2x^2 + 9)(ax + 1)^{3/2}(ax - 1)^{3/2} + 8(15a^{11}x^{11} - 60a^9x^9 + 92a^7x^7 \\ &- 63a^5x^5 + 15a^3x^3 + ax)(ax + 1)(ax - 1) + (48a^{12}x^{12} - 240a^{10}x^{10} + 484a^8x^8 - 484a^6x^6 + 243a^4x^4 - 58a^2x^2 + 7) \sqrt{ax + 1} \sqrt{ax - 1} + 8ax) / ((a^{13}x^{12} - 6a^{11}x^{10} + (ax + 1)^3(a \\ &ax - 1)^3a^7x^6 + 15a^9x^8 - 20a^7x^6 + 15a^5x^4 + 6(a^8x^7 - a^6x^5)(ax + 1)^{5/2}(ax - 1)^{5/2} + 15(a^9x^8 - 2a^7x^6 + a^5x^4) \\ &(ax + 1)^2(ax - 1)^2 - 6a^3x^2 + 20(a^{10}x^9 - 3a^8x^7 + 3a^6x^5 - a^4x^3)(ax + 1)^{3/2}(ax - 1)^{3/2} + 15(a^{11}x^{10} - 4a^9x^8 + 6a^7x^6 - 4a^5x^4 + a^3x^2)(ax + 1)(ax - 1) + 6(a^{12}x^{11} - 5a^{10}x^9 + 10a^8x^7 - 10a^6x^5 + 5a^4x^3 - a^2x) \sqrt{ax + 1} \sqrt{ax - 1} + a) \log(ax + \sqrt{ax + 1} \sqrt{ax - 1})), x) \end{aligned}$$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(ax)^4,x, algorithm="fricas")

[Out] integral(x/arccosh(ax)^4, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(ax)**4,x)

[Out] Integral(x/acosh(a*x)**4, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^4,x, algorithm="giac")

[Out] integrate(x/arccosh(a*x)^4, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x)^4,x)

[Out] int(x/acosh(a*x)^4, x)

$$3.69 \quad \int \frac{1}{\cosh^{-1}(ax)^4} dx$$

Optimal. Leaf size=86

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^3} - \frac{x}{6\cosh^{-1}(ax)^2} - \frac{\sqrt{-1+ax}\sqrt{1+ax}}{6a\cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{6a}$$

[Out] $-1/6*x/\text{arccosh}(a*x)^2 + 1/6*\text{Chi}(\text{arccosh}(a*x))/a - 1/3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\text{arccosh}(a*x)^3 - 1/6*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\text{arccosh}(a*x)$

Rubi [A]

time = 0.23, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5880, 5951, 5953, 3382}

$$\frac{\text{Chi}(\cosh^{-1}(ax))}{6a} - \frac{x}{6\cosh^{-1}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{6a\cosh^{-1}(ax)} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]^(-4),x]`

[Out] $-1/3*(\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(a*\text{ArcCosh}[a*x]^3) - x/(6*\text{ArcCosh}[a*x]^2) - (\text{Sqrt}[-1+a*x]*\text{Sqrt}[1+a*x])/(6*a*\text{ArcCosh}[a*x]) + \text{CoshIntegral}[\text{ArcCosh}[a*x]]/(6*a)$

Rule 3382

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]`

Rule 5880

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_), x_Symbol] := Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]`

Rule 5951

`Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_)*((f_.)*(x_))^ (m_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1}`

, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.)*((d1_) + (e1_.)*(x_.))^(p_.)*((d2_) + (e2_.)*(x_.))^(q_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-1 + c*x)^q], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\cosh^{-1}(ax)^4} dx &= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} + \frac{1}{3}a \int \frac{x}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^3} dx \\ &= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{x}{6 \cosh^{-1}(ax)^2} + \frac{1}{6} \int \frac{1}{\cosh^{-1}(ax)^2} dx \\ &= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{x}{6 \cosh^{-1}(ax)^2} - \frac{\sqrt{-1+ax} \sqrt{1+ax}}{6a \cosh^{-1}(ax)} + \frac{1}{6}a \int \frac{1}{\sqrt{-1+ax}} dx \\ &= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{x}{6 \cosh^{-1}(ax)^2} - \frac{\sqrt{-1+ax} \sqrt{1+ax}}{6a \cosh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} dx\right)}{6} \\ &= -\frac{\sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^3} - \frac{x}{6 \cosh^{-1}(ax)^2} - \frac{\sqrt{-1+ax} \sqrt{1+ax}}{6a \cosh^{-1}(ax)} + \frac{\text{Chi}(\cosh^{-1}(ax))}{6a} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 114, normalized size = 1.33

$$\frac{2-2a^2x^2-ax\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)+(1-a^2x^2) \cosh^{-1}(ax)^2}{\cosh^{-1}(ax)^3} + \sqrt{\frac{-1+ax}{1+ax}} (1+ax)\text{Chi}(\cosh^{-1}(ax))}{6a\sqrt{-1+ax} \sqrt{1+ax}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^(-4), x]

[Out] ((2 - 2*a^2*x^2 - a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + (1 - a^2*x^2)*ArcCosh[a*x]^2)/ArcCosh[a*x]^3 + Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*CoshIntegral[ArcCosh[a*x]])/(6*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])

Maple [A]

time = 1.83, size = 67, normalized size = 0.78

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{3\operatorname{arccosh}(ax)^3} - \frac{ax}{6\operatorname{arccosh}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{6\operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(\operatorname{arccosh}(ax))}{6}}{a}$	67
default	$\frac{-\frac{\sqrt{ax-1}\sqrt{ax+1}}{3\operatorname{arccosh}(ax)^3} - \frac{ax}{6\operatorname{arccosh}(ax)^2} - \frac{\sqrt{ax-1}\sqrt{ax+1}}{6\operatorname{arccosh}(ax)} + \frac{\operatorname{hyperbolicCosineIntegral}(\operatorname{arccosh}(ax))}{6}}{a}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/arccosh(a*x)^4,x,method=_RETURNVERBOSE)`

[Out] $1/a*(-1/3/\operatorname{arccosh}(a*x)^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}-1/6/\operatorname{arccosh}(a*x)^2*a*x-1/6/\operatorname{arccosh}(a*x)*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}+1/6*\operatorname{Chi}(\operatorname{arccosh}(a*x)))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^4,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/6*(2*a^{11}*x^{11} - 10*a^9*x^9 + 20*a^7*x^7 - 20*a^5*x^5 + 2*(a^6*x^6 - a^4 \\ & *x^4)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + 10*a^3*x^3 + 2*(5*a^7*x^7 - 9*a^5*x \\ & ^5 + 4*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^8*x^8 - 13*a^6*x^6 + 11*a^ \\ & 4*x^4 - 3*a^2*x^2)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 4*(5*a^9*x^9 - 17*a^7* \\ & x^7 + 21*a^5*x^5 - 11*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (a^{11}*x^{11} - 5 \\ & *a^9*x^9 + 10*a^7*x^7 - 10*a^5*x^5 + (a^6*x^6 - a^4*x^4 + 3*a^2*x^2 - 3)*(a \\ & *x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + 5*a^3*x^3 + (5*a^7*x^7 - 9*a^5*x^5 + 10*a^3 \\ & *x^3 - 6*a*x)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^8*x^8 - 26*a^6*x^6 + 22*a^4*x \\ & ^4 - 3*a^2*x^2 - 3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(5*a^9*x^9 - 17*a^7 \\ & *x^7 + 18*a^5*x^5 - 5*a^3*x^3 - a*x)*(a*x + 1)*(a*x - 1) + (5*a^{10}*x^{10} - 2 \\ & 1*a^8*x^8 + 31*a^6*x^6 - 20*a^4*x^4 + 6*a^2*x^2 - 1)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x \\ & - 1) - a*x)*\log(a*x + \operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1))^2 + 2*(5*a^{10}*x^{10} - 21 \\ & *a^8*x^8 + 34*a^6*x^6 - 26*a^4*x^4 + 9*a^2*x^2 - 1)*\operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x \\ & - 1) - 2*a*x + (a^{11}*x^{11} - 5*a^9*x^9 + 10*a^7*x^7 - 10*a^5*x^5 + (a^6*x^6 \\ & - a^2*x^2)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + 5*a^3*x^3 + (5*a^7*x^7 - 5*a^5 \\ & *x^5 - 2*a^3*x^3 + 2*a*x)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^8*x^8 - 20*a^6*x^ \\ & 6 + 10*a^4*x^4 + a^2*x^2 - 1)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(5*a^9*x^ \\ & 9 - 15*a^7*x^7 + 16*a^5*x^5 - 7*a^3*x^3 + a*x)*(a*x + 1)*(a*x - 1) + (5*a^1 \\ & 0*x^{10} - 20*a^8*x^8 + 31*a^6*x^6 - 23*a^4*x^4 + 8*a^2*x^2 - 1)*\operatorname{sqrt}(a*x + 1 \\ &)*\operatorname{sqrt}(a*x - 1) - a*x)*\log(a*x + \operatorname{sqrt}(a*x + 1)*\operatorname{sqrt}(a*x - 1)))/((a^{11}*x^{10} \\ & - 5*a^9*x^8 + (a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)}*a^6*x^5 + 10*a^7*x^6 - 10*a^5 \\ & *x^4 + 5*(a^7*x^6 - a^5*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 5*a^3*x^2 + 10*(a^8*x \\ & ^7 - 2*a^6*x^5 + a^4*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 10*(a^9*x^8 - \end{aligned}$$

```

3*a^7*x^6 + 3*a^5*x^4 - a^3*x^2)*(a*x + 1)*(a*x - 1) + 5*(a^10*x^9 - 4*a^8*
x^7 + 6*a^6*x^5 - 4*a^4*x^3 + a^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3 + integrate(1/6*(a^12*x^12 - 6*a^10*x^
10 + 15*a^8*x^8 - 20*a^6*x^6 + 15*a^4*x^4 + (a^6*x^6 + a^4*x^4 - 9*a^2*x^2
+ 15)*(a*x + 1)^3*(a*x - 1)^3 + (6*a^7*x^7 - a^5*x^5 - 31*a^3*x^3 + 33*a*x)
*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + (15*a^8*x^8 - 20*a^6*x^6 - 19*a^4*x^4 +
3*a^2*x^2 + 21)*(a*x + 1)^2*(a*x - 1)^2 + (20*a^9*x^9 - 50*a^7*x^7 + 54*a^5
*x^5 - 59*a^3*x^3 + 35*a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - 6*a^2*x^2 + (
15*a^10*x^10 - 55*a^8*x^8 + 101*a^6*x^6 - 90*a^4*x^4 + 22*a^2*x^2 + 7)*(a*x
+ 1)*(a*x - 1) + (6*a^11*x^11 - 29*a^9*x^9 + 65*a^7*x^7 - 66*a^5*x^5 + 23*
a^3*x^3 + a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/((a^12*x^12 - 6*a^10*x^10 +
(a*x + 1)^3*(a*x - 1)^3*a^6*x^6 + 15*a^8*x^8 - 20*a^6*x^6 + 15*a^4*x^4 + 6
*(a^7*x^7 - a^5*x^5)*(a*x + 1)^(5/2)*(a*x - 1)^(5/2) + 15*(a^8*x^8 - 2*a^6*
x^6 + a^4*x^4)*(a*x + 1)^2*(a*x - 1)^2 + 20*(a^9*x^9 - 3*a^7*x^7 + 3*a^5*x^
5 - a^3*x^3)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - 6*a^2*x^2 + 15*(a^10*x^10 -
4*a^8*x^8 + 6*a^6*x^6 - 4*a^4*x^4 + a^2*x^2)*(a*x + 1)*(a*x - 1) + 6*(a^11*
x^11 - 5*a^9*x^9 + 10*a^7*x^7 - 10*a^5*x^5 + 5*a^3*x^3 - a*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)

```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^(-4), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**4,x)
```

```
[Out] Integral(acosh(a*x)**(-4), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(-4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acosh(a*x)^4,x)
```

```
[Out] int(1/acosh(a*x)^4, x)
```

$$3.70 \quad \int \frac{1}{x \cosh^{-1}(ax)^4} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)^4}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^4,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^4} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^4),x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^4), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^4} dx = \int \frac{1}{x \cosh^{-1}(ax)^4} dx$$

Mathematica [A]

time = 7.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)^4} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^4),x]

[Out] Integrate[1/(x*ArcCosh[a*x]^4), x]

Maple [A]

time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccosh(a*x)^4,x)

[Out] int(1/x/arccosh(a*x)^4,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^4,x, algorithm="maxima")

[Out]
$$-1/6*(2*a^{13}*x^{13} - 10*a^{11}*x^{11} + 20*a^9*x^9 - 20*a^7*x^7 + 10*a^5*x^5 + 2*(a^8*x^8 - a^6*x^6)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} - 2*a^3*x^3 + 2*(5*a^9*x^9 - 9*a^7*x^7 + 4*a^5*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^{10}*x^{10} - 13*a^8*x^8 + 11*a^6*x^6 - 3*a^4*x^4)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 4*(5*a^{11}*x^{11} - 17*a^9*x^9 + 21*a^7*x^7 - 11*a^5*x^5 + 2*a^3*x^3)*(a*x + 1)*(a*x - 1) - (4*(a^6*x^6 - 3*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + (16*a^7*x^7 - 46*a^5*x^5 + 37*a^3*x^3 - 7*a*x)*(a*x + 1)^2*(a*x - 1)^2 + (24*a^8*x^8 - 66*a^6*x^6 + 59*a^4*x^4 - 19*a^2*x^2 + 2)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + (16*a^9*x^9 - 42*a^7*x^7 + 39*a^5*x^5 - 16*a^3*x^3 + 3*a*x)*(a*x + 1)*(a*x - 1) + (4*a^{10}*x^{10} - 10*a^8*x^8 + 9*a^6*x^6 - 4*a^4*x^4 + a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1))*sqrt(a*x - 1))^2 + 2*(5*a^{12}*x^{12} - 21*a^{10}*x^{10} + 34*a^8*x^8 - 26*a^6*x^6 + 9*a^4*x^4 - a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1) + (2*(a^6*x^6 - a^4*x^4)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + (8*a^7*x^7 - 13*a^5*x^5 + 5*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + (12*a^8*x^8 - 27*a^6*x^6 + 19*a^4*x^4 - 4*a^2*x^2)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + (8*a^9*x^9 - 23*a^7*x^7 + 23*a^5*x^5 - 9*a^3*x^3 + a*x)*(a*x + 1)*(a*x - 1) + (2*a^{10}*x^{10} - 7*a^8*x^8 + 9*a^6*x^6 - 5*a^4*x^4 + a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1))*sqrt(a*x - 1)))/((a^{13}*x^{13} - 5*a^{11}*x^{11} + (a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)}*a^8*x^8 + 10*a^9*x^9 - 10*a^7*x^7 + 5*a^5*x^5 - a^3*x^3 + 5*(a^9*x^9 - a^7*x^7)*(a*x + 1)^2*(a*x - 1)^2 + 10*(a^{10}*x^{10} - 2*a^8*x^8 + a^6*x^6)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 10*(a^{11}*x^{11} - 3*a^9*x^9 + 3*a^7*x^7 - a^5*x^5)*(a*x + 1)*(a*x - 1) + 5*(a^{12}*x^{12} - 4*a^{10}*x^{10} + 6*a^8*x^8 - 4*a^6*x^6 + a^4*x^4)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1))*sqrt(a*x - 1))^3 + integrate(1/6*(8*(a^7*x^7 - 6*a^5*x^5 + 6*a^3*x^3)*(a*x + 1)^3*(a*x - 1)^3 + (40*a^8*x^8 - 204*a^6*x^6 + 228*a^4*x^4 - 57*a^2*x^2)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + 2*(40*a^9*x^9 - 168*a^7*x^7 + 200*a^5*x^5 - 87*a^3*x^3 + 15*a*x)*(a*x + 1)^2*(a*x - 1)^2 + 2*(40*a^{10}*x^{10} - 132*a^8*x^8 + 156*a^6*x^6 - 91*a^4*x^4 + 30*a^2*x^2 - 3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(20*a^{11}*x^{11} - 48*a^9*x^9 + 48*a^7*x^7 - 35*a^5*x^5 + 18*a^3*x^3 - 3*a*x)*(a*x + 1)*(a*x - 1) + (8*a^{12}*x^{12} - 12*a^{10}*x^{10} + 4*a^8*x^8 - 5*a^6*x^6 + 6*a^4*x^4 - a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^{15}*x^{16} - 6*a^{13}*x^{14} + (a*x + 1)^3*(a*x - 1)^3*a^9*x^{10} + 15*a^{11}*x^{12} - 20*a^9*x^{10} + 15*a^7*x^8 - 6*a^5*x^6 + a^3*x^4 + 6*(a^{10}*x^{11} - a^8*x^9)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + 1$$

$5*(a^{11}*x^{12} - 2*a^9*x^{10} + a^7*x^8)*(a*x + 1)^2*(a*x - 1)^2 + 20*(a^{12}*x^{13} - 3*a^{10}*x^{11} + 3*a^8*x^9 - a^6*x^7)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 15*(a^{13}*x^{14} - 4*a^{11}*x^{12} + 6*a^9*x^{10} - 4*a^7*x^8 + a^5*x^6)*(a*x + 1)*(a*x - 1) + 6*(a^{14}*x^{15} - 5*a^{12}*x^{13} + 10*a^{10}*x^{11} - 10*a^8*x^9 + 5*a^6*x^7 - a^4*x^5)*\sqrt{a*x + 1}*\sqrt{a*x - 1})*\log(a*x + \sqrt{a*x + 1}*\sqrt{a*x - 1}))$, x)

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^4,x, algorithm="fricas")

[Out] integral(1/(x*arccosh(a*x)^4), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/acosh(a*x)**4,x)

[Out] Integral(1/(x*acosh(a*x)**4), x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)^4,x, algorithm="giac")

[Out] integrate(1/(x*arccosh(a*x)^4), x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*acosh(a*x)^4),x)

[Out] int(1/(x*acosh(a*x)^4), x)

$$3.71 \quad \int \frac{1}{x^2 \cosh^{-1}(ax)^4} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \cosh^{-1}(ax)^4}, x\right)$$

[Out] Unintegrable(1/x^2/arccosh(a*x)^4,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*ArcCosh[a*x]^4),x]

[Out] Defer[Int][1/(x^2*ArcCosh[a*x]^4), x]

Rubi steps

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} dx = \int \frac{1}{x^2 \cosh^{-1}(ax)^4} dx$$

Mathematica [A]

time = 9.92, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cosh^{-1}(ax)^4} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*ArcCosh[a*x]^4),x]

[Out] Integrate[1/(x^2*ArcCosh[a*x]^4), x]

Maple [A]

time = 1.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arccosh(a*x)^4,x)

[Out] int(1/x^2/arccosh(a*x)^4,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccosh(a*x)^4,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/6*(2*a^{13}*x^{13} - 10*a^{11}*x^{11} + 20*a^9*x^9 - 20*a^7*x^7 + 10*a^5*x^5 + 2 \\ & *(a^8*x^8 - a^6*x^6)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} - 2*a^3*x^3 + 2*(5*a^9 \\ & *x^9 - 9*a^7*x^7 + 4*a^5*x^5)*(a*x + 1)^2*(a*x - 1)^2 + 4*(5*a^{10}*x^{10} - 13 \\ & *a^8*x^8 + 11*a^6*x^6 - 3*a^4*x^4)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 4*(5*a \\ & ^{11}*x^{11} - 17*a^9*x^9 + 21*a^7*x^7 - 11*a^5*x^5 + 2*a^3*x^3)*(a*x + 1)*(a*x \\ & - 1) + (a^{13}*x^{13} - 5*a^{11}*x^{11} + 10*a^9*x^9 - 10*a^7*x^7 + 5*a^5*x^5 + (a \\ & ^8*x^8 - 13*a^6*x^6 + 27*a^4*x^4 - 15*a^2*x^2)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5 \\ & /2)} - a^3*x^3 + (5*a^9*x^9 - 57*a^7*x^7 + 124*a^5*x^5 - 90*a^3*x^3 + 18*a*x \\ &)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^{10}*x^{10} - 98*a^8*x^8 + 220*a^6*x^6 - 189* \\ & a^4*x^4 + 63*a^2*x^2 - 6)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(5*a^{11}*x^{11} \\ & - 41*a^9*x^9 + 93*a^7*x^7 - 89*a^5*x^5 + 38*a^3*x^3 - 6*a*x)*(a*x + 1)*(a*x \\ & - 1) + (5*a^{12}*x^{12} - 33*a^{10}*x^{10} + 73*a^8*x^8 - 74*a^6*x^6 + 36*a^4*x^4 \\ & - 7*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x \\ & - 1))^2 + 2*(5*a^{12}*x^{12} - 21*a^{10}*x^{10} + 34*a^8*x^8 - 26*a^6*x^6 + 9*a^4*x \\ & ^4 - a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1) - (a^{13}*x^{13} - 5*a^{11}*x^{11} + 10*a \\ & ^9*x^9 - 10*a^7*x^7 + 5*a^5*x^5 + (a^8*x^8 - 4*a^6*x^6 + 3*a^4*x^4)*(a*x + \\ & 1)^{(5/2)}*(a*x - 1)^{(5/2)} - a^3*x^3 + (5*a^9*x^9 - 21*a^7*x^7 + 24*a^5*x^5 - \\ & 8*a^3*x^3)*(a*x + 1)^2*(a*x - 1)^2 + (10*a^{10}*x^{10} - 44*a^8*x^8 + 64*a^6*x \\ & ^6 - 37*a^4*x^4 + 7*a^2*x^2)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + 2*(5*a^{11}*x^{11} \\ & - 23*a^9*x^9 + 39*a^7*x^7 - 30*a^5*x^5 + 10*a^3*x^3 - a*x)*(a*x + 1)*(a*x \\ & - 1) + (5*a^{12}*x^{12} - 24*a^{10}*x^{10} + 45*a^8*x^8 - 41*a^6*x^6 + 18*a^4*x^4 \\ & - 3*a^2*x^2)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x \\ & - 1)))/((a^{13}*x^{14} - 5*a^{11}*x^{12} + (a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)}*a^8*x^9 \\ & + 10*a^9*x^{10} - 10*a^7*x^8 + 5*a^5*x^6 - a^3*x^4 + 5*(a^9*x^{10} - a^7*x^8)* \\ & (a*x + 1)^2*(a*x - 1)^2 + 10*(a^{10}*x^{11} - 2*a^8*x^9 + a^6*x^7)*(a*x + 1)^{(3 \\ & /2)}*(a*x - 1)^{(3/2)} + 10*(a^{11}*x^{12} - 3*a^9*x^{10} + 3*a^7*x^8 - a^5*x^6)*(a*x \\ & + 1)*(a*x - 1) + 5*(a^{12}*x^{13} - 4*a^{10}*x^{11} + 6*a^8*x^9 - 4*a^6*x^7 + a^4 \\ & *x^5)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3 \\ &) - integrate(1/6*(a^{15}*x^{15} - 6*a^{13}*x^{13} + 15*a^{11}*x^{11} - 20*a^9*x^9 + 15 \\ & *a^7*x^7 - 6*a^5*x^5 + (a^9*x^9 - 39*a^7*x^7 + 135*a^5*x^5 - 105*a^3*x^3)*(\\ & a*x + 1)^3*(a*x - 1)^3 + (6*a^{10}*x^{10} - 201*a^8*x^8 + 677*a^6*x^6 - 663*a^4 \\ & *x^4 + 174*a^2*x^2)*(a*x + 1)^{(5/2)}*(a*x - 1)^{(5/2)} + a^3*x^3 + (15*a^{11}*x^{11} \\ & - 420*a^9*x^9 + 1373*a^7*x^7 - 1565*a^5*x^5 + 705*a^3*x^3 - 108*a*x)*(a*x \\ & + 1)^2*(a*x - 1)^2 + (20*a^{12}*x^{12} - 450*a^{10}*x^{10} + 1422*a^8*x^8 - 1787* \end{aligned}$$

$$a^6x^6 + 1059a^4x^4 - 288a^2x^2 + 24)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + (15a^{13}x^{13} - 255a^{11}x^{11} + 773a^9x^9 - 1026a^7x^7 + 714a^5x^5 - 257a^3x^3 + 36ax)(ax + 1)(ax - 1) + (6a^{14}x^{14} - 69a^{12}x^{12} + 197a^{10}x^{10} - 266a^8x^8 + 201a^6x^6 - 83a^4x^4 + 14a^2x^2)\sqrt{(ax + 1)\sqrt{(ax - 1)}}/((a^{15}x^{17} - 6a^{13}x^{15} + (ax + 1)^3(ax - 1)^3 a^9x^{11} + 15a^{11}x^{13} - 20a^9x^{11} + 15a^7x^9 - 6a^5x^7 + a^3x^5 + 6(a^{10}x^{12} - a^8x^{10})(ax + 1)^{(5/2)}(ax - 1)^{(5/2)} + 15(a^{11}x^{13} - 2a^9x^{11} + a^7x^9)(ax + 1)^2(ax - 1)^2 + 20(a^{12}x^{14} - 3a^{10}x^{12} + 3a^8x^{10} - a^6x^8)(ax + 1)^{(3/2)}(ax - 1)^{(3/2)} + 15(a^{13}x^{15} - 4a^{11}x^{13} + 6a^9x^{11} - 4a^7x^9 + a^5x^7)(ax + 1)(ax - 1) + 6(a^{14}x^{16} - 5a^{12}x^{14} + 10a^{10}x^{12} - 10a^8x^{10} + 5a^6x^8 - a^4x^6)\sqrt{(ax + 1)\sqrt{(ax - 1))} \cdot \log(ax + \sqrt{(ax + 1)\sqrt{(ax - 1))}), x)$$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccosh(a*x)^4,x, algorithm="fricas")

[Out] integral(1/(x^2*arccosh(a*x)^4), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acosh}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/acosh(a*x)**4,x)

[Out] Integral(1/(x**2*acosh(a*x)**4), x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccosh(a*x)^4,x, algorithm="giac")

[Out] integrate(1/(x^2*arccosh(a*x)^4), x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{acosh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acosh(a*x)^4),x)
```

```
[Out] int(1/(x^2*acosh(a*x)^4), x)
```

3.72 $\int x^4 \sqrt{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=182

$$\frac{1}{5}x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{320a^5}$$

[Out] $-1/1600*\operatorname{erf}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\pi^{(1/2)}/a^5-1/1600*\operatorname{erfi}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\pi^{(1/2)}/a^5-1/192*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\pi^{(1/2)}/a^5-1/192*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\pi^{(1/2)}/a^5-1/32*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\pi^{(1/2)}/a^5-1/32*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\pi^{(1/2)}/a^5+1/5*x^5*\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.32, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5884, 5953, 3393, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{320a^5} - \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erfi}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{320a^5} + \frac{1}{5}x^5 \sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]], x]$

[Out] $(x^5*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/5 - (\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(32*a^5) - (\operatorname{Sqrt}[\pi/3]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(64*a^5) - (\operatorname{Sqrt}[\pi/5]*\operatorname{Erf}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(320*a^5) - (\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(32*a^5) - (\operatorname{Sqrt}[\pi/3]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(64*a^5) - (\operatorname{Sqrt}[\pi/5]*\operatorname{Erfi}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(320*a^5)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$\$UseGamma$]

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /;$ Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 3393

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int x^4 \sqrt{\cosh^{-1}(ax)} dx &= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{1}{10} a \int \frac{x^5}{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}} dx \\
&= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{10a^5} \\
&= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{5 \cosh(x)}{8\sqrt{x}} + \frac{5 \cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{10a^5} \\
&= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{160a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{32a^5} \\
&= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{320a^5} - \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{320a^5} \\
&= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-5x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{160a^5} - \frac{\text{Subst}\left(\int e^{5x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{160a^5} \\
&= \frac{1}{5} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{64a^5}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 162, normalized size = 0.89

$$\frac{3\sqrt{5} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -5 \cosh^{-1}(ax)\right) + 25\sqrt{3} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -3 \cosh^{-1}(ax)\right) + 150 \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -\cosh^{-1}(ax)\right) + 150 \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, \cosh^{-1}(ax)\right) + 25\sqrt{3} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, 3 \cosh^{-1}(ax)\right) + 3\sqrt{5} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, 5 \cosh^{-1}(ax)\right)}{2400a^5 \sqrt{-\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*Sqrt[ArcCosh[a*x]], x]

[Out] (3*Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -5*ArcCosh[a*x]] + 25*Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -3*ArcCosh[a*x]] + 150*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -ArcCosh[a*x]] + 150*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, ArcCosh[a*x]] + 25*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, 3*ArcCosh[a*x]] + 3*Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, 5*ArcCosh[a*x]])/(2400*a^5*Sqrt[-ArcCosh[a*x]])

Maple [F]

time = 7.43, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\operatorname{arccosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccosh(a*x)^(1/2),x)`

[Out] `int(x^4*arccosh(a*x)^(1/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^4*sqrt(arccosh(a*x)), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acosh(a*x)**(1/2),x)`

[Out] `Integral(x**4*sqrt(acosh(a*x)), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^4*sqrt(arccosh(a*x)), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*acosh(a*x)^(1/2),x)`

[Out] `int(x^4*acosh(a*x)^(1/2), x)`

3.73 $\int x^3 \sqrt{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=139

$$-\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\pi}}{32a^4}$$

[Out] $-1/64*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4 - 1/64*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4 - 1/256*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4 - 1/256*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4 - 3/32*\operatorname{arccosh}(a*x)^{(1/2)}/a^4 + 1/4*x^4*\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.28, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5884, 5953, 3393, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi} \operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\pi} \operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]], x]$

[Out] $(-3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(32*a^4) + (x^4*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/4 - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(256*a^4) - (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(32*a^4) - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(256*a^4) - (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(32*a^4)$

Rule 2211

$\operatorname{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x_Symbol] :> \operatorname{Simp}[F^{a*}\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x_Symbol] :> \operatorname{Simp}[F^{a*}\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{NegQ}[b]$

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{\cosh^{-1}(ax)} dx &= \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{1}{8}a \int \frac{x^4}{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}} dx \\
&= \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^4} \\
&= \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{8a^4} \\
&= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{64a^4} \\
&= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{128a^4} \\
&= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-4x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{64a^4} \\
&= -\frac{3\sqrt{\cosh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\sqrt{\frac{\pi}{2}}\right)}{256a^4}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 101, normalized size = 0.73

$$\frac{\sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -4 \cosh^{-1}(ax)\right) + 4\sqrt{2} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -2 \cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)} \left(4\sqrt{2} \Gamma\left(\frac{3}{2}, 2 \cosh^{-1}(ax)\right) + \Gamma\left(\frac{3}{2}, 4 \cosh^{-1}(ax)\right)\right)}{128a^4 \sqrt{-\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Sqrt[ArcCosh[a*x]], x]`

```
[Out] (Sqrt[ArcCosh[a*x]]*Gamma[3/2, -4*ArcCosh[a*x]] + 4*Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(4*Sqrt[2]*Gamma[3/2, 2*ArcCosh[a*x]] + Gamma[3/2, 4*ArcCosh[a*x]]))/(128*a^4*Sqrt[-ArcCosh[a*x]])
```

Maple [A]

time = 9.00, size = 150, normalized size = 1.08

method	result
--------	--------

default	$-\frac{\sqrt{2} \left(-8\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} a^2 x^2 + 4\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} + \pi \operatorname{erf}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) + \pi \operatorname{erfi}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) \right)}{64\sqrt{\pi} a^4}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*2^(1/2)*(-8*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*a^2*x^2+4*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)+Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))+Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2))/Pi^(1/2)/a^4-1/256*(-64*arccosh(a*x)^(1/2)*Pi^(1/2)*a^4*x^4+64*arccosh(a*x)^(1/2)*Pi^(1/2)*a^2*x^2+Pi*erf(2*arccosh(a*x)^(1/2))+Pi*erfi(2*arccosh(a*x)^(1/2))-8*arccosh(a*x)^(1/2)*Pi^(1/2))/Pi^(1/2)/a^4
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(acosh(a*x)), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccosh(a*x)^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acosh(a*x)^(1/2),x)`

[Out] `int(x^3*acosh(a*x)^(1/2), x)`

3.74 $\int x^2 \sqrt{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=120

$$\frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3} - \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3}$$

[Out] -1/144*erf(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/144*erfi(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/16*erf(arccosh(a*x)^(1/2))*Pi^(1/2)/a^3-1/16*erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a^3+1/3*x^3*arccosh(a*x)^(1/2)

Rubi [A]

time = 0.27, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5884, 5953, 3393, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3} - \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{48a^3} + \frac{1}{3}x^3\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[ArcCosh[a*x]],x]

[Out] (x^3*Sqrt[ArcCosh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(48*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(48*a^3)

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
  := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
  I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
  f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
  t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f,
  m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[
  x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
  x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
  , x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
  _))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
  Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
  [x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
  x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
  2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{\cosh^{-1}(ax)} dx &= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{1}{6} a \int \frac{x^3}{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}} dx \\
&= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{6a^3} \\
&= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3 \cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{6a^3} \\
&= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{24a^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^3} \\
&= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{48a^3} - \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{48a^3} \\
&= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-3x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{24a^3} - \frac{\text{Subst}\left(\int e^{3x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{24a^3} \\
&= \frac{1}{3} x^3 \sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^3} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{48a^3}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 100, normalized size = 0.83

$$\frac{\sqrt{3} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -3 \cosh^{-1}(ax)\right) + 9 \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -\cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)} \left(9 \Gamma\left(\frac{3}{2}, \cosh^{-1}(ax)\right) + \sqrt{3} \Gamma\left(\frac{3}{2}, 3 \cosh^{-1}(ax)\right)\right)}{72a^3 \sqrt{-\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[ArcCosh[a*x]],x]

[Out] (Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -3*ArcCosh[a*x]] + 9*Sqrt[ArcCosh[a*x]]*Gamma[3/2, -ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(9*Gamma[3/2, ArcCosh[a*x]] + Sqrt[3]*Gamma[3/2, 3*ArcCosh[a*x]]))/(72*a^3*Sqrt[-ArcCosh[a*x]])

Maple [F]

time = 4.42, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\operatorname{arccosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccosh(a*x)^(1/2),x)`

[Out] `int(x^2*arccosh(a*x)^(1/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2*sqrt(arccosh(a*x)), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acosh(a*x)**(1/2),x)`

[Out] `Integral(x**2*sqrt(acosh(a*x)), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^2*sqrt(arccosh(a*x)), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acosh(a*x)^(1/2),x)`

[Out] `int(x^2*acosh(a*x)^(1/2), x)`

3.75 $\int x \sqrt{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2}$$

[Out] $-1/32*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2-1/32*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2-1/4*\operatorname{arccosh}(a*x)^{(1/2)}/a^{2+1/2}*x^{2+1/2}$

Rubi [A]

time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5884, 5953, 3393, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]], x]$

[Out] $-1/4*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]/a^2 + (x^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/2 - (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(16*a^2) - (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(16*a^2)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]))], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]))], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{NegQ}[b]$

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x
_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x \sqrt{\cosh^{-1}(ax)} dx &= \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{1}{4}a \int \frac{x^2}{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}} dx \\
&= \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^2} \\
&= \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cosh(2x)}{2\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{4a^2} \\
&= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^2} \\
&= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{16a^2} - \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{16a^2} \\
&= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^2} - \frac{\text{Subst}\left(\int e^{2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^2} \\
&= -\frac{\sqrt{\cosh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{16a^2}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 65, normalized size = 0.70

$$\frac{8\sqrt{\cosh^{-1}(ax)} \cosh(2\cosh^{-1}(ax)) - \sqrt{2\pi} \left(\operatorname{Erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right) + \operatorname{Erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right) \right)}{32a^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Sqrt[ArcCosh[a*x]], x]`

```
[Out] (8*Sqrt[ArcCosh[a*x]]*Cosh[2*ArcCosh[a*x]] - Sqrt[2*Pi]*(Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]))/(32*a^2)
```

Maple [A]

time = 3.49, size = 73, normalized size = 0.78

method	result
default	$ -\frac{\sqrt{2} \left(-8\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} a^2 x^2 + 4\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} + \pi \operatorname{erf}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) + \pi \operatorname{erfi}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) \right)}{32\sqrt{\pi} a^2} $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*2^(1/2)*(-8*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*a^2*x^2+4*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)+Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))+Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(acosh(a*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arccosh(a*x)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(a*x)^(1/2),x)
```

```
[Out] int(x*acosh(a*x)^(1/2), x)
```

3.76 $\int \sqrt{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=53

$$x\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a} - \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a}$$

[Out] $-1/4*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a-1/4*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a+x*\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.15, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5879, 5953, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a} - \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a} + x\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[ArcCosh[a*x]],x]`

[Out] $x*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]] - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a) - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a)$

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3388

`Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[`

$I/2, \text{Int}[(c + d*x)^m * E^{(I*k*Pi)} * E^{(I*(e + f*x))}, x], x] /;$ FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5879

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(n_.)}, x_Symbol] := \text{Simp}[x*(a + b*\text{ArcCosh}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[x*((a + b*\text{ArcCosh}[c*x])^{(n-1)})/(\text{Sqrt}[1 + c*x]*\text{Sqrt}[-1 + c*x])], x], x] /;$ FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5953

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)^{(m_.)}*((d1_.) + (e1_.)*(x_.))^{(p_.)}*((d2_.) + (e2_.)*(x_.))^{(p_.)}, x_Symbol] := \text{Dist}[(1/(b*c^{(m+1)}))*\text{Simp}[(d1 + e1*x)^p/(1 + c*x)^p]*\text{Simp}[(d2 + e2*x)^p/(-1 + c*x)^p], \text{Subst}[\text{Int}[x^n*\text{Cosh}[-a/b + x/b]^m*\text{Sinh}[-a/b + x/b]^{(2*p+1)}, x], x, a + b*\text{ArcCosh}[c*x]], x] /;$ FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{\cosh^{-1}(ax)} dx &= x \sqrt{\cosh^{-1}(ax)} - \frac{1}{2}a \int \frac{x}{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}} dx \\ &= x \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a} \\ &= x \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a} \\ &= x \sqrt{\cosh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a} - \frac{\text{Subst}\left(\int e^{x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a} \\ &= x \sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a} - \frac{\sqrt{\pi} \operatorname{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 45, normalized size = 0.85

$$\frac{\sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{3}{2}, -\cosh^{-1}(ax)\right)}{\sqrt{-\cosh^{-1}(ax)}} + \Gamma\left(\frac{3}{2}, \cosh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[ArcCosh[a*x]], x]

[Out] ((Sqrt[ArcCosh[a*x]]*Gamma[3/2, -ArcCosh[a*x]])/Sqrt[-ArcCosh[a*x]] + Gamma[3/2, ArcCosh[a*x]])/(2*a)

Maple [A]

time = 3.03, size = 43, normalized size = 0.81

method	result	size
default	$\frac{4\sqrt{\operatorname{arccosh}(ax)}\sqrt{\pi}ax - \pi \operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(ax)}\right) - \pi \operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right)}{4\sqrt{\pi}a}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^(1/2), x, method=_RETURNVERBOSE)

[Out] 1/4*(4*arccosh(a*x)^(1/2)*Pi^(1/2)*a*x - Pi*erfi(arccosh(a*x)^(1/2)) - Pi*erf(arccosh(a*x)^(1/2)))/Pi^(1/2)/a

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(arccosh(a*x)), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**(1/2),x)`

[Out] `Integral(sqrt(acosh(a*x)), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(arccosh(a*x)), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^(1/2),x)`

[Out] `int(acosh(a*x)^(1/2), x)`

$$3.77 \quad \int \frac{\sqrt{\cosh^{-1}(ax)}}{x} dx$$

Optimal. Leaf size=15

$$\text{Int} \left(\frac{\sqrt{\cosh^{-1}(ax)}}{x}, x \right)$$

[Out] Unintegrable(arccosh(a*x)^(1/2)/x,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[ArcCosh[a*x]]/x,x]

[Out] Defer[Int][Sqrt[ArcCosh[a*x]]/x, x]

Rubi steps

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} dx = \int \frac{\sqrt{\cosh^{-1}(ax)}}{x} dx$$

Mathematica [A]

time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{x} dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[ArcCosh[a*x]]/x,x]

[Out] Integrate[Sqrt[ArcCosh[a*x]]/x, x]

Maple [A]

time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{arccosh}(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/x,x)
```

```
[Out] int(arccosh(a*x)^(1/2)/x,x)
```

Maxima [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/x, x)
```

Fricas [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(acosh(a*x))/x, x)
```

Giac [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(a*x))/x, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^(1/2)/x,x)`

[Out] `int(acosh(a*x)^(1/2)/x, x)`

3.78 $\int x^4 \cosh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=345

$$\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4 \sqrt{-1+ax} \sqrt{1+ax}}{50a}$$

```
[Out] 1/5*x^5*arccosh(a*x)^(3/2)-3/16000*erf(5^(1/2)*arccosh(a*x)^(1/2))*5^(1/2)*
Pi^(1/2)/a^5+3/16000*erfi(5^(1/2)*arccosh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-
1/384*erf(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+1/384*erfi(3^(1/
2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-3/64*erf(arccosh(a*x)^(1/2))*Pi
^(1/2)/a^5+3/64*erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a^5-4/25*(a*x-1)^(1/2)*(a
*x+1)^(1/2)*arccosh(a*x)^(1/2)/a^5-2/25*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arc
cosh(a*x)^(1/2)/a^3-3/50*x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)^(1/2)
/a
```

Rubi [A]

time = 0.71, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5884, 5939, 5915, 5881, 3389, 2211, 2235, 2236, 5887, 5556}

$$\frac{3\sqrt{3}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^5} - \frac{3\sqrt{3}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5} - \frac{\sqrt{3}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{200a^5} - \frac{3\sqrt{3}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5} - \frac{3\sqrt{3}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^5} - \frac{3\sqrt{3}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5} - \frac{\sqrt{3}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{200a^5} - \frac{3\sqrt{3}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3200a^5} - \frac{4\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{2a^2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{25a^3} + \frac{1}{5}a^2\cosh^{-1}(ax)^{3/2} - \frac{3a^2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{50a}$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcCosh[a*x]^(3/2), x]

```
[Out] (-4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(25*a^5) - (2*x^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(25*a^3) - (3*x^4*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(50*a) + (x^5*ArcCosh[a*x]^(3/2))/5 -
(3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqr
t[ArcCosh[a*x]]])/(200*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]]
)/(3200*a^5) - (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3200*a^5) +
(3*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(64*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*S
qrt[ArcCosh[a*x]]])/(200*a^5) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x
]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(3200*a^5
)
```

Rule 2211

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]ⁿ*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := Dist[1/(b*c), Su
bst[Int[xⁿ*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((x_)^(m_.), x_Symbol] := Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])ⁿ/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((x_)^(m_.), x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[xⁿ*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((x_)^(p_)*((d1_.) + (e1_.)*(x_))^(p_)
((d2_.) + (e2_.)(x_))^(p_), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 +

$$e^{2x} \cdot (p+1) \cdot \left(\frac{a + b \operatorname{ArcCosh}[c x]}{2 e_1 e_2 (p+1)} \right)^n, x] - \operatorname{Dist}[b \cdot (n/(2 \cdot c \cdot (p+1))) \cdot \operatorname{Simp}[(d_1 + e_1 x)^p / (1 + c x)^p] \cdot \operatorname{Simp}[(d_2 + e_2 x)^p / (-1 + c x)^p], \operatorname{Int}[(1 + c x)^{(p+1/2)} \cdot (-1 + c x)^{(p+1/2)} \cdot (a + b \operatorname{ArcCosh}[c x])^{(n-1)}, x], x] /;$$

$$\text{FreeQ}\{a, b, c, d_1, e_1, d_2, e_2, p\}, x] \ \&\& \ \text{EqQ}[e_1, c \cdot d_1] \ \&\& \ \text{EqQ}[e_2, (-c) \cdot d_2] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$$

Rule 5939

$$\operatorname{Int}[\left((a_.) + \operatorname{ArcCosh}[(c_.) \cdot (x_)] \cdot (b_.) \right)^{(n_.)} \cdot \left((f_.) \cdot (x_.) \right)^{(m_.)} \cdot \left((d_1_.) + (e_1_.) \cdot (x_.) \right)^{(p_.)} \cdot \left((d_2_.) + (e_2_.) \cdot (x_.) \right)^{(p_.)}, x_Symbol] \ :> \operatorname{Simp}[f \cdot (f x)^{(m-1)} \cdot (d_1 + e_1 x)^{(p+1)} \cdot (d_2 + e_2 x)^{(p+1)} \cdot \left(\frac{a + b \operatorname{ArcCosh}[c x]}{e_1 e_2 (m + 2p + 1)} \right)^n, x] + (\operatorname{Dist}[f^2 \cdot (m-1) / (c^2 \cdot (m + 2p + 1))], \operatorname{Int}[(f x)^{(m-2)} \cdot (d_1 + e_1 x)^p \cdot (d_2 + e_2 x)^p \cdot (a + b \operatorname{ArcCosh}[c x])^n, x], x] - \operatorname{Dist}[b \cdot f \cdot (n / (c \cdot (m + 2p + 1))) \cdot \operatorname{Simp}[(d_1 + e_1 x)^p / (1 + c x)^p] \cdot \operatorname{Simp}[(d_2 + e_2 x)^p / (-1 + c x)^p], \operatorname{Int}[(f x)^{(m-1)} \cdot (1 + c x)^{(p+1/2)} \cdot (-1 + c x)^{(p+1/2)} \cdot (a + b \operatorname{ArcCosh}[c x])^{(n-1)}, x], x)] /;$$

$$\text{FreeQ}\{a, b, c, d_1, e_1, d_2, e_2, f, p\}, x] \ \&\& \ \text{EqQ}[e_1, c \cdot d_1] \ \&\& \ \text{EqQ}[e_2, (-c) \cdot d_2] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m + 2p + 1, 0]$$

Rubi steps

$$\begin{aligned}
\int x^4 \cosh^{-1}(ax)^{3/2} dx &= \frac{1}{5}x^5 \cosh^{-1}(ax)^{3/2} - \frac{1}{10}(3a) \int \frac{x^5 \sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{3x^4 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \cosh^{-1}(ax)^{3/2} + \frac{3}{100} \int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} - \frac{3x^4 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{50a} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3} \\
&= -\frac{4\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^5} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{25a^3}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 152, normalized size = 0.44

$$\frac{9\sqrt{5} \sqrt{-\cosh^{-1}(ax)} \Gamma(\frac{5}{2}, -5\cosh^{-1}(ax)) + 125\sqrt{3} \sqrt{-\cosh^{-1}(ax)} \Gamma(\frac{5}{2}, -3\cosh^{-1}(ax)) + 2250 \sqrt{-\cosh^{-1}(ax)} \Gamma(\frac{5}{2}, -\cosh^{-1}(ax)) + 2250\Gamma(\frac{5}{2}, \cosh^{-1}(ax)) + 125\sqrt{3}\Gamma(\frac{5}{2}, 3\cosh^{-1}(ax)) + 9\sqrt{5}\Gamma(\frac{5}{2}, 5\cosh^{-1}(ax))}{36000a^5}$$

Antiderivative was successfully verified.

`[In] Integrate[x^4*ArcCosh[a*x]^(3/2), x]`

```
[Out] ((9*sqrt[5]*sqrt[-ArcCosh[a*x]]*Gamma[5/2, -5*ArcCosh[a*x]])/sqrt[ArcCosh[a*x]] + (125*sqrt[3]*sqrt[-ArcCosh[a*x]]*Gamma[5/2, -3*ArcCosh[a*x]])/sqrt[ArcCosh[a*x]] + (2250*sqrt[-ArcCosh[a*x]]*Gamma[5/2, -ArcCosh[a*x]])/sqrt[ArcCosh[a*x]] + 2250*Gamma[5/2, ArcCosh[a*x]] + (125*sqrt[3]*sqrt[ArcCosh[a*x]]*Gamma[5/2, 3*ArcCosh[a*x]])/sqrt[ArcCosh[a*x]] + (9*sqrt[5]*sqrt[ArcCosh[a*x]]*Gamma[5/2, 5*ArcCosh[a*x]])/sqrt[ArcCosh[a*x]])/36000
```

$\text{rcCosh}[a*x]] + (2250*\text{Sqrt}[-\text{ArcCosh}[a*x]]*\text{Gamma}[5/2, -\text{ArcCosh}[a*x]])/\text{Sqrt}[\text{ArcCosh}[a*x]] + 2250*\text{Gamma}[5/2, \text{ArcCosh}[a*x]] + 125*\text{Sqrt}[3]*\text{Gamma}[5/2, 3*\text{ArcCosh}[a*x]] + 9*\text{Sqrt}[5]*\text{Gamma}[5/2, 5*\text{ArcCosh}[a*x]])/(36000*a^5)$

Maple [F]

time = 6.91, size = 0, normalized size = 0.00

$$\int x^4 \text{arccosh}(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccosh(a*x)^(3/2),x)`

[Out] `int(x^4*arccosh(a*x)^(3/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^4*arccosh(a*x)^(3/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \text{acosh}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acosh(a*x)**(3/2),x)`

[Out] `Integral(x**4*acosh(a*x)**(3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccosh(a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^4*arccosh(a*x)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{acosh}(a x)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*acosh(a*x)^(3/2),x)

[Out] int(x^4*acosh(a*x)^(3/2), x)

3.79 $\int x^3 \cosh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=209

$$\frac{9x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{32a} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4 \cosh^{-1}(ax)$$

[Out] $-3/32*\operatorname{arccosh}(a*x)^{(3/2)}/a^4+1/4*x^4*\operatorname{arccosh}(a*x)^{(3/2)}-3/256*\operatorname{erf}(2^{(1/2)}*a*\operatorname{rccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4+3/256*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4-3/2048*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4+3/2048*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4-9/64*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}/a^3-3/32*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}/a$

Rubi [A]

time = 0.55, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5884, 5939, 5893, 5887, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\operatorname{Erf}(2\sqrt{\cosh^{-1}(ax)})}{2048a^4} - \frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erf}(\sqrt{2}\sqrt{\cosh^{-1}(ax)})}{128a^4} + \frac{3\sqrt{\pi}\operatorname{Erfi}(2\sqrt{\cosh^{-1}(ax)})}{2048a^4} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erfi}(\sqrt{2}\sqrt{\cosh^{-1}(ax)})}{128a^4} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} - \frac{9x\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{64a^3} + \frac{1}{4}x^4\cosh^{-1}(ax)^{3/2} - \frac{3x^2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{32a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{ArcCosh}[a*x]^{(3/2)}, x]$

[Out] $(-9*x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(64*a^3) - (3*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(32*a) - (3*\operatorname{ArcCosh}[a*x]^{(3/2)})/(32*a^4) + (x^4*\operatorname{ArcCosh}[a*x]^{(3/2)})/4 - (3*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(2048*a^4) - (3*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(128*a^4) + (3*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(2048*a^4) + (3*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(128*a^4)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2211

$\operatorname{Int}[(F_)^{((g_)*(e_)+(f_)*(x_))}/\operatorname{Sqrt}[(c_)+(d_)*(x_)], x_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_)+(b_)*((c_)+(d_)*(x_))^2)}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]ⁿ*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((x_))^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^{n/(m + 1)}), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((x_))^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[xⁿ*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[f*(f*x)^{(m -}

$1) * (d1 + e1*x)^{(p+1)} * (d2 + e2*x)^{(p+1)} * ((a + b*ArcCosh[c*x])^n / (e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^{(m - 2)} * (d1 + e1*x)^p * (d2 + e2*x)^p * (a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^{(m - 1)} * (1 + c*x)^{(p + 1/2)} * (-1 + c*x)^{(p + 1/2)} * (a + b*ArcCosh[c*x])^{(n - 1)}, x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]$

Rubi steps

$$\begin{aligned}
 \int x^3 \cosh^{-1}(ax)^{3/2} dx &= \frac{1}{4} x^4 \cosh^{-1}(ax)^{3/2} - \frac{1}{8} (3a) \int \frac{x^4 \sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= -\frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} + \frac{1}{4} x^4 \cosh^{-1}(ax)^{3/2} + \frac{3}{64} \int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}} dx \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a} \\
 &= -\frac{9x \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{64a^3} - \frac{3x^3 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{32a}
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 101, normalized size = 0.48

$$\frac{\sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{5}{2}, -4 \cosh^{-1}(ax)\right) + 8\sqrt{2} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{5}{2}, -2 \cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)} \left(8\sqrt{2} \Gamma\left(\frac{5}{2}, 2 \cosh^{-1}(ax)\right) + \Gamma\left(\frac{5}{2}, 4 \cosh^{-1}(ax)\right)\right)}{512a^4 \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCosh[a*x]^(3/2), x]

[Out] (Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -4*ArcCosh[a*x]] + 8*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -2*ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(8*Sqrt[2]*Gamma[5/2, 2*ArcCosh[a*x]] + Gamma[5/2, 4*ArcCosh[a*x]]))/(512*a^4*Sqrt[ArcCosh[a*x]])

Maple [A]

time = 10.08, size = 242, normalized size = 1.16

method	result
default	$-\frac{\sqrt{2} \left(-32 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} a^2 x^2 + 24 \sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} ax + 16 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{\pi} a^4 \right)}{256 \sqrt{\pi} a^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arccosh(a*x)^(3/2), x, method=_RETURNVERBOSE)

[Out] -1/256*2^(1/2)*(-32*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+24*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+16*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))-3*Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^4-1/2048*(-512*arccosh(a*x)^(3/2)*Pi^(1/2)*a^4*x^4+192*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3+512*arccosh(a*x)^(3/2)*Pi^(1/2)*a^2*x^2-96*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-64*arccosh(a*x)^(3/2)*Pi^(1/2)+3*Pi*erf(2*arccosh(a*x)^(1/2))-3*Pi*erfi(2*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^4

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccosh(a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x^3*arccosh(a*x)^(3/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^3 \operatorname{acosh}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**(3/2), x)
```

Giac [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int x^3 \operatorname{acosh}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a*x)^(3/2),x)
```

```
[Out] int(x^3*acosh(a*x)^(3/2), x)
```

3.80 $\int x^2 \cosh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=189

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3} x^3 \cosh^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{32a^3} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{96a^3} + \frac{3\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3} + \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{96a^3} - \frac{\sqrt{ax-1} \sqrt{ax+1} \sqrt{\cosh^{-1}(ax)}}{3a^3} + \frac{1}{3} x^3 \cosh^{-1}(ax)^{3/2} - \frac{x^2 \sqrt{ax-1} \sqrt{ax+1} \sqrt{\cosh^{-1}(ax)}}{6a}$$

```
[Out] 1/3*x^3*arccosh(a*x)^(3/2)-1/288*erf(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/288*erfi(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-3/32*erf(arccosh(a*x)^(1/2))*Pi^(1/2)/a^3+3/32*erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a^3-1/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)^(1/2)/a^3-1/6*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)^(1/2)/a
```

Rubi [A]

time = 0.40, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5884, 5939, 5915, 5881, 3389, 2211, 2235, 2236, 5887, 5556}

$$\frac{3\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{96a^3} + \frac{3\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{32a^3} + \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{96a^3} - \frac{\sqrt{ax-1} \sqrt{ax+1} \sqrt{\cosh^{-1}(ax)}}{3a^3} + \frac{1}{3} x^3 \cosh^{-1}(ax)^{3/2} - \frac{x^2 \sqrt{ax-1} \sqrt{ax+1} \sqrt{\cosh^{-1}(ax)}}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a*x]^(3/2), x]
```

```
[Out] -1/3*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/a^3 - (x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(6*a) + (x^3*ArcCosh[a*x]^(3/2))/3 - (3*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(32*a^3) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(96*a^3) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(96*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_.) + (e1_.)*(x_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5939

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_.))^ (m_.)*((d1_.) + (e
1_.)*(x_.))^ (p_.)*((d2_.) + (e2_.)*(x_.))^ (p_.), x_Symbol] := Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(
n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p]
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int x^2 \cosh^{-1}(ax)^{3/2} dx &= \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} - \frac{1}{2}a \int \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} + \frac{1}{12} \int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2} \\
&= -\frac{\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{3a^3} - \frac{x^2 \sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{3/2}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 100, normalized size = 0.53

$$\frac{\sqrt{3} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{5}{2}, -3 \cosh^{-1}(ax)\right) + 27 \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{5}{2}, -\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)} \left(27 \Gamma\left(\frac{5}{2}, \cosh^{-1}(ax)\right) + \sqrt{3} \Gamma\left(\frac{5}{2}, 3 \cosh^{-1}(ax)\right)\right)}{216a^3 \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcCosh[a*x]^(3/2), x]

[Out] (Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -3*ArcCosh[a*x]] + 27*Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(27*Gamma[5/2, ArcCosh[a*x]] + Sqrt[3]*Gamma[5/2, 3*ArcCosh[a*x]]))/(216*a^3*Sqrt[ArcCosh[a*x]])

Maple [F]

time = 4.31, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccosh}(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arccosh(a*x)^(3/2), x)

[Out] int(x^2*arccosh(a*x)^(3/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x^2*arccosh(a*x)^(3/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acosh}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acosh(a*x)**(3/2),x)`

[Out] `Integral(x**2*acosh(a*x)**(3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^2*arccosh(a*x)^(3/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acosh(a*x)^(3/2),x)`

[Out] `int(x^2*acosh(a*x)^(3/2), x)`

3.81 $\int x \cosh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=127

$$-\frac{3x\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} - \frac{3\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2}$$

[Out] $-1/4*\operatorname{arccosh}(a*x)^{(3/2)}/a^2+1/2*x^2*\operatorname{arccosh}(a*x)^{(3/2)}-3/128*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2+3/128*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2-3/8*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}/a$

Rubi [A]

time = 0.30, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5884, 5939, 5893, 5887, 5556, 12, 3389, 2211, 2235, 2236}

$$-\frac{3\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2} + \frac{3\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a^2} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} - \frac{3x\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{8a}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCosh[a*x]^(3/2),x]`

[Out] $(-3*x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(8*a) - \operatorname{ArcCosh}[a*x]^{(3/2)}/(4*a^2) + (x^2*\operatorname{ArcCosh}[a*x]^{(3/2)})/2 - (3*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(64*a^2) + (3*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(64*a^2)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 2211

`Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e-c*(f/d))+f*g*(x^2/d)), x], x, Sqrt[c+d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c+d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]ⁿ*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)(x_)^(m_.), x_Symbol] := Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])ⁿ/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)(x_)^(m_.), x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[xⁿ*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/(Sqrt[(d1_.) + (e1_.)*(x_)]*Sq
rt[(d2_.) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
&& EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)((f_.)*(x_))^(m_.)((d1_.) + (e
1_.)*(x_))^(p_.)((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[f*(f*x)<sup>(m -
1)</sup>(d1 + e1*x)^(p + 1)(d2 + e2*x)^(p + 1)((a + b*ArcCosh[c*x])ⁿ/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f²*((m - 1)/(c²*(m + 2*p + 1))), Int[(f*x)<sup>(m
- 2)</sup>(d1 + e1*x)^p(d2 + e2*x)^p(a + b*ArcCosh[c*x])ⁿ, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-

$1 + c*x)^p]$, Int $[(f*x)^{(m-1)}*(1 + c*x)^{(p+1/2)}*(-1 + c*x)^{(p+1/2)}*(a + b*\text{ArcCosh}[c*x])^{(n-1)}, x]$, x) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
 \int x \cosh^{-1}(ax)^{3/2} dx &= \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} - \frac{1}{4}(3a) \int \frac{x^2 \sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} + \frac{3}{16} \int \frac{x}{\sqrt{\cosh^{-1}(ax)}} dx \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} + \dots \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} + \dots \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} + \dots \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} - \dots \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} - \dots \\
 &= -\frac{3x\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{8a} - \frac{\cosh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{3/2} - \dots
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 84, normalized size = 0.66

$$\frac{32 \cosh^{-1}(ax)^{3/2} \cosh(2 \cosh^{-1}(ax)) + 3\sqrt{2\pi} \left(-\text{Erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right) + \text{Erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right) \right) - 24\sqrt{\cosh^{-1}(ax)} \sinh(2 \cosh^{-1}(ax))}{128a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcCosh[a*x]^(3/2), x]

```
[Out] (32*ArcCosh[a*x]^(3/2)*Cosh[2*ArcCosh[a*x]] + 3*Sqrt[2*Pi]*(-Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]) - 24*Sqrt[ArcCosh[a*x]]*Sinh[2*ArcCosh[a*x]])/(128*a^2)
```

Maple [A]

time = 3.90, size = 105, normalized size = 0.83

method	result
default	$-\frac{\sqrt{2} \left(-32 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} a^2 x^2 + 24 \sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} ax + 16 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{\pi} a^2 \right)}{128 \sqrt{\pi} a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*2^(1/2)*(-32*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+24*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+16*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))-3*Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arccosh(a*x)^(3/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acosh}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acosh(a*x)**(3/2),x)`

[Out] `Integral(x*acosh(a*x)**(3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(x*arccosh(a*x)^(3/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acosh(a*x)^(3/2),x)`

[Out] `int(x*acosh(a*x)^(3/2), x)`

3.82 $\int \cosh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=86

$$-\frac{3\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{2a} + x \cosh^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a}$$

[Out] $x*\operatorname{arccosh}(a*x)^{(3/2)}-3/8*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a+3/8*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a-3/2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}/a$

Rubi [A]

time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5879, 5915, 5881, 3389, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a} + x \cosh^{-1}(ax)^{3/2} - \frac{3\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

[In] `Int[ArcCosh[a*x]^(3/2), x]`

[Out] $(-3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(2*a) + x*\operatorname{ArcCosh}[a*x]^{(3/2)} - (3*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]/(8*a) + (3*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]/(8*a)$

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p_) * ((d2_) + (e2_.)*(x_))^(p_), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(ax)^{3/2} dx &= x \cosh^{-1}(ax)^{3/2} - \frac{1}{2}(3a) \int \frac{x \sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{3\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{2a} + x \cosh^{-1}(ax)^{3/2} + \frac{3}{4} \int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{3\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{2a} + x \cosh^{-1}(ax)^{3/2} + \frac{3 \text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a} \\
&= -\frac{3\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{2a} + x \cosh^{-1}(ax)^{3/2} - \frac{3 \text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a} \\
&= -\frac{3\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{2a} + x \cosh^{-1}(ax)^{3/2} - \frac{3 \text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a} \\
&= -\frac{3\sqrt{-1+ax} \sqrt{1+ax} \sqrt{\cosh^{-1}(ax)}}{2a} + x \cosh^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 45, normalized size = 0.52

$$\frac{\sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{5}{2}, -\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \Gamma\left(\frac{5}{2}, \cosh^{-1}(ax)\right)$$

$$2a$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^(3/2), x]`

```
[Out] ((Sqrt[-ArcCosh[a*x]]*Gamma[5/2, -ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + Gamma[5/2, ArcCosh[a*x]])/(2*a)
```

Maple [A]

time = 3.51, size = 68, normalized size = 0.79

method	result
default	$-\frac{8 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{\pi} \sqrt{ax+12} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} + 3\pi \operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right) - 3\pi \operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(ax)}\right)}{8\sqrt{\pi} a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/8*(-8*\operatorname{arccosh}(a*x)^{(3/2)}*\pi^{(1/2)}*a*x+12*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}+3*\pi*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})-3*\pi*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)}))/\pi^{(1/2)}/a$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)^(3/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: `integrate: implementation incomplete (constant residues)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**(3/2),x)`

[Out] `Integral(acosh(a*x)**(3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(arccosh(a*x)^(3/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^(3/2), x)`

[Out] `int(acosh(a*x)^(3/2), x)`

$$3.83 \quad \int \frac{\cosh^{-1}(ax)^{3/2}}{x} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cosh^{-1}(ax)^{3/2}}{x}, x\right)$$

[Out] Unintegrable(arccosh(a*x)^(3/2)/x, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} dx$$

Verification is not applicable to the result.

[In] Int[ArcCosh[a*x]^(3/2)/x, x]

[Out] Defer[Int][ArcCosh[a*x]^(3/2)/x, x]

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} dx = \int \frac{\cosh^{-1}(ax)^{3/2}}{x} dx$$

Mathematica [A]

time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{x} dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCosh[a*x]^(3/2)/x, x]

[Out] Integrate[ArcCosh[a*x]^(3/2)/x, x]

Maple [A]

time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\text{arccosh}(ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^(3/2)/x,x)`

[Out] `int(arccosh(a*x)^(3/2)/x,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(3/2)/x,x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)^(3/2)/x, x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(3/2)/x,x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**(3/2)/x,x)`

[Out] `Integral(acosh(a*x)**(3/2)/x, x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(3/2)/x,x, algorithm="giac")`

[Out] `integrate(arccosh(a*x)^(3/2)/x, x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{acosh}(ax)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^(3/2)/x,x)

[Out] int(acosh(a*x)^(3/2)/x, x)

3.84 $\int x^4 \cosh^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=394

$$\frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{-1}}$$

```
[Out] 1/5*x^5*arccosh(a*x)^(5/2)-3/32000*erf(5^(1/2)*arccosh(a*x)^(1/2))*5^(1/2)*
Pi^(1/2)/a^5-3/32000*erfi(5^(1/2)*arccosh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-
5/2304*erf(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-5/2304*erfi(3^(
1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-15/128*erf(arccosh(a*x)^(1/2)
)*Pi^(1/2)/a^5-15/128*erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a^5-4/15*arccosh(a*
x)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5-2/15*x^2*arccosh(a*x)^(3/2)*(a*x-1
)^(1/2)*(a*x+1)^(1/2)/a^3-1/10*x^4*arccosh(a*x)^(3/2)*(a*x-1)^(1/2)*(a*x+1
)^(1/2)/a^2+5*x*arccosh(a*x)^(1/2)/a^4+1/15*x^3*arccosh(a*x)^(1/2)/a^2+3/100
*x^5*arccosh(a*x)^(1/2)
```

Rubi [A]

time = 1.13, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5884, 5939, 5915, 5879, 5953, 3388, 2211, 2235, 2236, 3393}

$$\frac{2x\sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3\sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{-1}}$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcCosh[a*x]^(5/2), x]

```
[Out] (2*x*Sqrt[ArcCosh[a*x]])/(5*a^4) + (x^3*Sqrt[ArcCosh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcCosh[a*x]])/100 - (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(
3/2))/(15*a^5) - (2*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(1
5*a^3) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(10*a) + (x^
5*ArcCosh[a*x]^(5/2))/5 - (15*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(128*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[S
qrt[3]*Sqrt[ArcCosh[a*x]]])/(1280*a^5) - (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[Arc
Cosh[a*x]]])/(6400*a^5) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(128*a^5)
- (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf
i[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(1280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt
[ArcCosh[a*x]]])/(6400*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 3393

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]ⁿ, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])ⁿ, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])ⁿ/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_.) + (e1_.)*(x_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])ⁿ/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq

Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_.))^(m_.)*((d1_) + (e1_.)*(x_.))^(p_.)*((d2_) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1)), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_.))^(p_.)*((d2_) + (e2_.)*(x_.))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int x^4 \cosh^{-1}(ax)^{5/2} dx &= \frac{1}{5} x^5 \cosh^{-1}(ax)^{5/2} - \frac{1}{2} a \int \frac{x^5 \cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{10a} + \frac{1}{5} x^5 \cosh^{-1}(ax)^{5/2} + \frac{3}{20} \int x^4 \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{2x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^3} - \frac{x^4 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^3} \\
&= \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5} \\
&= \frac{2x \sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5} \\
&= \frac{2x \sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5} \\
&= \frac{2x \sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5} \\
&= \frac{2x \sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5} \\
&= \frac{2x \sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5} \\
&= \frac{2x \sqrt{\cosh^{-1}(ax)}}{5a^4} + \frac{x^3 \sqrt{\cosh^{-1}(ax)}}{15a^2} + \frac{3}{100} x^5 \sqrt{\cosh^{-1}(ax)} - \frac{4\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{15a^5}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 162, normalized size = 0.41

$$\frac{27\sqrt{5} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -5 \cosh^{-1}(ax)\right) + 625\sqrt{3} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -3 \cosh^{-1}(ax)\right) + 33750 \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -\cosh^{-1}(ax)\right) + 33750 \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, \cosh^{-1}(ax)\right) + 625\sqrt{3} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, 3 \cosh^{-1}(ax)\right) + 27\sqrt{5} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, 5 \cosh^{-1}(ax)\right)}{54000a^5 \sqrt{-\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*ArcCosh[a*x]^(5/2), x]

[Out] (27*sqrt[5]*sqrt[ArcCosh[a*x]]*Gamma[7/2, -5*ArcCosh[a*x]] + 625*sqrt[3]*sqrt[ArcCosh[a*x]]*Gamma[7/2, -3*ArcCosh[a*x]] + 33750*sqrt[ArcCosh[a*x]]*Gamma[7/2, -ArcCosh[a*x]] + 33750*sqrt[-ArcCosh[a*x]]*Gamma[7/2, ArcCosh[a*x]] + 625*sqrt[3]*sqrt[-ArcCosh[a*x]]*Gamma[7/2, 3*ArcCosh[a*x]] + 27*sqrt[5]*sqrt[-ArcCosh[a*x]]*Gamma[7/2, 5*ArcCosh[a*x]])/54000a^5

```
ma[7/2, -ArcCosh[a*x]] + 33750*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, ArcCosh[a*x]]
+ 625*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, 3*ArcCosh[a*x]] + 27*Sqrt[5]*
Sqrt[-ArcCosh[a*x]]*Gamma[7/2, 5*ArcCosh[a*x]])/(540000*a^5*Sqrt[-ArcCosh[a
*x]])
```

Maple [F]

time = 7.27, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccosh}(ax)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^4*arccosh(a*x)^(5/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```


Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccosh(a*x)^(5/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
 eur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{acosh}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*acosh(a*x)^(5/2),x)

[Out] int(x^4*acosh(a*x)^(5/2), x)

3.85 $\int x^3 \cosh^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=257

$$-\frac{225\sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{15x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{64a^3} - \dots$$

[Out] $-3/32*\operatorname{arccosh}(a*x)^{(5/2)}/a^4+1/4*x^4*\operatorname{arccosh}(a*x)^{(5/2)}-15/1024*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4-15/1024*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4-15/16384*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4-15/16384*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4-15/64*x*\operatorname{arccosh}(a*x)^{(3/2)}*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3-5/32*x^3*\operatorname{arccosh}(a*x)^{(3/2)}*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-225/2048*\operatorname{arccosh}(a*x)^{(1/2)}/a^4+45/256*x^2*\operatorname{arccosh}(a*x)^{(1/2)}/a^2+15/256*x^4*\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.89, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5884, 5939, 5893, 5953, 3393, 3388, 2211, 2235, 2236}

$$\frac{15\sqrt{\pi}\operatorname{Erf}\left(\frac{2\sqrt{\cosh^{-1}(ax)}}{16384a^4}\right)}{16384a^4} - \frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\cosh^{-1}(ax)}}{512a^4}\right)}{512a^4} - \frac{15\sqrt{\pi}\operatorname{Erf}\left(\frac{2\sqrt{\cosh^{-1}(ax)}}{16384a^4}\right)}{16384a^4} - \frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\cosh^{-1}(ax)}}{512a^4}\right)}{512a^4} - \frac{3\cosh^{-1}(ax)^{3/2}}{32a^4} - \frac{225\sqrt{\cosh^{-1}(ax)}}{2048a^4} - \frac{15x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{64a^3} + \frac{45x^2\sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{1}{4}x^4\cosh^{-1}(ax)^{5/2} + \frac{15}{256}x^4\sqrt{\cosh^{-1}(ax)} - \frac{5x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{32a} - \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{ArcCosh}[a*x]^{(5/2)}, x]$

[Out] $(-225*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(2048*a^4) + (45*x^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(256*a^2) + (15*x^4*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/256 - (15*x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^{(3/2)})/(64*a^3) - (5*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^{(3/2)})/(32*a) - (3*\operatorname{ArcCosh}[a*x]^{(5/2)})/(32*a^4) + (x^4*\operatorname{ArcCosh}[a*x]^{(5/2)})/4 - (15*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]/(16384*a^4) - (15*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]/(512*a^4) - (15*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]/(16384*a^4) - (15*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]/(512*a^4)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_.)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+dx]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+dx)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 3393

Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + (f_.)*(x_)]^n, x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n*(x_)^m, x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n*((f_.)*(x_)^m)*((d1_) + (e1_.)*(x_)^p)*((d2_) + (e2_.)*(x_)^p), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(q_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-1 + c*x)^q], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cosh^{-1}(ax)^{5/2} dx &= \frac{1}{4}x^4 \cosh^{-1}(ax)^{5/2} - \frac{1}{8}(5a) \int \frac{x^4 \cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{5x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4 \cosh^{-1}(ax)^{5/2} + \frac{15}{64} \int x^3 \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} \\
&= \frac{45x^2 \sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} \\
&= \frac{45x^2 \sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} \\
&= -\frac{45 \sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2 \sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} \\
&= -\frac{225 \sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2 \sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} \\
&= -\frac{225 \sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2 \sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3} \\
&= -\frac{225 \sqrt{\cosh^{-1}(ax)}}{2048a^4} + \frac{45x^2 \sqrt{\cosh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4 \sqrt{\cosh^{-1}(ax)} - \frac{15x \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)}{64a^3}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 101, normalized size = 0.39

$$\frac{\sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -4 \cosh^{-1}(ax)\right) + 16\sqrt{2} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -2 \cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)} \left(16\sqrt{2} \Gamma\left(\frac{7}{2}, 2 \cosh^{-1}(ax)\right) + \Gamma\left(\frac{7}{2}, 4 \cosh^{-1}(ax)\right)\right)}{2048a^4 \sqrt{-\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCosh[a*x]^(5/2),x]

[Out] (Sqrt[ArcCosh[a*x]]*Gamma[7/2, -4*ArcCosh[a*x]] + 16*Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[7/2, -2*ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(16*Sqrt[2]*Gamma[7/2, 2*ArcCosh[a*x]] + Gamma[7/2, 4*ArcCosh[a*x]]))/(2048*a^4*Sqrt[-ArcCosh[a*x]])

Maple [A]

time = 10.07, size = 321, normalized size = 1.25

method	result
default	$\frac{\sqrt{2} \left(-128 \operatorname{arccosh}(ax)^{\frac{5}{2}} \sqrt{2} \sqrt{\pi} a^2 x^2 + 160 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} ax - 120 \sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)

[Out]
$$\frac{-1/1024*2^{(1/2)}*(-128*\operatorname{arccosh}(a*x)^{(5/2)}*2^{(1/2)}*\pi^{(1/2)}*a^2*x^2+160*\operatorname{arccosh}(a*x)^{(3/2)}*2^{(1/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x-120*2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*a^2*x^2+64*\operatorname{arccosh}(a*x)^{(5/2)}*2^{(1/2)}*\pi^{(1/2)}+60*2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}+15*\pi*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})+15*\pi*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}))/\pi^{(1/2)}/a^4-1/16384*(-4096*\operatorname{arccosh}(a*x)^{(5/2)}*\pi^{(1/2)}*a^4*x^4+2560*\operatorname{arccosh}(a*x)^{(3/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a^3*x^3-960*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*a^4*x^4+4096*\operatorname{arccosh}(a*x)^{(5/2)}*\pi^{(1/2)}*a^2*x^2-1280*\operatorname{arccosh}(a*x)^{(3/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x+960*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*a^2*x^2-512*\operatorname{arccosh}(a*x)^{(5/2)}*\pi^{(1/2)}-120*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}+15*\pi*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})+15*\pi*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)}))/\pi^{(1/2)}/a^4$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccosh(a*x)^(5/2),x, algorithm="maxima")

[Out] integrate(x^3*arccosh(a*x)^(5/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acosh}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**(5/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**(5/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acosh}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a*x)^(5/2),x)
```

```
[Out] int(x^3*acosh(a*x)^(5/2), x)
```

3.86 $\int x^2 \cosh^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=220

$$\frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{18a}$$

[Out] $\frac{1}{3}x^3\operatorname{arccosh}(ax)^{5/2} - \frac{5}{1728}\operatorname{erf}\left(3^{1/2}\operatorname{arccosh}(ax)^{1/2}\right)3^{1/2}\operatorname{Pi}^{1/2}/a^3 - \frac{5}{1728}\operatorname{erfi}\left(3^{1/2}\operatorname{arccosh}(ax)^{1/2}\right)3^{1/2}\operatorname{Pi}^{1/2}/a^3 - \frac{15}{64}\operatorname{erf}\left(\operatorname{arccosh}(ax)^{1/2}\right)\operatorname{Pi}^{1/2}/a^3 - \frac{15}{64}\operatorname{erfi}\left(\operatorname{arccosh}(ax)^{1/2}\right)\operatorname{Pi}^{1/2}/a^3 - \frac{5}{9}\operatorname{arccosh}(ax)^{3/2}(ax-1)^{1/2}(ax+1)^{1/2}/a^3 - \frac{5}{18}x^2\operatorname{arccosh}(ax)^{3/2}(ax-1)^{1/2}(ax+1)^{1/2}/a^3 + \frac{5}{6}x\operatorname{arccosh}(ax)^{1/2}/a^2 + \frac{5}{36}x^3\operatorname{arccosh}(ax)^{1/2}$

Rubi [A]

time = 0.69, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5884, 5939, 5915, 5879, 5953, 3388, 2211, 2235, 2236, 3393}

$$\frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^3} - \frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{576a^3} - \frac{15\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{64a^3} - \frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{576a^3} - \frac{5\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{9a^3} + \frac{5x\sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{1}{3}x^2\cosh^{-1}(ax)^{3/2} + \frac{5}{36}x^3\sqrt{\cosh^{-1}(ax)} - \frac{5x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{18a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2\operatorname{ArcCosh}[a*x]^{5/2}, x]$

[Out] $\frac{(5*x*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])}{(6*a^2)} + \frac{(5*x^3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])}{36} - \frac{(5*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^{3/2})}{(9*a^3)} - \frac{(5*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x]*\operatorname{ArcCosh}[a*x]^{3/2})}{(18*a)} + \frac{(x^3*\operatorname{ArcCosh}[a*x]^{5/2})}{3} - \frac{(15*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]}{(64*a^3)} - \frac{(5*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]}{(576*a^3)} - \frac{(15*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]}{(64*a^3)} - \frac{(5*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])]}{(576*a^3)}$

Rule 2211

$\operatorname{Int}[(F_)^((g_.)*((e_.)+(f_)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^(g*(e-c*(f/d))+f*g*(x^2/d)), x], x, \operatorname{Sqrt}[c+d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\amp; \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^((a_.)+(b_.)*((c_.)+(d_)*(x_))^2), x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \&\amp; \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)](n_.), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)m, Sin[e + f*x]n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.), x_Symbol] := Simp[x*(a + b*A
rcCosh[c*x])n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])(n - 1)/(sqrt[
1 + c*x]*sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)*x(m_.), x_Symbol] := Simp[
x(m + 1)*((a + b*ArcCosh[c*x])n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x(m + 1)*((a + b*ArcCosh[c*x])(n - 1)/(sqrt[1 + c*x]*sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)*x(p_.)*((d1_.) + (e1_.)*(x_))(p
_)*((d2_.) + (e2_.)*(x_))(p_.), x_Symbol] := Simp[(d1 + e1*x)(p + 1)*(d2 +
e2*x)(p + 1)*((a + b*ArcCosh[c*x])n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)p/(1 + c*x)p]*Simp[(d2 + e2*x)p/(-1 + c*x)
p], Int[(1 + c*x)(p + 1/2)*(-1 + c*x)(p + 1/2)*(a + b*ArcCosh[c*x])(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)*((f_.)*(x_))(m_.)*((d1_.) + (e
1_.)*(x_))(p_.)*((d2_.) + (e2_.)*(x_))(p_.), x_Symbol] := Simp[f*(f*x)(m -
1)*(d1 + e1*x)(p + 1)*(d2 + e2*x)(p + 1)*((a + b*ArcCosh[c*x])n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f2*((m - 1)/(c2*(m + 2*p + 1))), Int[(f*x)(m
```



```

- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]

```

Rule 5953

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_)*(x_)^(m_)*((d1_) + (e1_.)*(x
_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\int x^2 \cosh^{-1}(ax)^{5/2} dx &= \frac{1}{3}x^3 \cosh^{-1}(ax)^{5/2} - \frac{1}{6}(5a) \int \frac{x^3 \cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3 \cosh^{-1}(ax)^{5/2} + \frac{5}{12} \int x^2 \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} \\
&= \frac{5x \sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} \\
&= \frac{5x \sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} \\
&= \frac{5x \sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} \\
&= \frac{5x \sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} \\
&= \frac{5x \sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a} \\
&= \frac{5x \sqrt{\cosh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3 \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2 \sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{18a}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 100, normalized size = 0.45

$$\frac{\sqrt{3} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -3 \cosh^{-1}(ax)\right) + 81 \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -\cosh^{-1}(ax)\right) + \sqrt{-\cosh^{-1}(ax)} \left(81 \Gamma\left(\frac{7}{2}, \cosh^{-1}(ax)\right) + \sqrt{3} \Gamma\left(\frac{7}{2}, 3 \cosh^{-1}(ax)\right)\right)}{648a^3 \sqrt{-\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcCosh[a*x]^(5/2), x]

[Out] (Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[7/2, -3*ArcCosh[a*x]] + 81*Sqrt[ArcCosh[a*x]]*Gamma[7/2, -ArcCosh[a*x]] + Sqrt[-ArcCosh[a*x]]*(81*Gamma[7/2, ArcCosh[a*x]] + Sqrt[3]*Gamma[7/2, 3*ArcCosh[a*x]]))/(648*a^3*Sqrt[-ArcCosh[a*x]])

Maple [F]

time = 4.43, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccosh}(ax)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccosh(a*x)^(5/2),x)`[Out] `int(x^2*arccosh(a*x)^(5/2),x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x)^(5/2),x, algorithm="maxima")`[Out] `integrate(x^2*arccosh(a*x)^(5/2), x)`**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccosh(a*x)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acosh}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acosh(a*x)**(5/2),x)`[Out] `Integral(x**2*acosh(a*x)**(5/2), x)`**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int x^2 \operatorname{acosh}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(a*x)^(5/2),x)
```

```
[Out] int(x^2*acosh(a*x)^(5/2), x)
```

3.87 $\int x \cosh^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=157

$$-\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \cosh^{-1}(ax)$$

[Out] $-1/4*\operatorname{arccosh}(a*x)^{(5/2)}/a^2+1/2*x^2*\operatorname{arccosh}(a*x)^{(5/2)}-15/512*\operatorname{erf}(2^{(1/2)}*a*\operatorname{rccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2-15/512*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2-5/8*x*\operatorname{arccosh}(a*x)^{(3/2)}*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a-15/64*\operatorname{arccosh}(a*x)^{(1/2)}/a^2+15/32*x^2*\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.45, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5884, 5939, 5893, 5953, 3393, 3388, 2211, 2235, 2236}

$$-\frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a^2} - \frac{15\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a^2} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} - \frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{1}{2}x^2\cosh^{-1}(ax)^{5/2} + \frac{15}{32}x^2\sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{8a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{ArcCosh}[a*x]^{(5/2)}, x]$

[Out] $(-15*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(64*a^2) + (15*x^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/32 - (5*x*\operatorname{Sqrt}[-1 + a*x]*\operatorname{Sqrt}[1 + a*x]*\operatorname{ArcCosh}[a*x]^{(3/2)})/(8*a) - \operatorname{ArcCosh}[a*x]^{(5/2)}/(4*a^2) + (x^2*\operatorname{ArcCosh}[a*x]^{(5/2)})/2 - (15*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(256*a^2) - (15*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(256*a^2)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{NegQ}[b]$

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1]
&& EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] := Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
```

```
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x \cosh^{-1}(ax)^{5/2} dx &= \frac{1}{2}x^2 \cosh^{-1}(ax)^{5/2} - \frac{1}{4}(5a) \int \frac{x^2 \cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} + \frac{1}{2}x^2 \cosh^{-1}(ax)^{5/2} + \frac{15}{16} \int x \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2} \int x \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2} \int x \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} - \frac{\cosh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2} \int x \sqrt{\cosh^{-1}(ax)} dx \\
&= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} \\
&= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} \\
&= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a} \\
&= -\frac{15\sqrt{\cosh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2 \sqrt{\cosh^{-1}(ax)} - \frac{5x\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{8a}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 92, normalized size = 0.59

$$\frac{8\sqrt{\cosh^{-1}(ax)} (15 + 16 \cosh^{-1}(ax)^2) \cosh(2 \cosh^{-1}(ax)) - 15\sqrt{2\pi} \left(\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) + \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) \right) - 160 \cosh^{-1}(ax)^{3/2} \sinh(2 \cosh^{-1}(ax))}{512a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcCosh[a*x]^(5/2), x]

```
[Out] (8*sqrt[ArcCosh[a*x]]*(15 + 16*ArcCosh[a*x]^2)*Cosh[2*ArcCosh[a*x]] - 15*sqrt[2*Pi]*(Erf[Sqrt[2]*sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*sqrt[ArcCosh[a*x]]]) - 160*ArcCosh[a*x]^(3/2)*Sinh[2*ArcCosh[a*x]])/(512*a^2)
```

Maple [A]

time = 3.78, size = 139, normalized size = 0.89

method	result
default	$-\frac{\sqrt{2} \left(-128 \operatorname{arccosh}(ax)^{\frac{5}{2}} \sqrt{2} \sqrt{\pi} a^2 x^2 + 160 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} ax - 120 \sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \right)}{512 a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/512*2^(1/2)*(-128*arccosh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+160*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-120*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*a^2*x^2+64*arccosh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)+60*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)+15*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))+15*Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acosh}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**(5/2),x)
```

```
[Out] Integral(x*acosh(a*x)**(5/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(a*x)^(5/2),x)
```

```
[Out] int(x*acosh(a*x)^(5/2), x)
```

3.88 $\int \cosh^{-1}(ax)^{5/2} dx$

Optimal. Leaf size=99

$$\frac{15}{4}x\sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}{2a} + x\cosh^{-1}(ax)^{5/2} - \frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a}$$

[Out] x*arccosh(a*x)^(5/2)-15/16*erf(arccosh(a*x)^(1/2))*Pi^(1/2)/a-15/16*erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a-5/2*arccosh(a*x)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+15/4*x*arccosh(a*x)^(1/2)

Rubi [A]

time = 0.25, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5879, 5915, 5953, 3388, 2211, 2235, 2236}

$$-\frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a} - \frac{15\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a} + x\cosh^{-1}(ax)^{5/2} - \frac{5\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^{3/2}}{2a} + \frac{15}{4}x\sqrt{\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^(5/2), x]

[Out] (15*x*Sqrt[ArcCosh[a*x]])/4 - (5*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(2*a) + x*ArcCosh[a*x]^(5/2) - (15*Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/(16*a) - (15*Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a)

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c^n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p_)
*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(ax)^{5/2} dx &= x \cosh^{-1}(ax)^{5/2} - \frac{1}{2}(5a) \int \frac{x \cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax} \sqrt{1+ax}} dx \\
&= -\frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{2a} + x \cosh^{-1}(ax)^{5/2} + \frac{15}{4} \int \sqrt{\cosh^{-1}(ax)} dx \\
&= \frac{15}{4} x \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{2a} + x \cosh^{-1}(ax)^{5/2} - \frac{1}{8} (15a) \\
&= \frac{15}{4} x \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{2a} + x \cosh^{-1}(ax)^{5/2} - \frac{15\sqrt{\pi}}{8} \\
&= \frac{15}{4} x \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{2a} + x \cosh^{-1}(ax)^{5/2} - \frac{15\sqrt{\pi}}{8} \\
&= \frac{15}{4} x \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{2a} + x \cosh^{-1}(ax)^{5/2} - \frac{15\sqrt{\pi}}{8} \\
&= \frac{15}{4} x \sqrt{\cosh^{-1}(ax)} - \frac{5\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}}{2a} + x \cosh^{-1}(ax)^{5/2} - \frac{15\sqrt{\pi}}{8}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 45, normalized size = 0.45

$$\frac{\sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{7}{2}, -\cosh^{-1}(ax)\right)}{\sqrt{-\cosh^{-1}(ax)}} + \Gamma\left(\frac{7}{2}, \cosh^{-1}(ax)\right)$$

$$2a$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^(5/2), x]`

```
[Out] ((Sqrt[ArcCosh[a*x]]*Gamma[7/2, -ArcCosh[a*x]])/Sqrt[-ArcCosh[a*x]] + Gamma[7/2, ArcCosh[a*x]])/(2*a)
```

Maple [A]

time = 3.53, size = 81, normalized size = 0.82

method	result
default	$ -\frac{-16\operatorname{arccosh}(ax)^{\frac{5}{2}} \sqrt{\pi} \operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right) + 40\operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} - 60 \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} \operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right) + 15\pi \operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right)}{16\sqrt{\pi} a} $

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/16*(-16*\operatorname{arccosh}(a*x)^{(5/2)}*\pi^{(1/2)}*a*x+40*\operatorname{arccosh}(a*x)^{(3/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}-60*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*a*x+15*\pi*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})+15*\pi*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)}))/\pi^{(1/2)}/a$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(5/2),x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)^(5/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**(5/2),x)`

[Out] `Integral(acosh(a*x)**(5/2), x)`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2),x)
```

```
[Out] int(acosh(a*x)^(5/2), x)
```

$$3.89 \quad \int \frac{\cosh^{-1}(ax)^{5/2}}{x} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cosh^{-1}(ax)^{5/2}}{x}, x\right)$$

[Out] Unintegrable(arccosh(a*x)^(5/2)/x, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{x} dx$$

Verification is not applicable to the result.

[In] Int[ArcCosh[a*x]^(5/2)/x, x]

[Out] Defer[Int][ArcCosh[a*x]^(5/2)/x, x]

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{x} dx = \int \frac{\cosh^{-1}(ax)^{5/2}}{x} dx$$

Mathematica [A]

time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{x} dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCosh[a*x]^(5/2)/x, x]

[Out] Integrate[ArcCosh[a*x]^(5/2)/x, x]

Maple [A]

time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{\text{arccosh}(ax)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccosh(a*x)^(5/2)/x,x)`

[Out] `int(arccosh(a*x)^(5/2)/x,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(5/2)/x,x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)^(5/2)/x, x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(5/2)/x,x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{5}{2}}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acosh(a*x)**(5/2)/x,x)`

[Out] `Integral(acosh(a*x)**(5/2)/x, x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccosh(a*x)^(5/2)/x,x, algorithm="giac")`

[Out] `integrate(arccosh(a*x)^(5/2)/x, x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{acosh}(ax)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^(5/2)/x,x)

[Out] int(acosh(a*x)^(5/2)/x, x)

$$3.90 \quad \int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=163

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erfi}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}$$

[Out] -1/160*erf(5^(1/2)*arccosh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/160*erfi(5^(1/2)*arccosh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-1/16*erf(arccosh(a*x)^(1/2))*Pi^(1/2)/a^5+1/16*erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a^5-1/32*erf(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+1/32*erfi(3^(1/2)*arccosh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5

Rubi [A]

time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5887, 5556, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{3\pi} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}} \operatorname{Erfi}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[ArcCosh[a*x]], x]

[Out] -1/16*(Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/a^5 - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(32*a^5) - (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(32*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/(16*a^5) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcCosh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcCosh[a*x]]])/(32*a^5)

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\sqrt{\cosh^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^4(x) \sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{x}} + \frac{3\sinh(3x)}{16\sqrt{x}} + \frac{\sinh(5x)}{16\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
 &= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{3\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{16a^5} \\
 &= -\frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{32a^5} - \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{32a^5} \\
 &= -\frac{\text{Subst}\left(\int e^{-5x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{5x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\text{Subst}\left(\int e^{-3x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} \\
 &= -\frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi} \operatorname{erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{5}} \operatorname{erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{32a^5}
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 150, normalized size = 0.92

$$\frac{\sqrt{5} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -5 \cosh^{-1}(ax)\right) + 5\sqrt{3} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -3 \cosh^{-1}(ax)\right) + 10 \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + 10 \Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) + 5\sqrt{3} \Gamma\left(\frac{1}{2}, 3 \cosh^{-1}(ax)\right) + \sqrt{5} \Gamma\left(\frac{1}{2}, 5 \cosh^{-1}(ax)\right)}{160a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[ArcCosh[a*x]], x]

[Out] ((Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -5*ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]]) + (5*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -3*ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + (10*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + 10*Gamma[1/2, ArcCosh[a*x]] + 5*Sqrt[3]*Gamma[1/2, 3*ArcCosh[a*x]] + Sqrt[5]*Gamma[1/2, 5*ArcCosh[a*x]]/(160*a^5)

Maple [F]

time = 8.22, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{arccosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccosh(a*x)^(1/2), x)**[Out]** int(x^4/arccosh(a*x)^(1/2), x)**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(1/2), x, algorithm="maxima")**[Out]** integrate(x^4/sqrt(arccosh(a*x)), x)**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**4/acosh(a*x)**(1/2),x)``[Out] Integral(x**4/sqrt(acosh(a*x)), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^4/arccosh(a*x)^(1/2),x, algorithm="giac")``[Out] integrate(x^4/sqrt(arccosh(a*x)), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^4/acosh(a*x)^(1/2),x)``[Out] int(x^4/acosh(a*x)^(1/2), x)`

$$3.91 \quad \int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=109

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4}$$

[Out] -1/16*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+1/16*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/32*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)/a^4+1/32*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)/a^4

Rubi [A]

time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5887, 5556, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[ArcCosh[a*x]], x]

[Out] -1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcCosh[a*x]]])/a^4 - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a^4) + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(8*a^4)

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] & IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^4} \\
 &= \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{16a^4} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^4} \\
 &= \frac{\text{Subst}\left(\int e^{-4x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{4x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^4} - \frac{\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{8a^4} \\
 &= \frac{\sqrt{\pi} \operatorname{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi} \operatorname{erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a^4}
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 101, normalized size = 0.93

$$\frac{\sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -4 \cosh^{-1}(ax)\right) + 2\sqrt{2} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2 \cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)} \left(2\sqrt{2} \Gamma\left(\frac{1}{2}, 2 \cosh^{-1}(ax)\right) + \Gamma\left(\frac{1}{2}, 4 \cosh^{-1}(ax)\right)\right)}{32a^4 \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[ArcCosh[a*x]], x]

[Out] (Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] + 2*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(2*Sqrt[2]*Gamma[1/2, 2*ArcCosh[a*x]] + Gamma[1/2, 4*ArcCosh[a*x]]))/(32*a^4*Sqrt[ArcCosh[a*x]])

Maple [A]

time = 9.33, size = 67, normalized size = 0.61

method	result
default	$-\frac{\sqrt{\pi} \sqrt{2} \left(\operatorname{erf}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) - \operatorname{erfi}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) \right)}{16a^4} - \frac{\sqrt{\pi} \left(\operatorname{erf}\left(2\sqrt{\operatorname{arccosh}(ax)}\right) - \operatorname{erfi}\left(2\sqrt{\operatorname{arccosh}(ax)}\right) \right)}{32a^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arccosh(a*x)^(1/2), x, method=_RETURNVERBOSE)

[Out] -1/16*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arccosh(a*x)^(1/2))-erfi(2^(1/2)*arccosh(a*x)^(1/2)))/a^4-1/32*Pi^(1/2)*(erf(2*arccosh(a*x)^(1/2))-erfi(2*arccosh(a*x)^(1/2)))/a^4

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(x^3/sqrt(arccosh(a*x)), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/acosh(a*x)**(1/2),x)

[Out] Integral(x**3/sqrt(acosh(a*x)), x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acosh(a*x)^(1/2),x)

[Out] int(x^3/acosh(a*x)^(1/2), x)

$$3.92 \quad \int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=105

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{8a^3}$$

[Out] $-1/24*\operatorname{erf}(3^{1/2}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{1/2}*Pi^{(1/2)}/a^3+1/24*\operatorname{erfi}(3^{1/2})*\operatorname{arccosh}(a*x)^{(1/2)}*3^{1/2}*Pi^{(1/2)}/a^3-1/8*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*Pi^{(1/2)}/a^3+1/8*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*Pi^{(1/2)}/a^3$

Rubi [A]

time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5887, 5556, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{8a^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]], x]$

[Out] $-1/8*(\operatorname{Sqrt}[Pi]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/a^3 - (\operatorname{Sqrt}[Pi/3]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^3) + (\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^3) + (\operatorname{Sqrt}[Pi/3]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^3)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[Pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[Pi]*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{NegQ}[b]$

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] & IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{x}} + \frac{\sinh(3x)}{4\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
 &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^3} \\
 &= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^3} \\
 &= \frac{\text{Subst}\left(\int e^{-3x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^3} - \frac{\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\text{Subst}\left(\int e^{3x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^3} \\
 &= \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3} - \frac{\sqrt{\frac{\pi}{3}} \operatorname{erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\pi} \operatorname{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^3}
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 100, normalized size = 0.95

$$\frac{\sqrt{3} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -3 \cosh^{-1}(ax)\right) + 3 \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)} \left(3 \Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) + \sqrt{3} \Gamma\left(\frac{1}{2}, 3 \cosh^{-1}(ax)\right)\right)}{24a^3 \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[ArcCosh[a*x]], x]

[Out] (Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -3*ArcCosh[a*x]] + 3*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]] + Sqrt[ArcCosh[a*x]]*(3*Gamma[1/2, ArcCosh[a*x]] + Sqrt[3]*Gamma[1/2, 3*ArcCosh[a*x]]))/(24*a^3*Sqrt[ArcCosh[a*x]])

Maple [F]

time = 5.43, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{arccosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccosh(a*x)^(1/2), x)

[Out] int(x^2/arccosh(a*x)^(1/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(x^2/sqrt(arccosh(a*x)), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/acosh(a*x)**(1/2),x)`

[Out] `Integral(x**2/sqrt(acosh(a*x)), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/arccosh(a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^2/sqrt(arccosh(a*x)), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/acosh(a*x)^(1/2),x)`

[Out] `int(x^2/acosh(a*x)^(1/2), x)`

3.93

$$\int \frac{x}{\sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=63

$$-\frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{4a^2}$$

[Out] $-1/8*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2+1/8*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2$

Rubi [A]

time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5887, 5556, 12, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{4a^2} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

[In] `Int[x/Sqrt[ArcCosh[a*x]], x]`

[Out] $-1/4*(\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/a^2 + (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a^2)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr`

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{\sqrt{\cosh^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
 &= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
 &= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a^2} \\
 &= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^2} \\
 &= -\frac{\text{Subst}\left(\int e^{-2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^2} + \frac{\text{Subst}\left(\int e^{2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^2} \\
 &= -\frac{\sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{4a^2}
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 49, normalized size = 0.78

$$\frac{\sqrt{\frac{\pi}{2}} \left(-\operatorname{Erf} \left(\sqrt{2} \sqrt{\cosh^{-1}(ax)} \right) + \operatorname{Erfi} \left(\sqrt{2} \sqrt{\cosh^{-1}(ax)} \right) \right)}{4a^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x/Sqrt[ArcCosh[a*x]],x]``[Out] (Sqrt[Pi/2]*(-Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]))/(4*a^2)`**Maple [A]**

time = 3.98, size = 37, normalized size = 0.59

method	result	size
default	$-\frac{\sqrt{\pi} \sqrt{2} \left(\operatorname{erf} \left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \right) - \operatorname{erfi} \left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \right) \right)}{8a^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/arccosh(a*x)^(1/2),x,method=_RETURNVERBOSE)``[Out] -1/8*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arccosh(a*x)^(1/2))-erfi(2^(1/2)*arccosh(a*x)^(1/2)))/a^2`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/arccosh(a*x)^(1/2),x, algorithm="maxima")``[Out] integrate(x/sqrt(arccosh(a*x)), x)`**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/arccosh(a*x)^(1/2),x, algorithm="fricas")``[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(a*x)**(1/2),x)

[Out] Integral(x/sqrt(acosh(a*x)), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(1/2),x, algorithm="giac")

[Out] integrate(x/sqrt(arccosh(a*x)), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x)^(1/2),x)

[Out] int(x/acosh(a*x)^(1/2), x)

$$3.94 \quad \int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=43

$$-\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}$$

[Out] $-1/2*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a+1/2*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a$

Rubi [A]

time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5881, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a} - \frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[ArcCosh[a*x]],x]`

[Out] $-1/2*(\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/a + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(2*a)$

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`
`> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /;` `FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=` `Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /;` `FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=` `Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /;` `FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a} \\ &= -\frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a} \\ &= -\frac{\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{\text{Subst}\left(\int e^{x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a} \\ &= -\frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a} + \frac{\sqrt{\pi} \operatorname{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{2a} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 45, normalized size = 1.05

$$\frac{\sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right)}{\sqrt{\cosh^{-1}(ax)}} + \Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[ArcCosh[a*x]], x]
```

```
[Out] ((Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]])/Sqrt[ArcCosh[a*x]] + Gamma[1/2, ArcCosh[a*x]])/(2*a)
```

Maple [A]

time = 2.71, size = 26, normalized size = 0.60

method	result	size
default	$-\frac{\sqrt{\pi} \left(\operatorname{erf} \left(\sqrt{\operatorname{arccosh}(ax)} \right) - \operatorname{erfi} \left(\sqrt{\operatorname{arccosh}(ax)} \right) \right)}{2a}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Pi^(1/2)*(erf(arccosh(a*x)^(1/2))-erfi(arccosh(a*x)^(1/2)))/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(acosh(a*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(arccosh(a*x)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acosh(a*x)^(1/2),x)
```

```
[Out] int(1/acosh(a*x)^(1/2), x)
```

$$3.95 \quad \int \frac{1}{x \sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=15

$$\text{Int} \left(\frac{1}{x \sqrt{\cosh^{-1}(ax)}}, x \right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^(1/2),x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \sqrt{\cosh^{-1}(ax)}} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*Sqrt[ArcCosh[a*x]]),x]

[Out] Defer[Int][1/(x*Sqrt[ArcCosh[a*x]]), x]

Rubi steps

$$\int \frac{1}{x \sqrt{\cosh^{-1}(ax)}} dx = \int \frac{1}{x \sqrt{\cosh^{-1}(ax)}} dx$$

Mathematica [A]

time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{\cosh^{-1}(ax)}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*Sqrt[ArcCosh[a*x]]),x]

[Out] Integrate[1/(x*Sqrt[ArcCosh[a*x]]), x]

Maple [A]

time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{\text{arccosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/x/arccosh(a*x)^(1/2),x)
```

Maxima [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{x \sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(acosh(a*x))), x)
```

Giac [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(arccosh(a*x))), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \sqrt{\operatorname{acosh}(a x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*acosh(a*x)^(1/2)),x)

[Out] int(1/(x*acosh(a*x)^(1/2)), x)

$$3.96 \quad \int \frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}}, x\right)$$

[Out] Unintegrable(1/x^2/arccosh(a*x)^(1/2), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}} dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*Sqrt[ArcCosh[a*x]]), x]

[Out] Defer[Int][1/(x^2*Sqrt[ArcCosh[a*x]]), x]

Rubi steps

$$\int \frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}} dx = \int \frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}} dx$$

Mathematica [A]

time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\cosh^{-1}(ax)}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*Sqrt[ArcCosh[a*x]]), x]

[Out] Integrate[1/(x^2*Sqrt[ArcCosh[a*x]]), x]

Maple [A]

time = 3.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\text{arccosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/x^2/arccosh(a*x)^(1/2),x)
```

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arccosh(a*x))), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(acosh(a*x))), x)
```

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt(arccosh(a*x))), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x^2 \sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*acosh(a*x)^(1/2)),x)

[Out] int(1/(x^2*acosh(a*x)^(1/2)), x)

$$3.97 \quad \int \frac{x^4}{\cosh^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=193

$$-\frac{2x^4 \sqrt{-1+ax} \sqrt{1+ax}}{a \sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{5\pi} \operatorname{Erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{16a^5}$$

[Out] $1/8*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^5+1/8*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^5+3/16*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5+3/16*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5+1/16*\operatorname{erf}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5+1/16*\operatorname{erfi}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5-2*x^4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5885, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi} \operatorname{Erf}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{5\pi} \operatorname{Erf}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi} \operatorname{Erfi}\left(\sqrt{3} \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{5\pi} \operatorname{Erfi}\left(\sqrt{5} \sqrt{\cosh^{-1}(ax)}\right)}{16a^5} - \frac{2x^4 \sqrt{ax-1} \sqrt{ax+1}}{a \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4/\operatorname{ArcCosh}[a*x]^{(3/2)}, x]$

[Out] $(-2*x^4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^5) + (3*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(16*a^5) + (\operatorname{Sqrt}[5*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(16*a^5) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^5) + (3*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(16*a^5) + (\operatorname{Sqrt}[5*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(16*a^5)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_.)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+dx]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\amp; \ \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+dx)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\amp; \ \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))(n_.)(x_)(m_.), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b2*c(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x(n +
1), Cosh[-a/b + x/b](m - 1)*(m - (m + 1))*Cosh[-a/b + x/b]2], x], x], x, a
+ b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\cosh^{-1}(ax)^{3/2}} dx &= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} - \frac{2\text{Subst}\left(\int\left(-\frac{\cosh(x)}{8\sqrt{x}} - \frac{9\cosh(3x)}{16\sqrt{x}} - \frac{5\cosh(5x)}{16\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^5} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int\frac{\cosh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^5} + \frac{5\text{Subst}\left(\int\frac{\cosh(5x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^5} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^5} + \frac{\text{Subst}\left(\int\frac{e^x}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{8a^5} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^5} + \frac{\text{Subst}\left(\int e^{x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{4a^5} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{16a^5}
\end{aligned}$$

Mathematica [A]

time = 0.23, size = 201, normalized size = 1.04

$$\frac{4\sqrt{\frac{-1+ax}{1+ax}}(1+ax) - \sqrt{5}\sqrt{-\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, -5\cosh^{-1}(ax)\right) - 3\sqrt{3}\sqrt{-\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, -3\cosh^{-1}(ax)\right) - 2\sqrt{-\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + 2\sqrt{\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) + 3\sqrt{3}\sqrt{\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, 3\cosh^{-1}(ax)\right) + \sqrt{5}\sqrt{\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, 5\cosh^{-1}(ax)\right) + 6\sinh(3\cosh^{-1}(ax)) + 2\sinh(5\cosh^{-1}(ax))}{16a^5\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/ArcCosh[a*x]^(3/2), x]

[Out] -1/16*(4*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) - Sqrt[5]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -5*ArcCosh[a*x]] - 3*Sqrt[3]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -3*ArcCosh[a*x]] - 2*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]] + 2*Sqrt[ArcCosh[a*x]]*Gamma[1/2, ArcCosh[a*x]] + 3*Sqrt[3]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 3*ArcCosh[a*x]] + Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 5*ArcCosh[a*x]] + 6*Sinh[3*ArcCosh[a*x]] + 2*Sinh[5*ArcCosh[a*x]])/(a^5*Sqrt[ArcCosh[a*x]])

Maple [F]

time = 5.98, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arccosh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccosh(a*x)^(3/2), x)**[Out]** int(x^4/arccosh(a*x)^(3/2), x)**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(3/2), x, algorithm="maxima")**[Out]** integrate(x^4/arccosh(a*x)^(3/2), x)**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/acosh(a*x)**(3/2),x)

[Out] Integral(x**4/acosh(a*x)**(3/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^4/arccosh(a*x)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/acosh(a*x)^(3/2),x)

[Out] int(x^4/acosh(a*x)^(3/2), x)

$$3.98 \quad \int \frac{x^3}{\cosh^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=143

$$-\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{a \sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi} \operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4}$$

[Out] $1/4*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4+1/4*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4+1/4*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4+1/4*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4-2*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5885, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2} \sqrt{\cosh^{-1}(ax)}\right)}{2a^4} - \frac{2x^3 \sqrt{ax-1} \sqrt{ax+1}}{a \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/\operatorname{ArcCosh}[a*x]^{(3/2)}, x]$

[Out] $(-2*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(4*a^4) + (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(2*a^4) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(4*a^4) + (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(2*a^4)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erf}[(c+d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /;$ Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] & LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\cosh^{-1}(ax)^{3/2}} dx &= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} - \frac{2\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2\sqrt{x}} - \frac{\cosh(4x)}{2\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^4} + \frac{\text{Subst}\left(\int\frac{\cosh(4x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^4} \\
 &= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int\frac{e^{-4x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a^4} + \frac{\text{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a^4} \\
 &= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-4x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a^4} + \frac{\text{Subst}\left(\int e^{-2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a^4} \\
 &= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a^4} + \frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{2a^4}
 \end{aligned}$$

Mathematica [A]

time = 0.13, size = 124, normalized size = 0.87

$$-\frac{\sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -4 \cosh^{-1}(ax)\right) - \sqrt{2} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2 \cosh^{-1}(ax)\right) + \sqrt{2} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, 2 \cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, 4 \cosh^{-1}(ax)\right) + 2 \sinh(2 \cosh^{-1}(ax)) + \sinh(4 \cosh^{-1}(ax))}{4a^4 \sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCosh[a*x]^(3/2), x]

[Out]
$$-1/4 * (-\sqrt{-\text{ArcCosh}[a*x]} * \text{Gamma}[1/2, -4 * \text{ArcCosh}[a*x]]) - \sqrt{2} * \sqrt{-\text{ArcCosh}[a*x]} * \text{Gamma}[1/2, -2 * \text{ArcCosh}[a*x]] + \sqrt{2} * \sqrt{\text{ArcCosh}[a*x]} * \text{Gamma}[1/2, 2 * \text{ArcCosh}[a*x]] + \sqrt{\text{ArcCosh}[a*x]} * \text{Gamma}[1/2, 4 * \text{ArcCosh}[a*x]] + 2 * \text{Sinh}[2 * \text{ArcCosh}[a*x]] + \text{Sinh}[4 * \text{ArcCosh}[a*x]]) / (a^4 * \sqrt{\text{ArcCosh}[a*x]})$$

Maple [A]

time = 10.02, size = 191, normalized size = 1.34

method	result
default	$-\frac{\sqrt{2} \left(2\sqrt{2} \sqrt{\text{arccosh}(ax)} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} a^{ax-\text{arccosh}(ax)\pi} \text{erf}\left(\sqrt{2} \sqrt{\text{arccosh}(ax)}\right) - \text{arccosh}(ax) \right)}{4\sqrt{\pi} a^4 \text{arccosh}(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arccosh(a*x)^(3/2), x, method=_RETURNVERBOSE)

[Out]
$$-1/4 * 2^{(1/2)} * (2 * 2^{(1/2)} * \text{arccosh}(a*x)^{(1/2)} * \text{Pi}^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a^{ax-\text{arccosh}(a*x)} * \text{Pi} * \text{erf}(2^{(1/2)} * \text{arccosh}(a*x)^{(1/2)}) - \text{arccosh}(a*x) * \text{Pi} * \text{erfi}(2^{(1/2)} * \text{arccosh}(a*x)^{(1/2)})) / \text{Pi}^{(1/2)} / a^4 / \text{arccosh}(a*x) - 1/4 * (8 * \text{arccosh}(a*x)^{(1/2)} * \text{Pi}^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a^{3*x^3-4 * \text{arccosh}(a*x)^{(1/2)} * \text{Pi}^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a^{ax-\text{arccosh}(a*x)} * \text{Pi} * \text{erf}(2 * \text{arccosh}(a*x)^{(1/2)}) - \text{arccosh}(a*x) * \text{Pi} * \text{erfi}(2 * \text{arccosh}(a*x)^{(1/2)})) / \text{Pi}^{(1/2)} / a^4 / \text{arccosh}(a*x)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x^3/arccosh(a*x)^(3/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/acosh(a*x)**(3/2), x)
```

Giac [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

```
time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{x^3}{\operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/acosh(a*x)^(3/2),x)
```

```
[Out] int(x^3/acosh(a*x)^(3/2), x)
```

$$3.99 \quad \int \frac{x^2}{\cosh^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=135

$$-\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}$$

[Out] $1/4*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^3+1/4*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^3+1/4*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^3+1/4*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^3-2*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5885, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} - \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/\operatorname{ArcCosh}[a*x]^{(3/2)}, x]$

[Out] $(-2*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a^3) + (\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a^3) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a^3) + (\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(4*a^3)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_.)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]))], x] /;$ FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erf}[(c+d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]))], x] /;$ Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] & LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\cosh^{-1}(ax)^{3/2}} dx &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} - \frac{2\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4\sqrt{x}} - \frac{3\cosh(3x)}{4\sqrt{x}}\right) dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int\frac{\cosh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{2a^3} + \frac{3\text{Subst}\left(\int\frac{\cosh(3x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^3} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int\frac{e^x}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{4a^3} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^3} + \frac{\text{Subst}\left(\int e^{x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a^3} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{4a^3}
 \end{aligned}$$

Mathematica [A]

time = 0.14, size = 139, normalized size = 1.03

$$\frac{2\sqrt{\frac{-1+ax}{1+ax}}(1+ax) - \sqrt{3}\sqrt{-\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, -3\cosh^{-1}(ax)\right) - \sqrt{-\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + \sqrt{\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) + \sqrt{3}\sqrt{\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, 3\cosh^{-1}(ax)\right) + 2\sinh(3\cosh^{-1}(ax))}{4a^3\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/ArcCosh[a*x]^(3/2), x]

[Out]
$$\frac{-1/4*(2*\text{Sqrt}[(-1 + a*x)/(1 + a*x)]*(1 + a*x) - \text{Sqrt}[3]*\text{Sqrt}[-\text{ArcCosh}[a*x]]*\text{Gamma}[1/2, -3*\text{ArcCosh}[a*x]] - \text{Sqrt}[-\text{ArcCosh}[a*x]]*\text{Gamma}[1/2, -\text{ArcCosh}[a*x]] + \text{Sqrt}[\text{ArcCosh}[a*x]]*\text{Gamma}[1/2, \text{ArcCosh}[a*x]] + \text{Sqrt}[3]*\text{Sqrt}[\text{ArcCosh}[a*x]]*\text{Gamma}[1/2, 3*\text{ArcCosh}[a*x]] + 2*\text{Sinh}[3*\text{ArcCosh}[a*x]])}{(a^3*\text{Sqrt}[\text{ArcCosh}[a*x]])}$$

Maple [F]

time = 3.99, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{arccosh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccosh(a*x)^(3/2), x)

[Out] int(x^2/arccosh(a*x)^(3/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x^2/arccosh(a*x)^(3/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2/acosh(a*x)**(3/2), x)``[Out] Integral(x**2/acosh(a*x)**(3/2), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/arccosh(a*x)^(3/2), x, algorithm="giac")``[Out] integrate(x^2/arccosh(a*x)^(3/2), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/acosh(a*x)^(3/2), x)``[Out] int(x^2/acosh(a*x)^(3/2), x)`

3.100 $\int \frac{x}{\cosh^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=89

$$-\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}$$

[Out] 1/2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/2*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-2*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5885, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2} - \frac{2x\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCosh[a*x]^(3/2), x]

[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/a^2

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{\cosh^{-1}(ax)^{3/2}} dx &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{2\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
 &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a^2} \\
 &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{2\text{Subst}\left(\int e^{-2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a^2} + \frac{2\text{Subst}\left(\int e^{2x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a^2} \\
 &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a^2}
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 63, normalized size = 0.71

$$\frac{\sqrt{\frac{\pi}{2}} \left(\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) + \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) \right) - \frac{\sinh(2\cosh^{-1}(ax))}{\sqrt{\cosh^{-1}(ax)}}}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCosh[a*x]^(3/2), x]

[Out] (Sqrt[Pi/2]*(Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]) - Sinh[2*ArcCosh[a*x]]/Sqrt[ArcCosh[a*x]])/a^2

Maple [A]

time = 5.23, size = 85, normalized size = 0.96

method	result
default	$-\frac{\sqrt{2} \left(2\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} ax - \operatorname{arccosh}(ax) \pi \operatorname{erf}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) - \operatorname{arccosh}(ax) \right)}{2\sqrt{\pi} a^2 \operatorname{arccosh}(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccosh(a*x)^(3/2), x, method=_RETURNVERBOSE)

[Out] -1/2*2^(1/2)*(2*2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-arccosh(a*x)*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))-arccosh(a*x)*Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2)))/Pi^(1/2)/a^2/arccosh(a*x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x/arccosh(a*x)^(3/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(a*x)**(3/2),x)

[Out] Integral(x/acosh(a*x)**(3/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x/arccosh(a*x)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x)^(3/2),x)

[Out] int(x/acosh(a*x)^(3/2), x)

3.101 $\int \frac{1}{\cosh^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}$$

[Out] erf(arccosh(a*x)^(1/2))*Pi^(1/2)/a+erfi(arccosh(a*x)^(1/2))*Pi^(1/2)/a-2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(1/2)

Rubi [A]

time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5880, 5953, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a} - \frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^(-3/2), x]

[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[ArcCosh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcCosh[a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcCosh[a*x]]])/a

Rule 2211

Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x
_))^(p_.)*((d2_.) + (e2_.)*(x_))^(q_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-1 + c*x)^q], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^{3/2}} dx &= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + (2a) \int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a} + \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{a} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{2\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{2\text{Subst}\left(\int e^{x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{\cosh^{-1}(ax)}} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a} + \frac{\sqrt{\pi} \operatorname{erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 76, normalized size = 1.12

$$\frac{-2\sqrt{\frac{-1+ax}{1+ax}}(1+ax) + \sqrt{-\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) - \sqrt{\cosh^{-1}(ax)}\Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right)}{a\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[ArcCosh[a*x]^(-3/2), x]`

```
[Out] (-2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -ArcCosh[a*x]] - Sqrt[ArcCosh[a*x]]*Gamma[1/2, ArcCosh[a*x]])/(a*Sqrt[ArcCosh[a*x]])
```

Maple [A]

time = 3.67, size = 66, normalized size = 0.97

method	result
default	$\frac{-2\sqrt{\operatorname{arccosh}(ax)}\sqrt{\pi}\sqrt{ax+1}\sqrt{ax-1} + \operatorname{arccosh}(ax)\pi \operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right) + \operatorname{arccosh}(ax)\pi \operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(ax)}\right)}{\sqrt{\pi} a \operatorname{arccosh}(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/arccosh(a*x)^(3/2), x, method=_RETURNVERBOSE)`

```
[Out] (-2*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)+arccosh(a*x)*Pi*erf(arccosh(a*x)^(1/2))+arccosh(a*x)*Pi*erfi(arccosh(a*x)^(1/2)))/Pi^(1/2)/a/arccosh(a*x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/arccosh(a*x)^(3/2), x, algorithm="maxima")``[Out] integrate(arccosh(a*x)^(-3/2), x)`**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/acosh(a*x)**(3/2),x)`

[Out] `Integral(acosh(a*x)**(-3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(arccosh(a*x)^(-3/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/acosh(a*x)^(3/2),x)`

[Out] `int(1/acosh(a*x)^(3/2), x)`

$$3.102 \quad \int \frac{1}{x \cosh^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)^{3/2}}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^(3/2), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^(3/2)), x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^(3/2)), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} dx = \int \frac{1}{x \cosh^{-1}(ax)^{3/2}} dx$$

Mathematica [A]

time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^(3/2)), x]

[Out] Integrate[1/(x*ArcCosh[a*x]^(3/2)), x]

Maple [A]

time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/arccosh(a*x)^(3/2),x)`

[Out] `int(1/x/arccosh(a*x)^(3/2),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*arccosh(a*x)^(3/2)), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/acosh(a*x)**(3/2),x)`

[Out] `Integral(1/(x*acosh(a*x)**(3/2)), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(1/(x*arccosh(a*x)^(3/2)), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*acosh(a*x)^(3/2)),x)

[Out] int(1/(x*acosh(a*x)^(3/2)), x)

3.103 $\int \frac{x^4}{\cosh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=228

$$-\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5}$$

[Out] $-1/12*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^5+1/12*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^5-3/8*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5+3/8*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5-5/24*\operatorname{erf}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5+5/24*\operatorname{erfi}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5-2/3*x^4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(3/2)}+16/3*x^3/a^2/\operatorname{arccosh}(a*x)^{(1/2)}-20/3*x^5/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.58, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5886, 5951, 5887, 5556, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} - \frac{5\sqrt{5\pi}\operatorname{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{24a^5} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a^5} + \frac{3\sqrt{3\pi}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{8a^5} + \frac{5\sqrt{5\pi}\operatorname{Erfi}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{24a^5} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4/\operatorname{ArcCosh}[a*x]^{(5/2)}, x]$

[Out] $(-2*x^4*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a*\operatorname{ArcCosh}[a*x]^{(3/2)}) + (16*x^3)/(3*a^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (20*x^5)/(3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(12*a^5) - (3*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^5) - (5*\operatorname{Sqrt}[5*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(24*a^5) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(12*a^5) + (3*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(8*a^5) + (5*\operatorname{Sqrt}[5*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(24*a^5)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_)*(e_)+(f_)*(x_))}/\operatorname{Sqrt}[(c_)+(d_)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

$\operatorname{Int}[(F_)^{((a_)+(b_)*((c_)+(d_)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))m*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]p*((c_.) + (d_.)*(x_))m*Sinh[(a_.) +
(b_.)*(x_)]n, x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]n*Cosh[a + b*x]p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))n*((c_.) + (d_.)*(x_))m, x_Symbol] := Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])n+1/(b*c*(n + 1)
)), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)n+1/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])n+1/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))n*((c_.) + (d_.)*(x_))m, x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))n*((f_.)*(x_))m)/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])n+1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])n+1, x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\cosh^{-1}(ax)^{5/2}} dx &= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{8\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}} dx}{3a} + \frac{1}{3}(10a) \int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} + \frac{100}{3} \int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{16\text{Subst}\left(\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx\right)}{3} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{16\text{Subst}\left(\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx\right)}{3} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} + \frac{25\text{Subst}\left(\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx\right)}{3} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{25\text{Subst}\left(\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx\right)}{3} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{25\text{Subst}\left(\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx\right)}{3} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{25\text{Subst}\left(\int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx\right)}{3} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a}
\end{aligned}$$

Mathematica [A]

time = 1.14, size = 278, normalized size = 1.22

$\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{16x^3}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{12a}$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/ArcCosh[a*x]^(5/2), x]

[Out] -1/24*(2*sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + (2*ArcCosh[a*x])/E^ArcCosh[a*x] + 2*E^ArcCosh[a*x]*ArcCosh[a*x] + 2*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -

```
ArcCosh[a*x]] - 2*ArcCosh[a*x]^(3/2)*Gamma[1/2, ArcCosh[a*x]] + 5*ArcCosh[a
*x]*(E^(-5*ArcCosh[a*x]) + E^(5*ArcCosh[a*x]) - Sqrt[5]*Sqrt[-ArcCosh[a*x]]
*Gamma[1/2, -5*ArcCosh[a*x]] - Sqrt[5]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 5*ArcC
osh[a*x]]) + 3*((3*ArcCosh[a*x])/E^(3*ArcCosh[a*x]) + 3*E^(3*ArcCosh[a*x])*
ArcCosh[a*x] + 3*Sqrt[3]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -3*ArcCosh[a*x]]
- 3*Sqrt[3]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 3*ArcCosh[a*x]] + Sinh[3*ArcCosh[
a*x]]) + Sinh[5*ArcCosh[a*x]])/(a^5*ArcCosh[a*x]^(3/2))
```

Maple [F]

time = 6.43, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arccosh}(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)^(5/2),x)
```

```
[Out] int(x^4/arccosh(a*x)^(5/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acosh}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/acosh(a*x)**(5/2),x)

[Out] Integral(x**4/acosh(a*x)**(5/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(5/2),x, algorithm="giac")

[Out] integrate(x^4/arccosh(a*x)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\operatorname{acosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/acosh(a*x)^(5/2),x)

[Out] int(x^4/acosh(a*x)^(5/2), x)

3.104 $\int \frac{x^3}{\cosh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=172

$$\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{\pi}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}$$

[Out] $-2/3*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4+2/3*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4-1/3*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4+1/3*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4-2/3*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(3/2)}+4*x^2/a^2/\operatorname{arccosh}(a*x)^{(1/2)}-16/3*x^4/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.51, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5886, 5951, 5887, 5556, 3389, 2211, 2235, 2236, 12}

$$-\frac{2\sqrt{\pi}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4} + \frac{2\sqrt{\pi}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4} + \frac{\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^4} + \frac{4x^2}{a^2\sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/\operatorname{ArcCosh}[a*x]^{(5/2)}, x]$

[Out] $(-2*x^3*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a*\operatorname{ArcCosh}[a*x]^{(3/2)}) + (4*x^2)/(a^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (16*x^4)/(3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (2*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a^4) - (\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a^4) + (2*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a^4) + (\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a^4)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]ⁿ*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((x_)^(m_.), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((x_)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[xⁿ*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5951

Int((((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\cosh^{-1}(ax)^{5/2}} dx &= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} - \frac{2 \int \frac{x^2}{\sqrt{-1+ax} \sqrt{1+ax} \cosh^{-1}(ax)^{3/2}} dx}{a} + \frac{1}{3}(8a) \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} + \frac{64}{3} \int \frac{x^3}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} - \frac{8 \operatorname{Subst}\left(\int \frac{\cosh(x)}{\sqrt{\cosh(x)}} dx\right)}{3} \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} - \frac{8 \operatorname{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} + \frac{8 \operatorname{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} - \frac{4 \operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} - \frac{8 \operatorname{Subst}\left(\int e^{-4x^2} dx\right)}{3} \\
&= -\frac{2x^3 \sqrt{-1+ax} \sqrt{1+ax}}{3a \cosh^{-1}(ax)^{3/2}} + \frac{4x^2}{a^2 \sqrt{\cosh^{-1}(ax)}} - \frac{16x^4}{3 \sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{\pi} \operatorname{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a^4}
\end{aligned}$$

Mathematica [A]

time = 0.55, size = 175, normalized size = 1.02

$$\frac{-4 \cosh^{-1}(ax) \left(e^{-4 \cosh^{-1}(ax)} + e^{4 \cosh^{-1}(ax)} - 2 \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -4 \cosh^{-1}(ax)\right) - 2 \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, 4 \cosh^{-1}(ax)\right) \right) - 2 \operatorname{Subst}\left(\int \frac{e^{-2 \cosh^{-1}(ax)} \left(e^{-2 \cosh^{-1}(ax)} + e^{2 \cosh^{-1}(ax)} - \sqrt{2} \sqrt{-\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2 \cosh^{-1}(ax)\right) - \sqrt{2} \sqrt{\cosh^{-1}(ax)} \Gamma\left(\frac{1}{2}, 2 \cosh^{-1}(ax)\right) \right) + \sinh(2 \cosh^{-1}(ax)) \right) - \sinh(4 \cosh^{-1}(ax))}{12 a^4 \cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCosh[a*x]^(5/2), x]

```
[Out] (-4*ArcCosh[a*x]*(E^(-4*ArcCosh[a*x])) + E^(4*ArcCosh[a*x]) - 2*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] - 2*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]]) - 2*(2*ArcCosh[a*x]*(E^(-2*ArcCosh[a*x])) + E^(2*ArcCosh[a*x]) - Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] - Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]]) + Sinh[2*ArcCosh[a*x]]) - Sinh[4*ArcCosh[a*x]])/(12*a^4*ArcCosh[a*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(132) = 264$.

time = 9.90, size = 277, normalized size = 1.61

method	result
default	$-\frac{\sqrt{2} \left(4 \operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} a^2 x^2 + \sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} a x + 2 \operatorname{arccosh}(ax)^2 \pi \operatorname{erf} \left(\frac{2 \sqrt{2} \sqrt{\operatorname{arccosh}(ax)}}{\sqrt{\pi}} \right) \right)}{6 \sqrt{\pi} a^4 \operatorname{arccosh}(ax)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*(4*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+2^(1/2)*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+2*arccosh(a*x)^2*Pi*erf(2^(1/2)*arccosh(a*x)^(1/2))-2*arccosh(a*x)^2*Pi*erfi(2^(1/2)*arccosh(a*x)^(1/2))-2*arccosh(a*x)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^4/arccosh(a*x)^2-1/3*(16*arccosh(a*x)^(3/2)*Pi^(1/2)*a^4*x^4+2*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^3*x^3-16*arccosh(a*x)^(3/2)*Pi^(1/2)*a^2*x^2-arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+2*arccosh(a*x)^2*Pi*erf(2*arccosh(a*x)^(1/2))-2*arccosh(a*x)^2*Pi*erfi(2*arccosh(a*x)^(1/2))+2*Pi^(1/2)*arccosh(a*x)^(3/2))/Pi^(1/2)/a^4/arccosh(a*x)^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arccosh(a*x)^(5/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acosh}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/acosh(a*x)**(5/2),x)

[Out] Integral(x**3/acosh(a*x)**(5/2), x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{acosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acosh(a*x)^(5/2),x)

[Out] int(x^3/acosh(a*x)^(5/2), x)

3.105 $\int \frac{x^2}{\cosh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=166

$$-\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3} - \frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3}$$

[Out] $-1/6*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^3+1/6*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^3-1/2*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^3+1/2*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^3-2/3*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(3/2)}+8/3*x/a^2/\operatorname{arccosh}(a*x)^{(1/2)}-4*x^3/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.43, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5886, 5951, 5887, 5556, 3389, 2211, 2235, 2236, 5881}

$$-\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3} - \frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{2a^3} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{2a^3} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} - \frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/\operatorname{ArcCosh}[a*x]^{(5/2)}, x]$

[Out] $(-2*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a*\operatorname{ArcCosh}[a*x]^{(3/2)}) + (8*x)/(3*a^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (4*x^3)/\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]] - (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(6*a^3) - (\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(2*a^3) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(6*a^3) + (\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(2*a^3)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d)}, x], x, \operatorname{Sqrt}[c+d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^{a*}\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_.) + (b_.)*(x_)^(p_.)*((c_.) + (d_.)*(x_)^m)*Sinh[(a_.) +
(b_.)*(x_)^(n_.)], x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n, x_Symbol] := Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n*(x_)^m, x_Symbol] := Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1
)), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n*(x_)^m, x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^n*((f_.)*(x_)^m)/(Sqrt[(d1
_ + (e1_.)*(x_)^2]*Sqrt[(d2_ + (e2_.)*(x_)^2])), x_Symbol] := Simp[(f*x)^m*((
a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1

, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\cosh^{-1}(ax)^{5/2}} dx &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}} dx}{3a} + (2a) \int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}} dx \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + 12\int \frac{x^2}{\sqrt{\cosh^{-1}(ax)}} dx \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} - \frac{8\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}} dx\right)}{3a^2} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx\right)}{3a^2} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{8\text{Subst}\left(\int e^{-x^2} dx\right)}{3a^2} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi}\operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a^3} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi}\operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a^3} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{8x}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{6a^3}
 \end{aligned}$$

Mathematica [A]

time = 0.50, size = 194, normalized size = 1.17

$$\frac{-\sqrt{\frac{-1+ax}{1+ax}}(1+ax) - 3e^{-3\cosh^{-1}(ax)}\cosh^{-1}(ax) - e^{-\cosh^{-1}(ax)}\cosh^{-1}(ax) - e^{\cosh^{-1}(ax)}\cosh^{-1}(ax) - 3e^{3\cosh^{-1}(ax)}\cosh^{-1}(ax) - 3\sqrt{3}(-\cosh^{-1}(ax))^{3/2}\Gamma\left(\frac{1}{2}, -3\cosh^{-1}(ax)\right) - (-\cosh^{-1}(ax))^{3/2}\Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + \cosh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) + 3\sqrt{3}\cosh^{-1}(ax)^{3/2}\Gamma\left(\frac{1}{2}, 3\cosh^{-1}(ax)\right) - \sinh(3\cosh^{-1}(ax))}{6a^3\cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/ArcCosh[a*x]^(5/2),x]

[Out] $(-\sqrt{(-1 + ax)/(1 + ax)}(1 + ax)) - (3\text{ArcCosh}[ax])/E^{(3\text{ArcCosh}[ax])} - \text{ArcCosh}[ax]/E^{\text{ArcCosh}[ax]} - E^{\text{ArcCosh}[ax]}\text{ArcCosh}[ax] - 3E^{(3\text{ArcCosh}[ax])}\text{ArcCosh}[ax] - 3\sqrt{3}(-\text{ArcCosh}[ax])^{(3/2)}\Gamma[1/2, -3\text{ArcCosh}[ax]] - (-\text{ArcCosh}[ax])^{(3/2)}\Gamma[1/2, -\text{ArcCosh}[ax]] + \text{ArcCosh}[ax]^{(3/2)}\Gamma[1/2, \text{ArcCosh}[ax]] + 3\sqrt{3}\text{ArcCosh}[ax]^{(3/2)}\Gamma[1/2, 3\text{ArcCosh}[ax]] - \text{Sinh}[3\text{ArcCosh}[ax]])/(6a^3\text{ArcCosh}[ax]^{(3/2)})$

Maple [F]

time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{arccosh}(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccosh(a*x)^(5/2),x)

[Out] int(x^2/arccosh(a*x)^(5/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(5/2),x, algorithm="maxima")

[Out] integrate(x^2/arccosh(a*x)^(5/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{acosh}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/acosh(a*x)**(5/2),x)`

[Out] `Integral(x**2/acosh(a*x)**(5/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/arccosh(a*x)^(5/2),x, algorithm="giac")`

[Out] `integrate(x^2/arccosh(a*x)^(5/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{acosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/acosh(a*x)^(5/2),x)`

[Out] `int(x^2/acosh(a*x)^(5/2), x)`

3.106 $\int \frac{x}{\cosh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=123

$$-\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2} + \frac{2\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2}$$

[Out] $-2/3*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2+2/3*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^2-2/3*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(3/2)}+4/3/a^2/\operatorname{arccosh}(a*x)^{(1/2)}-8/3*x^2/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.30, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5886, 5951, 5887, 5556, 12, 3389, 2211, 2235, 2236, 5893}

$$-\frac{2\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2} + \frac{2\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a^2} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x/\operatorname{ArcCosh}[a*x]^{(5/2)}, x]$

[Out] $(-2*x*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a*\operatorname{ArcCosh}[a*x]^{(3/2)}) + 4/(3*a^2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (8*x^2)/(3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (2*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a^2) + (2*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a^2)$

Rule 12

$\operatorname{Int}[(a_*)*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2211

$\operatorname{Int}[(F_)^{((g_*)*((e_*) + (f_*)*(x_)))/\operatorname{Sqrt}[(c_*) + (d_*)*(x_)]}, x_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_*) + (b_*)*((c_*) + (d_*)*(x_))^{(2)}), x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{PosQ}[b]$

Rule 2236

Int[(F_)^((a_) + (b_)*(c_) + (d_)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_) + (b_)*(x_)]^(p_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5886

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)^m, x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5887

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)^m, x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5893

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)/(Sqrt[(d1_) + (e1_)*(x_)]*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5951

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)/(Sqrt[(d1_) + (e1_)*(x_)]*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]

```

]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cosh^{-1}(ax)^{5/2}} dx &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}} dx}{3a} + \frac{1}{3}(4a) \int \frac{x}{\sqrt{-1+ax}} dx \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{16}{3} \int \frac{x}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{16\text{Subst}\left(\int \frac{\cosh^{-1}(2x)}{2\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{16\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} + \frac{8\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{4\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}} dx\right)}{3} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{8\text{Subst}\left(\int e^{-2x^2} dx\right)}{3} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{4}{3a^2\sqrt{\cosh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 83, normalized size = 0.67

$$\frac{4\cosh(2\cosh^{-1}(ax))}{\sqrt{\cosh^{-1}(ax)}} + 2\sqrt{2\pi}\left(\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) - \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)\right) + \frac{\sinh(2\cosh^{-1}(ax))}{\cosh^{-1}(ax)^{3/2}}$$

$3a^2$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCosh[a*x]^(5/2),x]

[Out] $-\frac{1}{3} \left(\frac{4 \cosh[2 \operatorname{ArcCosh}[a x]]}{\sqrt{\operatorname{ArcCosh}[a x]}} + 2 \sqrt{2 \pi} \left(\operatorname{Erf}[\sqrt{2} \sqrt{\operatorname{ArcCosh}[a x]}] - \operatorname{Erfi}[\sqrt{2} \sqrt{\operatorname{ArcCosh}[a x]}] \right) + \frac{\sinh[2 \operatorname{ArcCosh}[a x]]}{\operatorname{ArcCosh}[a x]^{3/2}} \right) / a^2$

Maple [A]

time = 3.84, size = 122, normalized size = 0.99

method	result
default	$-\frac{\sqrt{2} \left(4 \operatorname{arccosh}(a x)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} a^2 x^2 + \sqrt{2} \sqrt{\operatorname{arccosh}(a x)} \sqrt{\pi} \sqrt{a x + 1} \sqrt{a x - 1} a x + 2 \operatorname{arccosh}(a x)^2 \pi \operatorname{erf} \left(\sqrt{2} \sqrt{\operatorname{arccosh}(a x)} \right) - 2 \operatorname{arccosh}(a x)^2 \pi \operatorname{erfi} \left(\sqrt{2} \sqrt{\operatorname{arccosh}(a x)} \right) \right)}{3 \sqrt{\pi} a^2 \operatorname{arccosh}(a x)^{5/2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)

[Out] $-\frac{1}{3} 2^{1/2} \left(4 \operatorname{arccosh}(a x)^{3/2} 2^{1/2} \pi^{1/2} a^2 x^2 + 2^{1/2} \operatorname{arccosh}(a x)^{1/2} \pi^{1/2} (a x + 1)^{1/2} (a x - 1)^{1/2} a x + 2 \operatorname{arccosh}(a x)^2 \pi \operatorname{erf} \left(2^{1/2} \operatorname{arccosh}(a x)^{1/2} \right) - 2 \operatorname{arccosh}(a x)^2 \pi \operatorname{erfi} \left(2^{1/2} \operatorname{arccosh}(a x)^{1/2} \right) \right) / \pi^{1/2} / a^2 \operatorname{arccosh}(a x)^{5/2}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(5/2),x, algorithm="maxima")

[Out] integrate(x/arccosh(a*x)^(5/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}^{\frac{5}{2}}(a x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acosh(a*x)**(5/2),x)

[Out] Integral(x/acosh(a*x)**(5/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccosh(a*x)^(5/2),x, algorithm="giac")

[Out] integrate(x/arccosh(a*x)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{acosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acosh(a*x)^(5/2),x)

[Out] int(x/acosh(a*x)^(5/2), x)

3.107 $\int \frac{1}{\cosh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a} + \frac{2\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}$$

[Out] $-2/3*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a+2/3*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a-2/3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(3/2)}-4/3*x/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5880, 5951, 5881, 3389, 2211, 2235, 2236}

$$-\frac{2\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a} + \frac{2\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcCosh}[a*x]^{-5/2}, x]$

[Out] $(-2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(3*a*\operatorname{ArcCosh}[a*x]^{(3/2)}) - (4*x)/(3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (2*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a) + (2*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(3*a)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*(e_.)+(f_.)*(x_))}/\operatorname{Sqrt}[(c_.)+(d_.)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2))}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2))}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erf}[(c+d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{NegQ}[b]$

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_), x_Symbol] := Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_)*((f_.)*(x_))^(m_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^{5/2}} dx &= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} + \frac{1}{3}(2a) \int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}} dx \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} + \frac{4}{3} \int \frac{1}{\sqrt{\cosh^{-1}(ax)}} dx \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{3a} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}} dx, x, \cosh^{-1}(ax)\right)}{3a} + \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{4\text{Subst}\left(\int e^{-x^2} dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{3a} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\cosh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a} + \frac{2\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{3a}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 121, normalized size = 1.36

$$\frac{2e^{-\cosh^{-1}(ax)} \left(e^{\cosh^{-1}(ax)} \sqrt{\frac{-1+ax}{1+ax}} (1+ax) + \cosh^{-1}(ax) + e^{2\cosh^{-1}(ax)} \cosh^{-1}(ax) + e^{\cosh^{-1}(ax)} (-\cosh^{-1}(ax))^{3/2} \Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) - e^{\cosh^{-1}(ax)} \cosh^{-1}(ax)^{3/2} \Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) \right)}{3a \cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^(-5/2), x]

[Out] (-2*(E^ArcCosh[a*x]*Sqrt[(-1+a*x)/(1+a*x)]*(1+a*x) + ArcCosh[a*x] + E^(2*ArcCosh[a*x])*ArcCosh[a*x] + E^ArcCosh[a*x]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -ArcCosh[a*x]] - E^ArcCosh[a*x]*ArcCosh[a*x]^(3/2)*Gamma[1/2, ArcCosh[a*x]])/(3*a*E^ArcCosh[a*x]*ArcCosh[a*x]^(3/2))

Maple [A]

time = 3.49, size = 84, normalized size = 0.94

method	result
--------	--------

default	$-\frac{2\left(2\operatorname{arccosh}(ax)^{\frac{3}{2}}\sqrt{\pi}ax+\operatorname{arccosh}(ax)^2\pi\operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right)-\operatorname{arccosh}(ax)^2\pi\operatorname{erfi}\left(\sqrt{\operatorname{arccosh}(ax)}\right)+\sqrt{\operatorname{arccosh}(ax)}\right)}{3\sqrt{\pi}a\operatorname{arccosh}(ax)^2}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/arccosh(a*x)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $-2/3*(2*\operatorname{arccosh}(a*x)^{(3/2)}*\pi^{(1/2)}*a*x+\operatorname{arccosh}(a*x)^2*\pi*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})-\operatorname{arccosh}(a*x)^2*\pi*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})+\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)})/\pi^{(1/2)}/a/\operatorname{arccosh}(a*x)^2$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^(5/2),x, algorithm="maxima")`

[Out] `integrate(arccosh(a*x)^(-5/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccosh(a*x)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acosh}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/acosh(a*x)**(5/2),x)`

[Out] `Integral(acosh(a*x)**(-5/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(-5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acosh(a*x)^(5/2),x)
```

```
[Out] int(1/acosh(a*x)^(5/2), x)
```

$$3.108 \quad \int \frac{1}{x \cosh^{-1}(ax)^{5/2}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)^{5/2}}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^(5/2), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^(5/2)), x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^(5/2)), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} dx = \int \frac{1}{x \cosh^{-1}(ax)^{5/2}} dx$$

Mathematica [A]

time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^(5/2)), x]

[Out] Integrate[1/(x*ArcCosh[a*x]^(5/2)), x]

Maple [A]

time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/arccosh(a*x)^(5/2),x)`

[Out] `int(1/x/arccosh(a*x)^(5/2),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(5/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*arccosh(a*x)^(5/2)), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/acosh(a*x)**(5/2),x)`

[Out] `Integral(1/(x*acosh(a*x)**(5/2)), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(5/2),x, algorithm="giac")`

[Out] `integrate(1/(x*arccosh(a*x)^(5/2)), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{acosh}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*acosh(a*x)^(5/2)),x)`

[Out] `int(1/(x*acosh(a*x)^(5/2)), x)`

3.109 $\int \frac{x^4}{\cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=300

$$-\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{40x^4\sqrt{-1-ax}}{3a\sqrt{\cosh^{-1}(ax)}}$$

[Out] $16/15*x^3/a^2/\operatorname{arccosh}(a*x)^{(3/2)} - 4/3*x^5/\operatorname{arccosh}(a*x)^{(3/2)} + 1/30*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^5 + 1/30*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^5 + 9/20*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5 + 9/20*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5 + 5/12*\operatorname{erf}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5 + 5/12*\operatorname{erfi}(5^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*5^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^5 - 2/5*x^4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(5/2)} + 32/5*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3/\operatorname{arccosh}(a*x)^{(1/2)} - 40/3*x^4*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.57, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5886, 5951, 5885, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{30a^5} + \frac{9\sqrt{3\pi} \operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{20a^5} + \frac{5\sqrt{5\pi} \operatorname{Erf}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{12a^5} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{30a^5} + \frac{9\sqrt{3\pi} \operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{20a^5} + \frac{5\sqrt{5\pi} \operatorname{Erfi}\left(\sqrt{5}\sqrt{\cosh^{-1}(ax)}\right)}{12a^5} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} - \frac{40x^4\sqrt{-1-ax}\sqrt{ax+1}}{3a\sqrt{\cosh^{-1}(ax)}} - \frac{2x^4\sqrt{-1-ax}\sqrt{ax+1}}{5a\cosh^{-1}(ax)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4/\operatorname{ArcCosh}[a*x]^{(7/2)}, x]$

[Out] $(-2*x^4*\operatorname{Sqrt}[-1+ax]*\operatorname{Sqrt}[1+ax])/(5*a*\operatorname{ArcCosh}[a*x]^{(5/2)}) + (16*x^3)/(15*a^2*\operatorname{ArcCosh}[a*x]^{(3/2)}) - (4*x^5)/(3*\operatorname{ArcCosh}[a*x]^{(3/2)}) + (32*x^2*\operatorname{Sqrt}[-1+ax]*\operatorname{Sqrt}[1+ax])/(5*a^3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (40*x^4*\operatorname{Sqrt}[-1+ax]*\operatorname{Sqrt}[1+ax])/(3*a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(30*a^5) + (9*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(20*a^5) + (5*\operatorname{Sqrt}[5*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(12*a^5) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(30*a^5) + (9*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(20*a^5) + (5*\operatorname{Sqrt}[5*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(12*a^5)$

Rule 2211

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_.)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^{(a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])}, x] /;$ $\operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_))^{m_}*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^{n_}*(x_)^{m_}, x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^{n + 1}/(b*c*(n + 1))), x] + Dist[1/(b²*c^{m + 1}*(n + 1)), Subst[Int[ExpandTrigReduce[x^{n + 1}, Cosh[-a/b + x/b]^{m - 1}*(m - (m + 1)*Cosh[-a/b + x/b]²), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^{n_}*(x_)^{m_}, x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^{n + 1}/(b*c*(n + 1))), x] + (-Dist[c*(m + 1)/(b*(n + 1))), Int[x^{m + 1}*((a + b*ArcCosh[c*x])^{n + 1}/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^{m - 1}*((a + b*ArcCosh[c*x])^{n + 1}/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^{n_}*((f_.)*(x_))^{m_})/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^{n + 1}/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^{m - 1}*(a + b*ArcCosh[c*x])^{n + 1}, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\cosh^{-1}(ax)^{7/2}} dx &= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{8\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}} dx}{5a} + (2a) \int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}} dx \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{20}{3} \int \frac{x}{\cosh^{-1}(ax)^{3/2}} dx \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^4\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{16x^3}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\cosh^{-1}(ax)^{3/2}} + \frac{32x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}}
\end{aligned}$$

Mathematica [A]

time = 1.49, size = 374, normalized size = 1.25

Warning: Unable to verify antiderivative.

`[In] Integrate[x^4/ArcCosh[a*x]^(7/2), x]`

```

[Out] (-6*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) - (2*ArcCosh[a*x])/E^ArcCosh[a*x]
- 2*E^ArcCosh[a*x]*ArcCosh[a*x] + (4*ArcCosh[a*x]^2)/E^ArcCosh[a*x] - 4*E^A
rcCosh[a*x]*ArcCosh[a*x]^2 + 4*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -ArcCosh[a*
x]] - 4*ArcCosh[a*x]^(5/2)*Gamma[1/2, ArcCosh[a*x]] - 5*ArcCosh[a*x]*((1 -
10*ArcCosh[a*x])/E^(5*ArcCosh[a*x]) + E^(5*ArcCosh[a*x])*(1 + 10*ArcCosh[a*
x]) + 10*Sqrt[5]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -5*ArcCosh[a*x]] + 10*Sqr
t[5]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 5*ArcCosh[a*x]]) - (9*(ArcCosh[a*x] + E^

```

$$(6*\text{ArcCosh}[a*x])*\text{ArcCosh}[a*x] - 6*\text{ArcCosh}[a*x]^2 + 6*E^{(6*\text{ArcCosh}[a*x])}*\text{ArcCosh}[a*x]^2 - 6*\text{Sqrt}[3]*E^{(3*\text{ArcCosh}[a*x])}*(-\text{ArcCosh}[a*x])^{(5/2)}*\text{Gamma}[1/2, -3*\text{ArcCosh}[a*x]] + 6*\text{Sqrt}[3]*E^{(3*\text{ArcCosh}[a*x])}*\text{ArcCosh}[a*x]^{(5/2)}*\text{Gamma}[1/2, 3*\text{ArcCosh}[a*x]] + E^{(3*\text{ArcCosh}[a*x])}*\text{Sinh}[3*\text{ArcCosh}[a*x]])/E^{(3*\text{ArcCosh}[a*x])} - 3*\text{Sinh}[5*\text{ArcCosh}[a*x]]/(120*a^5*\text{ArcCosh}[a*x]^{(5/2)})$$

Maple [F]

time = 6.22, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\text{arccosh}(ax)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccosh(a*x)^(7/2),x)

[Out] int(x^4/arccosh(a*x)^(7/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(7/2),x, algorithm="maxima")

[Out] integrate(x^4/arccosh(a*x)^(7/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\text{acosh}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/acosh(a*x)**(7/2),x)

[Out] Integral(x**4/acosh(a*x)**(7/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccosh(a*x)^(7/2),x, algorithm="giac")

[Out] integrate(x^4/arccosh(a*x)^(7/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\operatorname{acosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/acosh(a*x)^(7/2),x)

[Out] int(x^4/acosh(a*x)^(7/2), x)

3.110 $\int \frac{x^3}{\cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=244

$$-\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}\sqrt{1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} - \frac{128x^3\sqrt{-1+ax}}{15a\sqrt{\cosh^{-1}(ax)}}$$

[Out] $4/5*x^2/a^2/\operatorname{arccosh}(a*x)^{(3/2)} - 16/15*x^4/\operatorname{arccosh}(a*x)^{(3/2)} + 16/15*\operatorname{erf}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4 + 16/15*\operatorname{erfi}(2*\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a^4 + 4/15*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4 + 4/15*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/a^4 - 2/5*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(5/2)} + 16/5*x*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3/\operatorname{arccosh}(a*x)^{(1/2)} - 128/15*x^3*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.49, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5886, 5951, 5885, 3388, 2211, 2235, 2236}

$$\frac{16\sqrt{\pi}\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{15a^4} + \frac{4\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^4} + \frac{16\sqrt{\pi}\operatorname{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{15a^4} + \frac{4\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^4} + \frac{16x\sqrt{ax-1}\sqrt{ax+1}}{5a^3\sqrt{\cosh^{-1}(ax)}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} - \frac{128x^3\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} - \frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/\operatorname{ArcCosh}[a*x]^{(7/2)}, x]$

[Out] $(-2*x^3*\operatorname{Sqrt}[-1+ax]*\operatorname{Sqrt}[1+ax])/(5*a*\operatorname{ArcCosh}[a*x]^{(5/2)}) + (4*x^2)/(5*a^2*\operatorname{ArcCosh}[a*x]^{(3/2)}) - (16*x^4)/(15*\operatorname{ArcCosh}[a*x]^{(3/2)}) + (16*x*\operatorname{Sqrt}[-1+ax]*\operatorname{Sqrt}[1+ax])/(5*a^3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (128*x^3*\operatorname{Sqrt}[-1+ax]*\operatorname{Sqrt}[1+ax])/(15*a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (16*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(15*a^4) + (4*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(15*a^4) + (16*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[2*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(15*a^4) + (4*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]])/(15*a^4)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))m_*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))n_*(x_)m_, x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])n + 1/(b*c*(n + 1)
)), x] + Dist[1/(b2*cm + 1*(n + 1)), Subst[Int[ExpandTrigReduce[xn + 1
], Cosh[-a/b + x/b]m - 1*(m - (m + 1)*Cosh[-a/b + x/b]2), x], x], x, a
+ b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))n_*(x_)m_, x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])n + 1/(b*c*(n + 1)
)), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[xm + 1*((a + b*ArcCosh[c*x]
)n + 1/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[xm - 1*((a + b*ArcCosh[c*x])n + 1/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))n_*((f_.)*(x_)m_)/(Sqrt[(d1
_) + (e1_.)*(x_)])*(Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)m*((a
+ b*ArcCosh[c*x])n + 1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)m - 1*(a + b*ArcCosh[c*x])n + 1, x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\cosh^{-1}(ax)^{7/2}} dx &= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{6\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}} dx}{5a} + \frac{1}{5}(8a)\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}} dx \\
&= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{64}{15}\int \frac{x^3}{\cosh^{-1}(ax)^{5/2}} dx \\
&= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4x^2}{5a^2\cosh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\cosh^{-1}(ax)^{3/2}} + \frac{16x\sqrt{-1+ax}}{5a^3\sqrt{\cosh^{-1}(ax)}}
\end{aligned}$$

Mathematica [A]

time = 0.41, size = 291, normalized size = 1.19

$$\frac{3 - 3e^{8\operatorname{ArcCosh}[ax]} - 8\operatorname{ArcCosh}[ax] - 8e^{8\operatorname{ArcCosh}[ax]}\operatorname{ArcCosh}[ax] + 64\operatorname{ArcCosh}[ax]^2 - 64e^{8\operatorname{ArcCosh}[ax]}\operatorname{ArcCosh}[ax]^2 + 128e^{4\operatorname{ArcCosh}[ax]}(-\operatorname{ArcCosh}[ax])^{5/2}\Gamma[1/2, -4\operatorname{ArcCosh}[ax]] - 8e^{2\operatorname{ArcCosh}[ax]}(3ae^{2\operatorname{ArcCosh}[ax]}x\sqrt{(-1+ax)/(1+ax)}(1+ax) + \operatorname{ArcCosh}[ax] + e^{4\operatorname{ArcCosh}[ax]}\operatorname{ArcCosh}[ax] - 4\operatorname{ArcCosh}[ax]^2 + 4e^{4\operatorname{ArcCosh}[ax]}\operatorname{ArcCosh}[ax]^2 - 4\sqrt{2}e^{2\operatorname{ArcCosh}[ax]}(-\operatorname{ArcCosh}[ax])^{5/2}\Gamma[1/2, 2\operatorname{ArcCosh}[ax]]) - 128e^{2\operatorname{ArcCosh}[ax]}\operatorname{ArcCosh}[ax]^{5/2}\Gamma[1/2, 4\operatorname{ArcCosh}[ax]]}{128a^3\cosh^{-1}(ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/ArcCosh[a*x]^(7/2), x]

[Out] (3 - 3*E^(8*ArcCosh[a*x]) - 8*ArcCosh[a*x] - 8*E^(8*ArcCosh[a*x])*ArcCosh[a*x] + 64*ArcCosh[a*x]^2 - 64*E^(8*ArcCosh[a*x])*ArcCosh[a*x]^2 + 128*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -4*ArcCosh[a*x]] - 8*E^(2*ArcCosh[a*x])*(3*a*E^(2*ArcCosh[a*x])*x*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + ArcCosh[a*x] + E^(4*ArcCosh[a*x])*ArcCosh[a*x] - 4*ArcCosh[a*x]^2 + 4*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^2 - 4*Sqrt[2]*E^(2*ArcCosh[a*x])*(-ArcCosh[a*x])^(5/2)*Gamma[1/2, -2*ArcCosh[a*x]] + 4*Sqrt[2]*E^(2*ArcCosh[a*x])*ArcCosh[

$$a*x]^{(5/2)}*\text{Gamma}[1/2, 2*\text{ArcCosh}[a*x]] - 128*E^{(4*\text{ArcCosh}[a*x])}*\text{ArcCosh}[a*x]^{(5/2)}*\text{Gamma}[1/2, 4*\text{ArcCosh}[a*x]]/(120*a^4*E^{(4*\text{ArcCosh}[a*x])}*\text{ArcCosh}[a*x]^{(5/2)})$$

Maple [A]

time = 9.39, size = 366, normalized size = 1.50

method	result
default	$\frac{\sqrt{2} \left(-16 \operatorname{arccosh}(ax)^{\frac{5}{2}} \sqrt{2} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} a x^{-4 \operatorname{arccosh}(ax)^{\frac{3}{2}}} \sqrt{2} \sqrt{\pi} a^2 x^2 - 3 \sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arccosh(a*x)^(7/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{30} * 2^{(1/2)} * (-16 * \operatorname{arccosh}(a*x)^{(5/2)} * 2^{(1/2)} * \pi^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a*x - 4 * \operatorname{arccosh}(a*x)^{(3/2)} * 2^{(1/2)} * \pi^{(1/2)} * a^2 * x^2 - 3 * 2^{(1/2)} * \operatorname{arccosh}(a*x)^{(1/2)} * \pi^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a*x + 8 * \operatorname{arccosh}(a*x)^3 * \pi * \operatorname{erf}(2^{(1/2)} * \operatorname{arccosh}(a*x)^{(1/2)}) + 8 * \operatorname{arccosh}(a*x)^3 * \pi * \operatorname{erfi}(2^{(1/2)} * \operatorname{arccosh}(a*x)^{(1/2)}) + 2 * \operatorname{arccosh}(a*x)^{(3/2)} * 2^{(1/2)} * \pi^{(1/2)}) / \pi^{(1/2)} / a^4 / \operatorname{arccosh}(a*x)^3 + 1/15 * (-128 * (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)} * \pi^{(1/2)} * \operatorname{arccosh}(a*x)^{(5/2)} * a^3 * x^3 - 16 * \operatorname{arccosh}(a*x)^{(3/2)} * \pi^{(1/2)} * a^4 * x^4 - 6 * \operatorname{arccosh}(a*x)^{(1/2)} * \pi^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a^3 * x^3 + 64 * (a*x-1)^{(1/2)} * (a*x+1)^{(1/2)} * \pi^{(1/2)} * \operatorname{arccosh}(a*x)^{(5/2)} * a*x + 16 * \operatorname{arccosh}(a*x)^{(3/2)} * \pi^{(1/2)} * a^2 * x^2 + 3 * \operatorname{arccosh}(a*x)^{(1/2)} * \pi^{(1/2)} * (a*x+1)^{(1/2)} * (a*x-1)^{(1/2)} * a*x + 16 * \operatorname{arccosh}(a*x)^3 * \pi * \operatorname{erf}(2 * \operatorname{arccosh}(a*x)^{(1/2)}) + 16 * \operatorname{arccosh}(a*x)^3 * \pi * \operatorname{erfi}(2 * \operatorname{arccosh}(a*x)^{(1/2)}) - 2 * \pi^{(1/2)} * \operatorname{arccosh}(a*x)^{(3/2)}) / \pi^{(1/2)} / a^4 / \operatorname{arccosh}(a*x)^3$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccosh(a*x)^(7/2),x, algorithm="maxima")`

[Out] `integrate(x^3/arccosh(a*x)^(7/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccosh(a*x)^(7/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acosh}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/acosh(a*x)**(7/2),x)

[Out] Integral(x**3/acosh(a*x)**(7/2), x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccosh(a*x)^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\operatorname{acosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acosh(a*x)^(7/2),x)

[Out] int(x^3/acosh(a*x)^(7/2), x)

3.111 $\int \frac{x^2}{\cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=237

$$-\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}\sqrt{1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} - \frac{24x^2\sqrt{-1+ax}}{5a\sqrt{\cosh^{-1}(ax)}}$$

[Out] $8/15*x/a^2/\operatorname{arccosh}(a*x)^{(3/2)} - 4/5*x^3/\operatorname{arccosh}(a*x)^{(3/2)} + 1/15*\operatorname{erf}(\operatorname{arccosh}(a*x))^{(1/2)}*Pi^{(1/2)}/a^3 + 1/15*\operatorname{erfi}(\operatorname{arccosh}(a*x))^{(1/2)}*Pi^{(1/2)}/a^3 + 3/5*\operatorname{erf}(3^{(1/2)}*\operatorname{arccosh}(a*x))^{(1/2)}*3^{(1/2)}*Pi^{(1/2)}/a^3 + 3/5*\operatorname{erfi}(3^{(1/2)}*\operatorname{arccosh}(a*x))^{(1/2)}*3^{(1/2)}*Pi^{(1/2)}/a^3 - 2/5*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(5/2)} + 16/15*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a^3/\operatorname{arccosh}(a*x)^{(1/2)} - 24/5*x^2*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.52, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5886, 5951, 5885, 3388, 2211, 2235, 2236, 5880, 5953}

$$\frac{\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a^2} + \frac{3\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{5a^2} + \frac{\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a^2} + \frac{3\sqrt{3\pi}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\cosh^{-1}(ax)}\right)}{5a^2} + \frac{16\sqrt{ax-1}\sqrt{ax+1}}{15a^3\sqrt{\cosh^{-1}(ax)}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} - \frac{24x^2\sqrt{ax-1}\sqrt{ax+1}}{5a\sqrt{\cosh^{-1}(ax)}} - \frac{2x^2\sqrt{ax-1}\sqrt{ax+1}}{5a\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/\operatorname{ArcCosh}[a*x]^{(7/2)}, x]$

[Out] $(-2*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(5*a*\operatorname{ArcCosh}[a*x]^{(5/2)}) + (8*x)/(15*a^2*\operatorname{ArcCosh}[a*x]^{(3/2)}) - (4*x^3)/(5*\operatorname{ArcCosh}[a*x]^{(3/2)}) + (16*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(15*a^3*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) - (24*x^2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(5*a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (\operatorname{Sqrt}[Pi]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(15*a^3) + (3*\operatorname{Sqrt}[3*Pi]*\operatorname{Erf}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(5*a^3) + (\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(15*a^3) + (3*\operatorname{Sqrt}[3*Pi]*\operatorname{Erfi}[\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(5*a^3)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_)*(e_)+(f_)*(x_))}/\operatorname{Sqrt}[(c_)+(d_)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

$\operatorname{Int}[(F_)^{((a_)+(b_)*((c_)+(d_)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[Pi]*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E(I*k*Pi)*E(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E(I*k*Pi)*E(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_), x_Symbol] := Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_)*(x_)(m_.), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b2*c(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x(n +
1), Cosh[-a/b + x/b](m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]2), x], x], x, a
+ b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_)*(x_)(m_.), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*(m + 1)/(b*(n + 1))), Int[x(m + 1)*((a + b*ArcCosh[c*x]
)(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x(m - 1)*((a + b*ArcCosh[c*x])(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int((((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_)*((f_.)*(x_))(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)m*((a
+ b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)(m - 1)*(a + b*ArcCosh[c*x])(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
```

, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\cosh^{-1}(ax)^{7/2}} dx &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}} dx}{5a} + \frac{1}{5}(6a)\int \frac{x}{\cosh^{-1}(ax)^{5/2}} dx \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{12}{5}\int \frac{x}{\cosh^{-1}(ax)^{5/2}} dx \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}} \\
 &= -\frac{2x^2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{8x}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\cosh^{-1}(ax)^{3/2}} + \frac{16\sqrt{-1+ax}}{15a^3\sqrt{\cosh^{-1}(ax)}}
 \end{aligned}$$

Mathematica [A]

time = 0.58, size = 286, normalized size = 1.21

Integrate[...]

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/ArcCosh[a*x]^(7/2),x]

[Out] $(-(E^{(2 \operatorname{ArcCosh}[a*x])} * (3 * E^{\operatorname{ArcCosh}[a*x]} * \operatorname{Sqrt}[(-1 + a*x)/(1 + a*x)] * (1 + a*x) + \operatorname{ArcCosh}[a*x] + E^{(2 \operatorname{ArcCosh}[a*x])} * \operatorname{ArcCosh}[a*x] - 2 * \operatorname{ArcCosh}[a*x]^2 + 2 * E^{(2 \operatorname{ArcCosh}[a*x])} * \operatorname{ArcCosh}[a*x]^2 - 2 * E^{\operatorname{ArcCosh}[a*x]} * (-\operatorname{ArcCosh}[a*x])^{(5/2)} * \Gamma[1/2, -\operatorname{ArcCosh}[a*x]] + 2 * E^{\operatorname{ArcCosh}[a*x]} * \operatorname{ArcCosh}[a*x]^{(5/2)} * \Gamma[1/2, \operatorname{ArcCosh}[a*x]])) - 3 * (\operatorname{ArcCosh}[a*x] + E^{(6 \operatorname{ArcCosh}[a*x])} * \operatorname{ArcCosh}[a*x] - 6 * \operatorname{ArcCosh}[a*x]^2 + 6 * E^{(6 \operatorname{ArcCosh}[a*x])} * \operatorname{ArcCosh}[a*x]^2 - 6 * \operatorname{Sqrt}[3] * E^{(3 \operatorname{ArcCosh}[a*x])} * (-\operatorname{ArcCosh}[a*x])^{(5/2)} * \Gamma[1/2, -3 * \operatorname{ArcCosh}[a*x]] + 6 * \operatorname{Sqrt}[3] * E^{(3 \operatorname{ArcCosh}[a*x])} * \operatorname{ArcCosh}[a*x]^{(5/2)} * \Gamma[1/2, 3 * \operatorname{ArcCosh}[a*x]] + E^{(3 \operatorname{ArcCosh}[a*x])} * \operatorname{Sinh}[3 * \operatorname{ArcCosh}[a*x]])) / (30 * a^3 * E^{(3 \operatorname{ArcCosh}[a*x])} * \operatorname{ArcCosh}[a*x]^{(5/2)})$

Maple [F]

time = 4.20, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arccosh}(ax)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccosh(a*x)^(7/2),x)

[Out] int(x^2/arccosh(a*x)^(7/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(7/2),x, algorithm="maxima")

[Out] integrate(x^2/arccosh(a*x)^(7/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccosh(a*x)^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acosh}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2/acosh(a*x)**(7/2), x)``[Out] Integral(x**2/acosh(a*x)**(7/2), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/arccosh(a*x)^(7/2), x, algorithm="giac")``[Out] integrate(x^2/arccosh(a*x)^(7/2), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\operatorname{acosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/acosh(a*x)^(7/2), x)``[Out] int(x^2/acosh(a*x)^(7/2), x)`

3.112 $\int \frac{x}{\cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=157

$$-\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{8\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2}$$

[Out] 4/15/a^2/arccosh(a*x)^(3/2)-8/15*x^2/arccosh(a*x)^(3/2)+8/15*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+8/15*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-2/5*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(5/2)-32/15*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(1/2)

Rubi [A]

time = 0.29, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5886, 5951, 5885, 3388, 2211, 2235, 2236, 5893}

$$\frac{8\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2} + \frac{8\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{15a^2} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{ax-1}\sqrt{ax+1}}{15a\sqrt{\cosh^{-1}(ax)}} - \frac{2x\sqrt{ax-1}\sqrt{ax+1}}{5a\cosh^{-1}(ax)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCosh[a*x]^(7/2), x]

[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(5*a*ArcCosh[a*x]^(5/2)) + 4/(15*a^2*ArcCosh[a*x]^(3/2)) - (8*x^2)/(15*ArcCosh[a*x]^(3/2)) - (32*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*Sqrt[ArcCosh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(15*a^2) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(15*a^2)

Rule 2211

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[{\$UseGamma}

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rule 5886

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/(Sqrt[(d1_.) + (e1_.)*(x_)])*Sqrt[(d2_.) + (e2_.)*(x_)], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))/(Sqrt[(d1_.) + (e1_.)*(x_)])*Sqrt[(d2_.) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\cosh^{-1}(ax)^{7/2}} dx &= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{2\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}} dx}{5a} + \frac{1}{5}(4a) \int \frac{1}{\sqrt{-1+ax}} dx \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} + \frac{16}{15} \int \frac{x}{\cosh^{-1}(ax)^{5/2}} dx \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} \\
&= -\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} + \frac{4}{15a^2\cosh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\cosh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{-1+ax}}{15a\sqrt{\cosh^{-1}(ax)}}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 91, normalized size = 0.58

$$-\frac{\frac{4\cosh(2\cosh^{-1}(ax))}{\cosh^{-1}(ax)^{3/2}} - 8\sqrt{2\pi} \left(\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) + \operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) \right) + \frac{(3+16\cosh^{-1}(ax)^2)\sinh(2\cosh^{-1}(ax))}{\cosh^{-1}(ax)^{5/2}}}{15a^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x/ArcCosh[a*x]^(7/2), x]`

```
[Out] -1/15*((4*Cosh[2*ArcCosh[a*x]])/ArcCosh[a*x]^(3/2) - 8*Sqrt[2*Pi]*(Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]) + ((3 + 16*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]])/ArcCosh[a*x]^(5/2))/a^2
```

Maple [A]

time = 4.13, size = 153, normalized size = 0.97

method	result
default	$-\frac{\sqrt{2} \left(16\operatorname{arccosh}(ax)^{\frac{5}{2}} \sqrt{2} \sqrt{\pi} \sqrt{ax+1} \sqrt{ax-1} ax+4\operatorname{arccosh}(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} a^2x^2+3\sqrt{2} \sqrt{\operatorname{arccosh}(ax)} \right)}{15a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/arccosh(a*x)^(7/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/15*2^{(1/2)}*(16*\operatorname{arccosh}(a*x)^{(5/2)}*2^{(1/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x+4*\operatorname{arccosh}(a*x)^{(3/2)}*2^{(1/2)}*\pi^{(1/2)}*a^2*x^2+3*2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)}*\pi^{(1/2)}*(a*x+1)^{(1/2)}*(a*x-1)^{(1/2)}*a*x-8*\operatorname{arccosh}(a*x)^3*\pi*\operatorname{erf}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})-8*\operatorname{arccosh}(a*x)^3*\pi*\operatorname{erfi}(2^{(1/2)}*\operatorname{arccosh}(a*x)^{(1/2)})-2*\operatorname{arccosh}(a*x)^{(3/2)}*2^{(1/2)}*\pi^{(1/2)})/\pi^{(1/2)}/a^2/\operatorname{arccosh}(a*x)^3$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/arccosh(a*x)^(7/2),x, algorithm="maxima")`

[Out] `integrate(x/arccosh(a*x)^(7/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/arccosh(a*x)^(7/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acosh}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/acosh(a*x)**(7/2),x)`

[Out] `Integral(x/acosh(a*x)**(7/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/arccosh(a*x)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{acosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/acosh(a*x)^(7/2),x)
```

```
[Out] int(x/acosh(a*x)^(7/2), x)
```

3.113 $\int \frac{1}{\cosh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=122

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a\cosh^{-1}(ax)^{5/2}} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a}$$

[Out] $-4/15*x/\operatorname{arccosh}(a*x)^{(3/2)}+4/15*\operatorname{erf}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a+4/15*\operatorname{erfi}(\operatorname{arccosh}(a*x)^{(1/2)})*\operatorname{Pi}^{(1/2)}/a-2/5*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(5/2)}-8/15*(a*x-1)^{(1/2)}*(a*x+1)^{(1/2)}/a/\operatorname{arccosh}(a*x)^{(1/2)}$

Rubi [A]

time = 0.26, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5880, 5951, 5953, 3388, 2211, 2235, 2236}

$$\frac{4\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15a} - \frac{4x}{15\cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{ax-1}\sqrt{ax+1}}{15a\sqrt{\cosh^{-1}(ax)}} - \frac{2\sqrt{ax-1}\sqrt{ax+1}}{5a\cosh^{-1}(ax)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcCosh}[a*x]^{-7/2}, x]$

[Out] $(-2*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(5*a*\operatorname{ArcCosh}[a*x]^{(5/2)}) - (4*x)/(15*\operatorname{ArcCosh}[a*x]^{(3/2)}) - (8*\operatorname{Sqrt}[-1+a*x]*\operatorname{Sqrt}[1+a*x])/(15*a*\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]) + (4*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(15*a) + (4*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{ArcCosh}[a*x]]])/(15*a)$

Rule 2211

$\operatorname{Int}[(F_)^{((g_)*(e_)+(f_)*(x_))}/\operatorname{Sqrt}[(c_)+(d_)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_)+(b_)*((c_)+(d_)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}*(\operatorname{Erfi}[(c+d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]))], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_)+(b_)*((c_)+(d_)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}*(\operatorname{Erf}[(c+d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]))], x] /;$ Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_)^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5880

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_), x_Symbol] := Simp[Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)), Int[x*(a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

Rule 5951

Int[(((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_)*((f_.)*(x_)^(m_.))/(Sqrt[(d1_) + (e1_.)*(x_)])*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_)^(p_.))*((d2_) + (e2_.)*(x_)^(p_.)), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cosh^{-1}(ax)^{7/2}} dx &= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} + \frac{1}{5}(2a) \int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}} dx \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} - \frac{4x}{15 \cosh^{-1}(ax)^{3/2}} + \frac{4}{15} \int \frac{1}{\cosh^{-1}(ax)^{3/2}} dx \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} - \frac{4x}{15 \cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{1}{15}(8a) \int \frac{1}{\cosh^{-1}(ax)} dx \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} - \frac{4x}{15 \cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{8\text{Subst}\left(\int \frac{1}{\cosh^{-1}(ax)} dx\right)}{15} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} - \frac{4x}{15 \cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \frac{1}{\cosh^{-1}(ax)} dx\right)}{15} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} - \frac{4x}{15 \cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{8\text{Subst}\left(\int \frac{1}{\cosh^{-1}(ax)} dx\right)}{15} \\
&= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{5a \cosh^{-1}(ax)^{5/2}} - \frac{4x}{15 \cosh^{-1}(ax)^{3/2}} - \frac{8\sqrt{-1+ax}\sqrt{1+ax}}{15a\sqrt{\cosh^{-1}(ax)}} + \frac{4\sqrt{\pi} \operatorname{erf}\left(\sqrt{\cosh^{-1}(ax)}\right)}{15}
\end{aligned}$$

Mathematica [A]

time = 0.14, size = 147, normalized size = 1.20

$$\frac{2e^{-\cosh^{-1}(ax)} \left(3e^{\cosh^{-1}(ax)} \sqrt{\frac{-1+ax}{1+ax}} (1+ax) + \cosh^{-1}(ax) + e^{2\cosh^{-1}(ax)} \cosh^{-1}(ax) - 2\cosh^{-1}(ax)^2 + 2e^{2\cosh^{-1}(ax)} \cosh^{-1}(ax)^2 - 2e^{\cosh^{-1}(ax)} (-\cosh^{-1}(ax))^{5/2} \Gamma\left(\frac{1}{2}, -\cosh^{-1}(ax)\right) + 2e^{\cosh^{-1}(ax)} \cosh^{-1}(ax)^{5/2} \Gamma\left(\frac{1}{2}, \cosh^{-1}(ax)\right) \right)}{15a \cosh^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCosh[a*x]^(-7/2), x]

[Out] $(-2*(3*E^{\text{ArcCosh}[a*x]}*\text{Sqrt}[(-1+a*x)/(1+a*x)]*(1+a*x) + \text{ArcCosh}[a*x] + E^{(2*\text{ArcCosh}[a*x])}*\text{ArcCosh}[a*x] - 2*\text{ArcCosh}[a*x]^2 + 2*E^{(2*\text{ArcCosh}[a*x])}*\text{ArcCosh}[a*x]^2 - 2*E^{\text{ArcCosh}[a*x]}*(-\text{ArcCosh}[a*x])^{5/2}*\text{Gamma}[1/2, -\text{ArcCosh}[a*x]] + 2*E^{\text{ArcCosh}[a*x]}*\text{ArcCosh}[a*x]^{5/2}*\text{Gamma}[1/2, \text{ArcCosh}[a*x]]))/(15*a*E^{\text{ArcCosh}[a*x]}*\text{ArcCosh}[a*x]^{5/2})$

Maple [A]

time = 3.67, size = 111, normalized size = 0.91

method	result
default	$\frac{-\frac{8\sqrt{ax-1}\sqrt{ax+1}\operatorname{arccosh}(ax)^{\frac{5}{2}}\sqrt{\pi}}{15} - \frac{4\operatorname{arccosh}(ax)^{\frac{3}{2}}\sqrt{\pi}}{15} + \frac{4\operatorname{arccosh}(ax)^3\pi\operatorname{erf}\left(\sqrt{\operatorname{arccosh}(ax)}\right)}{15} + \frac{4\operatorname{arccosh}(ax)^3\pi}{\sqrt{\pi}\operatorname{arccosh}(ax)^3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(-4*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)^(5/2)*Pi^(1/2)-2*arccosh(a*x)^(3/2)*Pi^(1/2)*a*x+2*arccosh(a*x)^3*Pi*erf(arccosh(a*x)^(1/2))+2*arccosh(a*x)^3*Pi*erfi(arccosh(a*x)^(1/2))-3*arccosh(a*x)^(1/2)*Pi^(1/2)*(a*x+1)^(1/2)*(a*x-1)^(1/2))/Pi^(1/2)/a/arccosh(a*x)^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(-7/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acosh}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**(7/2),x)
```

```
[Out] Integral(acosh(a*x)**(-7/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/arccosh(a*x)^(7/2),x, algorithm="giac")``[Out] integrate(arccosh(a*x)^(-7/2), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/acosh(a*x)^(7/2),x)``[Out] int(1/acosh(a*x)^(7/2), x)`

$$3.114 \quad \int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \cosh^{-1}(ax)^{7/2}}, x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)^(7/2), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Verification is not applicable to the result.

[In] Int[1/(x*ArcCosh[a*x]^(7/2)), x]

[Out] Defer[Int][1/(x*ArcCosh[a*x]^(7/2)), x]

Rubi steps

$$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx = \int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Mathematica [A]

time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x \cosh^{-1}(ax)^{7/2}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*ArcCosh[a*x]^(7/2)), x]

[Out] Integrate[1/(x*ArcCosh[a*x]^(7/2)), x]

Maple [A]

time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/arccosh(a*x)^(7/2),x)`

[Out] `int(1/x/arccosh(a*x)^(7/2),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(7/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*arccosh(a*x)^(7/2)), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(7/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acosh}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/acosh(a*x)**(7/2),x)`

[Out] `Integral(1/(x*acosh(a*x)**(7/2)), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccosh(a*x)^(7/2),x, algorithm="giac")`

[Out] `integrate(1/(x*arccosh(a*x)^(7/2)), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{acosh}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*acosh(a*x)^(7/2)),x)

[Out] int(1/(x*acosh(a*x)^(7/2)), x)

3.115 $\int x^m \cosh^{-1}(ax)^4 dx$

Optimal. Leaf size=59

$$\frac{x^{1+m} \cosh^{-1}(ax)^4}{1+m} - \frac{4a \operatorname{Int}\left(\frac{x^{1+m} \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}}, x\right)}{1+m}$$

[Out] $x^{(1+m)} \operatorname{arccosh}(a*x)^4 / (1+m) - 4*a * \operatorname{Unintegrable}(x^{(1+m)} \operatorname{arccosh}(a*x)^3 / (a*x - 1)^{(1/2)} / (a*x + 1)^{(1/2)}, x) / (1+m)$

Rubi [A]

time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cosh^{-1}(ax)^4 dx$$

Verification is not applicable to the result.

[In] `Int[x^m*ArcCosh[a*x]^4,x]`

[Out] $(x^{(1+m)} \operatorname{ArcCosh}[a*x]^4) / (1+m) - (4*a * \operatorname{Defer}[\operatorname{Int}[(x^{(1+m)} \operatorname{ArcCosh}[a*x]^3) / (\operatorname{Sqrt}[-1+a*x] * \operatorname{Sqrt}[1+a*x]), x]) / (1+m)$

Rubi steps

$$\int x^m \cosh^{-1}(ax)^4 dx = \frac{x^{1+m} \cosh^{-1}(ax)^4}{1+m} - \frac{(4a) \int \frac{x^{1+m} \cosh^{-1}(ax)^3}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{1+m}$$

Mathematica [A]

time = 1.77, size = 0, normalized size = 0.00

$$\int x^m \cosh^{-1}(ax)^4 dx$$

Verification is not applicable to the result.

[In] `Integrate[x^m*ArcCosh[a*x]^4,x]`

[Out] `Integrate[x^m*ArcCosh[a*x]^4,x]`

Maple [A]

time = 5.30, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*arccosh(a*x)^4,x)`

[Out] `int(x^m*arccosh(a*x)^4,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^4,x, algorithm="maxima")`

[Out] $x^m \log(a x + \sqrt{a x + 1} \sqrt{a x - 1})^4 / (m + 1) - \text{integrate}(4 * (\sqrt{a x + 1} \sqrt{a x - 1} * a^2 x^2 x^m + (a^3 x^3 - a x) x^m) * \log(a x + \sqrt{a x + 1} \sqrt{a x - 1})^3 / (a^3 (m + 1) x^3 - a (m + 1) x + (a^2 (m + 1) x^2 - m - 1) \sqrt{a x + 1} \sqrt{a x - 1}), x)$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^4,x, algorithm="fricas")`

[Out] `integral(x^m*arccosh(a*x)^4, x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acosh}^4(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*acosh(a*x)**4,x)`

[Out] `Integral(x**m*acosh(a*x)**4, x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^4,x, algorithm="giac")`

[Out] integrate(x^m*arccosh(a*x)⁴, x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \operatorname{acosh}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*acosh(a*x)⁴,x)

[Out] int(x^m*acosh(a*x)⁴, x)

3.116 $\int x^m \cosh^{-1}(ax)^3 dx$

Optimal. Leaf size=59

$$\frac{x^{1+m} \cosh^{-1}(ax)^3}{1+m} - \frac{3a \operatorname{Int}\left(\frac{x^{1+m} \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}}, x\right)}{1+m}$$

[Out] $x^{(1+m)} \operatorname{arccosh}(a*x)^3 / (1+m) - 3*a * \operatorname{Unintegrable}(x^{(1+m)} \operatorname{arccosh}(a*x)^2 / (a*x-1)^{(1/2)} / (a*x+1)^{(1/2)}, x) / (1+m)$

Rubi [A]

time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cosh^{-1}(ax)^3 dx$$

Verification is not applicable to the result.

[In] `Int[x^m*ArcCosh[a*x]^3,x]`

[Out] $(x^{(1+m)} \operatorname{ArcCosh}[a*x]^3) / (1+m) - (3*a * \operatorname{Defer}[\operatorname{Int}[(x^{(1+m)} \operatorname{ArcCosh}[a*x]^2) / (\operatorname{Sqrt}[-1+a*x] * \operatorname{Sqrt}[1+a*x]), x]) / (1+m)$

Rubi steps

$$\int x^m \cosh^{-1}(ax)^3 dx = \frac{x^{1+m} \cosh^{-1}(ax)^3}{1+m} - \frac{(3a) \int \frac{x^{1+m} \cosh^{-1}(ax)^2}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{1+m}$$

Mathematica [A]

time = 1.68, size = 0, normalized size = 0.00

$$\int x^m \cosh^{-1}(ax)^3 dx$$

Verification is not applicable to the result.

[In] `Integrate[x^m*ArcCosh[a*x]^3,x]`

[Out] `Integrate[x^m*ArcCosh[a*x]^3, x]`

Maple [A]

time = 4.39, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*arccosh(a*x)^3,x)`

[Out] `int(x^m*arccosh(a*x)^3,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^3,x, algorithm="maxima")`

[Out] `x*x^m*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(m + 1) - integrate(3*(sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^2*x^m + (a^3*x^3 - a*x)*x^m)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*(m + 1)*x^3 - a*(m + 1)*x + (a^2*(m + 1)*x^2 - m - 1)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^3,x, algorithm="fricas")`

[Out] `integral(x^m*arccosh(a*x)^3, x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acosh}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*acosh(a*x)**3,x)`

[Out] `Integral(x**m*acosh(a*x)**3, x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^3,x, algorithm="giac")`

[Out] integrate(x^m*arccosh(a*x)^3, x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \operatorname{acosh}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*acosh(a*x)^3,x)

[Out] int(x^m*acosh(a*x)^3, x)

3.117 $\int x^m \cosh^{-1}(ax)^2 dx$

Optimal. Leaf size=154

$$\frac{x^{1+m} \cosh^{-1}(ax)^2}{1+m} - \frac{2ax^{2+m} \sqrt{1-ax} \cosh^{-1}(ax) {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{(2+3m+m^2) \sqrt{-1+ax}} - \frac{2a^2x^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2, \frac{5}{2} + \frac{m}{2}\right)}{6+11m+6m^2+m^3}$$

[Out] $x^{(1+m)} \operatorname{arccosh}(a*x)^2 / (1+m) - 2*a^2*x^{(3+m)} \operatorname{hypergeom}([1, 3/2+1/2*m, 3/2+1/2*m], [2+1/2*m, 5/2+1/2*m], a^2*x^2) / (m^3+6*m^2+11*m+6) - 2*a*x^{(2+m)} \operatorname{arccosh}(a*x) \operatorname{hypergeom}([1/2, 1+1/2*m], [2+1/2*m], a^2*x^2) * (-a*x+1)^{(1/2)} / (m^2+3*m+2) / (a*x-1)^{(1/2)}$

Rubi [A]

time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5883, 5949}

$$-\frac{2a^2x^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; a^2x^2\right)}{m^3+6m^2+11m+6} - \frac{2a\sqrt{1-ax} x^{m+2} \cosh^{-1}(ax) {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)}{(m^2+3m+2) \sqrt{ax-1}} + \frac{x^{m+1} \cosh^{-1}(ax)^2}{m+1}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^m \operatorname{ArcCosh}[a*x]^2, x]$

[Out] $(x^{(1+m)} \operatorname{ArcCosh}[a*x]^2) / (1+m) - (2*a*x^{(2+m)} \operatorname{Sqrt}[1-a*x] \operatorname{ArcCosh}[a*x] \operatorname{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, a^2*x^2]) / ((2+3*m+m^2) \operatorname{Sqrt}[-1+a*x]) - (2*a^2*x^{(3+m)} \operatorname{HypergeometricPFQ}[\{1, 3/2+m/2, 3/2+m/2\}, \{2+m/2, 5/2+m/2\}, a^2*x^2]) / (6+11*m+6*m^2+m^3)$

Rule 5883

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*((a+b*\operatorname{ArcCosh}[c*x])^n/(d*(m+1))), x] - \operatorname{Dist}[b*c*(n/(d*(m+1))), \operatorname{Int}[(d*x)^{(m+1)}*((a+b*\operatorname{ArcCosh}[c*x])^{(n-1)})/(\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[-1+c*x])], x, x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

Rule 5949

$\operatorname{Int}[(a_.) + \operatorname{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(m_.)} / (\operatorname{Sqrt}[(d1_.) + (e1_.)*(x_.)]*\operatorname{Sqrt}[(d2_.) + (e2_.)*(x_.)]), x_Symbol] \rightarrow \operatorname{Simp}[(f*x)^{(m+1)} / (f*(m+1)) * \operatorname{Simp}[\operatorname{Sqrt}[1-c^2*x^2] / (\operatorname{Sqrt}[d1+e1*x]*\operatorname{Sqrt}[d2+e2*x])], x] + \operatorname{Simp}[b*c*((f*x)^{(m+2)} / (f^2*(m+1)*(m+2))) * \operatorname{Simp}[\operatorname{Sqrt}[1+c*x] / \operatorname{Sqrt}[d1+e1*x]], x] + \operatorname{Simp}[\operatorname{Sqrt}[-1+c*x] / \operatorname{Sqrt}[d2+e2*x]] * \operatorname{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2*x^2], x] /; \operatorname{FreeQ}\{a, b, c, d1, e1, d2, e2, f, m\}, x] \&\& \operatorname{EqQ}[e1, c*d1] \&\& \operatorname{EqQ}[e2, (-c)*d2] \&\& \operatorname{IntegerQ}[m]$

Rubi steps

$$\int x^m \cosh^{-1}(ax)^2 dx = \frac{x^{1+m} \cosh^{-1}(ax)^2}{1+m} - \frac{(2a) \int \frac{x^{1+m} \cosh^{-1}(ax)}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{1+m}$$

$$= \frac{x^{1+m} \cosh^{-1}(ax)^2}{1+m} - \frac{2ax^{2+m} \sqrt{1-a^2x^2} \cosh^{-1}(ax) {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{(2+3m+m^2) \sqrt{-1+ax} \sqrt{1+ax}} - \frac{2a^2x^3}{1+m}$$

Mathematica [A]

time = 0.25, size = 143, normalized size = 0.93

$$\frac{x^{1+m} \left(\cosh^{-1}(ax)^2 - \frac{2ax \left(\frac{\sqrt{1-a^2x^2} \cosh^{-1}(ax) {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{\sqrt{-1+ax} \sqrt{1+ax}} + \frac{{}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; a^2x^2\right)}{3+m}\right)}{2+m} \right)}{1+m}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*ArcCosh[a*x]^2,x]

[Out] (x^(1+m)*(ArcCosh[a*x]^2 - (2*a*x*((Sqrt[1 - a^2*x^2]*ArcCosh[a*x]*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, a^2*x^2])/(Sqrt[-1+a*x]*Sqrt[1+a*x]) + (a*x*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, a^2*x^2])/(3+m)))/(2+m)))/(1+m)

Maple [F]

time = 4.73, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*arccosh(a*x)^2,x)

[Out] int(x^m*arccosh(a*x)^2,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arccosh(a*x)^2,x, algorithm="maxima")

```
[Out] x*x^m*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(m + 1) - integrate(2*(sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x^2*x^m + (a^3*x^3 - a*x)*x^m)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/(a^3*(m + 1)*x^3 - a*(m + 1)*x + (a^2*(m + 1)*x^2 - m - 1)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccosh(a*x)^2, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acosh}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acosh(a*x)**2,x)
```

```
[Out] Integral(x**m*acosh(a*x)**2, x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccosh(a*x)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{acosh}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acosh(a*x)^2,x)
```

```
[Out] int(x^m*acosh(a*x)^2, x)
```

3.118 $\int x^m \cosh^{-1}(ax) dx$

Optimal. Leaf size=91

$$\frac{x^{1+m} \cosh^{-1}(ax)}{1+m} - \frac{ax^{2+m} \sqrt{1-a^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{(2+3m+m^2) \sqrt{-1+ax} \sqrt{1+ax}}$$

[Out] $x^{(1+m)} \operatorname{arccosh}(a*x)/(1+m) - a*x^{(2+m)} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1+1/2*m\right], \left[2+1/2*m\right], a^2*x^2\right) * (-a^2*x^2+1)^{(1/2)} / (m^2+3*m+2) / (a*x-1)^{(1/2)} / (a*x+1)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,

Rules used = {5883, 127, 372, 371}

$$\frac{x^{m+1} \cosh^{-1}(ax)}{m+1} - \frac{a \sqrt{1-a^2x^2} x^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)}{(m^2+3m+2) \sqrt{ax-1} \sqrt{ax+1}}$$

Antiderivative was successfully verified.

[In] `Int[x^m*ArcCosh[a*x],x]`

[Out] $(x^{(1+m)} \operatorname{ArcCosh}[a*x]) / (1+m) - (a*x^{(2+m)} \operatorname{Sqrt}[1-a^2*x^2] \operatorname{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, a^2*x^2]) / ((2+3*m+m^2) \operatorname{Sqrt}[-1+a*x] \operatorname{Sqrt}[1+a*x])$

Rule 127

`Int[((f_)*(x_))^(p_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]`

Rule 371

`Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] :> Simp[a^p * ((c*x)^(m+1)/(c*(m+1))) * Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

Rule 372

`Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^m*(1 + b*(x^n/a))^p, x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])`

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x]))], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x^m \cosh^{-1}(ax) dx &= \frac{x^{1+m} \cosh^{-1}(ax)}{1+m} - \frac{a \int \frac{x^{1+m}}{\sqrt{-1+ax} \sqrt{1+ax}} dx}{1+m} \\ &= \frac{x^{1+m} \cosh^{-1}(ax)}{1+m} - \frac{\left(a\sqrt{-1+a^2x^2}\right) \int \frac{x^{1+m}}{\sqrt{-1+a^2x^2}} dx}{(1+m)\sqrt{-1+ax} \sqrt{1+ax}} \\ &= \frac{x^{1+m} \cosh^{-1}(ax)}{1+m} - \frac{\left(a\sqrt{1-a^2x^2}\right) \int \frac{x^{1+m}}{\sqrt{1-a^2x^2}} dx}{(1+m)\sqrt{-1+ax} \sqrt{1+ax}} \\ &= \frac{x^{1+m} \cosh^{-1}(ax)}{1+m} - \frac{ax^{2+m} \sqrt{1-a^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{(2+3m+m^2) \sqrt{-1+ax} \sqrt{1+ax}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 82, normalized size = 0.90

$$\frac{x^{1+m} \left(\cosh^{-1}(ax) - \frac{ax \sqrt{1-a^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{(2+m)\sqrt{-1+ax} \sqrt{1+ax}} \right)}{1+m}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*ArcCosh[a*x],x]

[Out] (x^(1+m)*(ArcCosh[a*x] - (a*x*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, a^2*x^2])/((2+m)*Sqrt[-1+a*x]*Sqrt[1+a*x]))/(1+m)

Maple [F]

time = 4.65, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccosh}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*arccosh(a*x),x)`

[Out] `int(x^m*arccosh(a*x),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x),x, algorithm="maxima")`

[Out] $-a^2 \int \frac{x^2 x^m}{(a^2(m+1)x^2 - m - 1)} dx + a \int \frac{x x^m}{(a^3(m+1)x^3 - a(m+1)x + (a^2(m+1)x^2 - m - 1)\sqrt{ax+1}\sqrt{ax-1})} dx + x x^m \log(ax + \sqrt{ax+1}\sqrt{ax-1}) / (m+1)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x),x, algorithm="fricas")`

[Out] `integral(x^m*arccosh(a*x), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acosh}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*acosh(a*x),x)`

[Out] `Integral(x**m*acosh(a*x), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x),x, algorithm="giac")`

[Out] `integrate(x^m*arccosh(a*x), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{acosh}(a x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*acosh(a*x),x)`

[Out] `int(x^m*acosh(a*x), x)`

$$3.119 \quad \int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x^m}{\cosh^{-1}(ax)}, x\right)$$

[Out] Unintegrable(x^m/arccosh(a*x), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Int[x^m/ArcCosh[a*x], x]

[Out] Defer[Int][x^m/ArcCosh[a*x], x]

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)} dx = \int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Mathematica [A]

time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/ArcCosh[a*x], x]

[Out] Integrate[x^m/ArcCosh[a*x], x]

Maple [A]

time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arccosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/arccosh(a*x),x)`

[Out] `int(x^m/arccosh(a*x),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x),x, algorithm="maxima")`

[Out] `integrate(x^m/arccosh(a*x), x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x),x, algorithm="fricas")`

[Out] `integral(x^m/arccosh(a*x), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/acosh(a*x),x)`

[Out] `Integral(x**m/acosh(a*x), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x),x, algorithm="giac")`

[Out] `integrate(x^m/arccosh(a*x), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^m}{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/acosh(a*x), x)
```

```
[Out] int(x^m/acosh(a*x), x)
```

$$3.120 \quad \int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x^m}{\cosh^{-1}(ax)^2}, x\right)$$

[Out] Unintegrable(x^m/arccosh(a*x)², x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

[In] Int[x^m/ArcCosh[a*x]², x]

[Out] Defer[Int][x^m/ArcCosh[a*x]², x]

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx = \int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Mathematica [A]

time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\cosh^{-1}(ax)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/ArcCosh[a*x]², x]

[Out] Integrate[x^m/ArcCosh[a*x]², x]

Maple [A]

time = 3.88, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arccosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/arccosh(a*x)^2,x)`

[Out] `int(x^m/arccosh(a*x)^2,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x)^2,x, algorithm="maxima")`

[Out]
$$-((a^2x^2 - 1)\sqrt{ax + 1}\sqrt{ax - 1}x^m + (a^3x^3 - ax)x^m)/((a^3x^2 + \sqrt{ax + 1}\sqrt{ax - 1}a^2x - a)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})) + \int ((a^3(m + 1)x^3 - a(m - 1)x)(ax + 1)(ax - 1)x^m + (2a^4(m + 1)x^4 - a^2(3m + 1)x^2 + m)\sqrt{ax + 1}\sqrt{ax - 1}x^m + (a^5(m + 1)x^5 - 2a^3(m + 1)x^3 + a(m + 1)x)x^m)/((a^5x^5 + (ax + 1)(ax - 1)a^3x^3 - 2a^3x^3 + 2(a^4x^4 - a^2x^2)\sqrt{ax + 1}\sqrt{ax - 1} + ax)\log(ax + \sqrt{ax + 1}\sqrt{ax - 1})), x)$$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x)^2,x, algorithm="fricas")`

[Out] `integral(x^m/arccosh(a*x)^2, x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acosh}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/acosh(a*x)**2,x)`

[Out] `Integral(x**m/acosh(a*x)**2, x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccosh(a*x)^2, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^m}{\operatorname{acosh}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/acosh(a*x)^2,x)
```

```
[Out] int(x^m/acosh(a*x)^2, x)
```

$$3.121 \quad \int \frac{x^m}{\cosh^{-1}(ax)^3} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x^m}{\cosh^{-1}(ax)^3}, x\right)$$

[Out] Unintegrable(x^m/arccosh(a*x)^3, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

[In] Int[x^m/ArcCosh[a*x]^3, x]

[Out] Defer[Int][x^m/ArcCosh[a*x]^3, x]

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)^3} dx = \int \frac{x^m}{\cosh^{-1}(ax)^3} dx$$

Mathematica [A]

time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\cosh^{-1}(ax)^3} dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/ArcCosh[a*x]^3, x]

[Out] Integrate[x^m/ArcCosh[a*x]^3, x]

Maple [A]

time = 3.95, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arccosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m/\text{arccosh}(a*x)^3, x)$

[Out] $\text{int}(x^m/\text{arccosh}(a*x)^3, x)$

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m/\text{arccosh}(a*x)^3, x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & -1/2*((a^5*x^5 - a^3*x^3)*(a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)}*x^m + (3*a^6*x^6 \\ & - 5*a^4*x^4 + 2*a^2*x^2)*(a*x + 1)*(a*x - 1)*x^m + (3*a^7*x^7 - 7*a^5*x^5 + \\ & 5*a^3*x^3 - a*x)*\text{sqrt}(a*x + 1)*\text{sqrt}(a*x - 1)*x^m + (a^8*x^8 - 3*a^6*x^6 + \\ & 3*a^4*x^4 - a^2*x^2)*x^m + ((a^5*(m + 1)*x^5 - 2*a^3*m*x^3 + a*(m - 1)*x)*(\\ & a*x + 1)^{(3/2)}*(a*x - 1)^{(3/2)}*x^m + (3*a^6*(m + 1)*x^6 - a^4*(7*m + 3)*x^4 \\ & + 5*a^2*m*x^2 - m)*(a*x + 1)*(a*x - 1)*x^m + (3*a^7*(m + 1)*x^7 - 2*a^5*(4 \\ & *m + 3)*x^5 + a^3*(7*m + 4)*x^3 - a*(2*m + 1)*x)*\text{sqrt}(a*x + 1)*\text{sqrt}(a*x - 1 \\ &)*x^m + (a^8*(m + 1)*x^8 - 3*a^6*(m + 1)*x^6 + 3*a^4*(m + 1)*x^4 - a^2*(m + \\ & 1)*x^2)*x^m*\log(a*x + \text{sqrt}(a*x + 1)*\text{sqrt}(a*x - 1)))/((a^8*x^7 + (a*x + 1) \\ &)^{(3/2)}*(a*x - 1)^{(3/2)}*a^5*x^4 - 3*a^6*x^5 + 3*a^4*x^3 + 3*(a^6*x^5 - a^4*x \\ & ^3)*(a*x + 1)*(a*x - 1) - a^2*x + 3*(a^7*x^6 - 2*a^5*x^4 + a^3*x^2)*\text{sqrt}(a* \\ & x + 1)*\text{sqrt}(a*x - 1))*\log(a*x + \text{sqrt}(a*x + 1)*\text{sqrt}(a*x - 1))^2 + \text{integrate} \\ & (1/2*((m^2 + 2*m + 1)*a^6*x^6 - 2*(m^2 - m)*a^4*x^4 + (m^2 - 4*m + 3)*a^2* \\ & x^2)*(a*x + 1)^2*(a*x - 1)^2*x^m + (4*(m^2 + 2*m + 1)*a^7*x^7 - 2*(5*m^2 + \\ & m + 2)*a^5*x^5 + (8*m^2 - 11*m + 3)*a^3*x^3 - (2*m^2 - 5*m)*a*x)*(a*x + 1)^ \\ & (3/2)*(a*x - 1)^{(3/2)}*x^m + (6*(m^2 + 2*m + 1)*a^8*x^8 - 6*(3*m^2 + 3*m + 2) \\ &)*a^6*x^6 + (19*m^2 + 2*m + 3)*a^4*x^4 - (8*m^2 - 5*m - 3)*a^2*x^2 + m^2 - \\ & m)*(a*x + 1)*(a*x - 1)*x^m + (4*(m^2 + 2*m + 1)*a^9*x^9 - 2*(7*m^2 + 11*m + \\ & 6)*a^7*x^7 + 3*(6*m^2 + 7*m + 3)*a^5*x^5 - (10*m^2 + 8*m + 1)*a^3*x^3 + (2 \\ & *m^2 + m)*a*x)*\text{sqrt}(a*x + 1)*\text{sqrt}(a*x - 1)*x^m + ((m^2 + 2*m + 1)*a^{10}*x^{10} \\ & - 4*(m^2 + 2*m + 1)*a^8*x^8 + 6*(m^2 + 2*m + 1)*a^6*x^6 - 4*(m^2 + 2*m + 1) \\ &)*a^4*x^4 + (m^2 + 2*m + 1)*a^2*x^2)*x^m)/((a^{10}*x^{10} + (a*x + 1)^2*(a*x - \\ & 1)^2*a^6*x^6 - 4*a^8*x^8 + 6*a^6*x^6 - 4*a^4*x^4 + 4*(a^7*x^7 - a^5*x^5)*(a \\ & *x + 1)^{(3/2)}*(a*x - 1)^{(3/2)} + a^2*x^2 + 6*(a^8*x^8 - 2*a^6*x^6 + a^4*x^4) \\ & *(a*x + 1)*(a*x - 1) + 4*(a^9*x^9 - 3*a^7*x^7 + 3*a^5*x^5 - a^3*x^3)*\text{sqrt}(a \\ & *x + 1)*\text{sqrt}(a*x - 1))*\log(a*x + \text{sqrt}(a*x + 1)*\text{sqrt}(a*x - 1))), x \end{aligned}$$

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m/\text{arccosh}(a*x)^3, x, \text{algorithm}="fricas")$

[Out] integral(x^m/arccosh(a*x)³, x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acosh}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m/acosh(a*x)**3,x)

[Out] Integral(x**m/acosh(a*x)**3, x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccosh(a*x)³,x, algorithm="giac")

[Out] integrate(x^m/arccosh(a*x)³, x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^m}{\operatorname{acosh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/acosh(a*x)³,x)

[Out] int(x^m/acosh(a*x)³, x)

3.122 $\int x^m \cosh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=15

$$\text{Int}(x^m \cosh^{-1}(ax)^{3/2}, x)$$

[Out] Unintegrable($x^m \cdot \text{arccosh}(a \cdot x)^{(3/2)}$, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,

Rules used = {}

$$\int x^m \cosh^{-1}(ax)^{3/2} dx$$

Verification is not applicable to the result.

[In] Int [$x^m \cdot \text{ArcCosh}[a \cdot x]^{(3/2)}$, x]

[Out] Defer[Int] [$x^m \cdot \text{ArcCosh}[a \cdot x]^{(3/2)}$, x]

Rubi steps

$$\int x^m \cosh^{-1}(ax)^{3/2} dx = \int x^m \cosh^{-1}(ax)^{3/2} dx$$

Mathematica [A]

time = 2.96, size = 0, normalized size = 0.00

$$\int x^m \cosh^{-1}(ax)^{3/2} dx$$

Verification is not applicable to the result.

[In] Integrate [$x^m \cdot \text{ArcCosh}[a \cdot x]^{(3/2)}$, x]

[Out] Integrate [$x^m \cdot \text{ArcCosh}[a \cdot x]^{(3/2)}$, x]

Maple [A]

time = 1.16, size = 0, normalized size = 0.00

$$\int x^m \text{arccosh}(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int ($x^m \cdot \text{arccosh}(a \cdot x)^{(3/2)}$, x)

[Out] $\text{int}(x^m \operatorname{arccosh}(ax)^{3/2}, x)$

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m \operatorname{arccosh}(ax)^{3/2}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(x^m \operatorname{arccosh}(ax)^{3/2}, x)$

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m \operatorname{arccosh}(ax)^{3/2}, x, \text{algorithm}="fricas")$

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acosh}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m \operatorname{acosh}(ax)^{3/2}, x)$

[Out] $\text{Integral}(x^m \operatorname{acosh}(ax)^{3/2}, x)$

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m \operatorname{arccosh}(ax)^{3/2}, x, \text{algorithm}="giac")$

[Out] Timed out

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int x^m \operatorname{acosh}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m \operatorname{acosh}(ax)^{3/2}, x)$

[Out] $\text{int}(x^m \operatorname{acosh}(ax)^{3/2}, x)$

$$\mathbf{3.123} \quad \int x^m \sqrt{\cosh^{-1}(ax)} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \sqrt{\cosh^{-1}(ax)}, x\right)$$

[Out] Unintegrable($x^m \cdot \text{arccosh}(a \cdot x)^{(1/2)}$, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{\cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Int [$x^m \cdot \text{Sqrt}[\text{ArcCosh}[a \cdot x]]$], x]

[Out] Defer[Int] [$x^m \cdot \text{Sqrt}[\text{ArcCosh}[a \cdot x]]$], x]

Rubi steps

$$\int x^m \sqrt{\cosh^{-1}(ax)} dx = \int x^m \sqrt{\cosh^{-1}(ax)} dx$$

Mathematica [A]

time = 2.89, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\cosh^{-1}(ax)} dx$$

Verification is not applicable to the result.

[In] Integrate [$x^m \cdot \text{Sqrt}[\text{ArcCosh}[a \cdot x]]$], x]

[Out] Integrate [$x^m \cdot \text{Sqrt}[\text{ArcCosh}[a \cdot x]]$], x]

Maple [A]

time = 1.07, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\text{arccosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*arccosh(a*x)^(1/2),x)`

[Out] `int(x^m*arccosh(a*x)^(1/2),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^m*sqrt(arccosh(a*x)), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*acosh(a*x)**(1/2),x)`

[Out] `Integral(x**m*sqrt(acosh(a*x)), x)`

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccosh(a*x)^(1/2),x, algorithm="giac")`

[Out] Timed out

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int x^m \sqrt{\operatorname{acosh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*acosh(a*x)^(1/2),x)`

[Out] `int(x^m*acosh(a*x)^(1/2), x)`

$$3.124 \quad \int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x^m}{\sqrt{\cosh^{-1}(ax)}}, x\right)$$

[Out] Unintegrable(x^m/arccosh(a*x)^(1/2), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}} dx$$

Verification is not applicable to the result.

[In] Int[x^m/Sqrt[ArcCosh[a*x]], x]

[Out] Defer[Int][x^m/Sqrt[ArcCosh[a*x]], x]

Rubi steps

$$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}} dx = \int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}} dx$$

Mathematica [A]

time = 2.75, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\cosh^{-1}(ax)}} dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/Sqrt[ArcCosh[a*x]], x]

[Out] Integrate[x^m/Sqrt[ArcCosh[a*x]], x]

Maple [A]

time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\text{arccosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccosh(a*x)^(1/2),x)
```

```
[Out] int(x^m/arccosh(a*x)^(1/2),x)
```

Maxima [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(arccosh(a*x)), x)
```

Fricas [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^m}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(acosh(a*x)), x)
```

Giac [A]

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(arccosh(a*x)), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m}{\sqrt{\operatorname{acosh}(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/acosh(a*x)^(1/2),x)

[Out] int(x^m/acosh(a*x)^(1/2), x)

$$3.125 \quad \int \frac{x^m}{\cosh^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x^m}{\cosh^{-1}(ax)^{3/2}}, x\right)$$

[Out] Unintegrable(x^m/arccosh(a*x)^(3/2), x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Int[x^m/ArcCosh[a*x]^(3/2), x]

[Out] Defer[Int][x^m/ArcCosh[a*x]^(3/2), x]

Rubi steps

$$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}} dx = \int \frac{x^m}{\cosh^{-1}(ax)^{3/2}} dx$$

Mathematica [A]

time = 2.85, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\cosh^{-1}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/ArcCosh[a*x]^(3/2), x]

[Out] Integrate[x^m/ArcCosh[a*x]^(3/2), x]

Maple [A]

time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arccosh}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/arccosh(a*x)^(3/2),x)`

[Out] `int(x^m/arccosh(a*x)^(3/2),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^m/arccosh(a*x)^(3/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acosh}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/acosh(a*x)**(3/2),x)`

[Out] `Integral(x**m/acosh(a*x)**(3/2), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/arccosh(a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^m/arccosh(a*x)^(3/2), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m}{\operatorname{acosh}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/acosh(a*x)^(3/2),x)

[Out] int(x^m/acosh(a*x)^(3/2), x)

3.126 $\int (dx)^m \cosh^{-1}(ax)^n dx$

Optimal. Leaf size=15

$$\text{Int}((dx)^m \cosh^{-1}(ax)^n, x)$$

[Out] Unintegrable((d*x)^m*arccosh(a*x)^n,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,

Rules used = {}

$$\int (dx)^m \cosh^{-1}(ax)^n dx$$

Verification is not applicable to the result.

[In] Int[(d*x)^m*ArcCosh[a*x]^n,x]

[Out] Defer[Int][(d*x)^m*ArcCosh[a*x]^n, x]

Rubi steps

$$\int (dx)^m \cosh^{-1}(ax)^n dx = \int (dx)^m \cosh^{-1}(ax)^n dx$$

Mathematica [A]

time = 1.43, size = 0, normalized size = 0.00

$$\int (dx)^m \cosh^{-1}(ax)^n dx$$

Verification is not applicable to the result.

[In] Integrate[(d*x)^m*ArcCosh[a*x]^n,x]

[Out] Integrate[(d*x)^m*ArcCosh[a*x]^n, x]

Maple [A]

time = 0.66, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{arccosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*arccosh(a*x)^n,x)

[Out] `int((d*x)^m*arccosh(a*x)^n,x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*arccosh(a*x)^n,x, algorithm="maxima")`

[Out] `integrate((d*x)^m*arccosh(a*x)^n, x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*arccosh(a*x)^n,x, algorithm="fricas")`

[Out] `integral((d*x)^m*arccosh(a*x)^n, x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{acosh}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*acosh(a*x)**n,x)`

[Out] `Integral((d*x)**m*acosh(a*x)**n, x)`

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*arccosh(a*x)^n,x, algorithm="giac")`

[Out] Timed out

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \operatorname{acosh}(ax)^n (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acosh(a*x)^n*(d*x)^m,x)`

[Out] `int(acosh(a*x)^n*(d*x)^m, x)`

3.127 $\int x^4 \cosh^{-1}(ax)^n dx$

Optimal. Leaf size=173

$$\frac{5^{-1-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -5 \cosh^{-1}(ax))}{32a^5} + \frac{3^{-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -3 \cosh^{-1}(ax))}{32a^5}$$

[Out] 1/32*5^(-1-n)*arccosh(a*x)^n*GAMMA(1+n,-5*arccosh(a*x))/a^5/((-arccosh(a*x))^n)+1/32*arccosh(a*x)^n*GAMMA(1+n,-3*arccosh(a*x))/(3^n)/a^5/((-arccosh(a*x))^n)+1/16*arccosh(a*x)^n*GAMMA(1+n,-arccosh(a*x))/a^5/((-arccosh(a*x))^n)+1/16*GAMMA(1+n,arccosh(a*x))/a^5+1/32*GAMMA(1+n,3*arccosh(a*x))/(3^n)/a^5+1/32*5^(-1-n)*GAMMA(1+n,5*arccosh(a*x))/a^5

Rubi [A]

time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5887, 5556, 3389, 2212}

$$\frac{5^{-n-1} \cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \Gamma(n+1, -5 \cosh^{-1}(ax))}{32a^5} + \frac{3^{-n} \cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \Gamma(n+1, -3 \cosh^{-1}(ax))}{32a^5} + \frac{\cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \Gamma(n+1, -\cosh^{-1}(ax))}{16a^5} + \frac{\Gamma(n+1, \cosh^{-1}(ax))}{16a^5} + \frac{3^{-n} \Gamma(n+1, 3 \cosh^{-1}(ax))}{32a^5} + \frac{5^{-n-1} \Gamma(n+1, 5 \cosh^{-1}(ax))}{32a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcCosh[a*x]^n,x]

[Out] (5^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -5*ArcCosh[a*x]])/(32*a^5*(-ArcCosh[a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(32*3^n*a^5*(-ArcCosh[a*x])^n) + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(16*a^5*(-ArcCosh[a*x])^n) + Gamma[1 + n, ArcCosh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcCosh[a*x]]/(32*3^n*a^5) + (5^(-1 - n)*Gamma[1 + n, 5*ArcCosh[a*x]])/(32*a^5)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

$b*x]^n*\text{Cosh}[a + b*x]^p, x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&$
 $\& \text{IGtQ}[p, 0]$

Rule 5887

$\text{Int}[(a_.) + \text{ArcCosh}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)^{(m_.)}, x_Symbol] \text{:> Dist}[$
 $1/(b*c^{(m + 1)}), \text{Subst}[\text{Int}[x^n*\text{Cosh}[-a/b + x/b]^m*\text{Sinh}[-a/b + x/b], x], x,$
 $a + b*\text{ArcCosh}[c*x]], x] /; \text{FreeQ}[\{a, b, c, n\}, x] \&\& \text{IGtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int x^4 \cosh^{-1}(ax)^n dx &= \frac{\text{Subst}(\int x^n \cosh^4(x) \sinh(x) dx, x, \cosh^{-1}(ax))}{a^5} \\ &= \frac{\text{Subst}(\int (\frac{1}{8}x^n \sinh(x) + \frac{3}{16}x^n \sinh(3x) + \frac{1}{16}x^n \sinh(5x)) dx, x, \cosh^{-1}(ax))}{a^5} \\ &= \frac{\text{Subst}(\int x^n \sinh(5x) dx, x, \cosh^{-1}(ax))}{16a^5} + \frac{\text{Subst}(\int x^n \sinh(x) dx, x, \cosh^{-1}(ax))}{8a^5} + \\ &= -\frac{\text{Subst}(\int e^{-5x}x^n dx, x, \cosh^{-1}(ax))}{32a^5} + \frac{\text{Subst}(\int e^{5x}x^n dx, x, \cosh^{-1}(ax))}{32a^5} - \frac{\text{Subst}(\int e^x x^n dx, x, \cosh^{-1}(ax))}{32a^5} \\ &= \frac{5^{-1-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -5 \cosh^{-1}(ax))}{32a^5} + \frac{3^{-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax))}{32a^5} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 144, normalized size = 0.83

$$\frac{5^{-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -5 \cosh^{-1}(ax)) + 5 \cdot 3^{-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax)) + 10(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax)) + 10\Gamma(1+n, \cosh^{-1}(ax)) + 5 \cdot 3^{-n} \Gamma(1+n, 3 \cosh^{-1}(ax)) + 5^{-n} \Gamma(1+n, 5 \cosh^{-1}(ax))}{160a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*ArcCosh[a*x]^n,x]

[Out] ((ArcCosh[a*x]^n*Gamma[1 + n, -5*ArcCosh[a*x]])/(5^n*(-ArcCosh[a*x])^n) + (5*ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(3^n*(-ArcCosh[a*x])^n) + (10*ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(-ArcCosh[a*x])^n + 10*Gamma[1 + n, ArcCosh[a*x]] + (5*Gamma[1 + n, 3*ArcCosh[a*x]])/3^n + Gamma[1 + n, 5*ArcCosh[a*x]]/5^n)/(160*a^5)

Maple [F]

time = 3.31, size = 0, normalized size = 0.00

$$\int x^4 \text{arccosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccosh(a*x)^n,x)`

[Out] `int(x^4*arccosh(a*x)^n,x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^n,x, algorithm="maxima")`

[Out] `integrate(x^4*arccosh(a*x)^n, x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^n,x, algorithm="fricas")`

[Out] `integral(x^4*arccosh(a*x)^n, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{acosh}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acosh(a*x)**n,x)`

[Out] `Integral(x**4*acosh(a*x)**n, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccosh(a*x)^n,x, algorithm="giac")`

[Out] `integrate(x^4*arccosh(a*x)^n, x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acosh(a*x)^n,x)
```

```
[Out] int(x^4*acosh(a*x)^n, x)
```

3.128 $\int x^3 \cosh^{-1}(ax)^n dx$

Optimal. Leaf size=117

$$\frac{2^{-2(3+n)}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -4 \cosh^{-1}(ax))}{a^4} + \frac{2^{-4-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, 2 \cosh^{-1}(ax))}{a^4}$$

[Out] arccosh(a*x)^n*GAMMA(1+n,-4*arccosh(a*x))/(2^(6+2*n))/a^4/((-arccosh(a*x))^n)+2^(-4-n)*arccosh(a*x)^n*GAMMA(1+n,-2*arccosh(a*x))/a^4/((-arccosh(a*x))^n)+2^(-4-n)*GAMMA(1+n,2*arccosh(a*x))/a^4+GAMMA(1+n,4*arccosh(a*x))/(2^(6+2*n))/a^4

Rubi [A]

time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5887, 5556, 3389, 2212}

$$\frac{2^{-2(n+3)} \cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \Gamma(n+1, -4 \cosh^{-1}(ax))}{a^4} + \frac{2^{-n-4} \cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \Gamma(n+1, -2 \cosh^{-1}(ax))}{a^4} + \frac{2^{-n-4} \Gamma(n+1, 2 \cosh^{-1}(ax))}{a^4} + \frac{2^{-2(n+3)} \Gamma(n+1, 4 \cosh^{-1}(ax))}{a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcCosh[a*x]^n,x]

[Out] (ArcCosh[a*x]^n*Gamma[1+n,-4*ArcCosh[a*x]]/(2^(2*(3+n))*a^4*(-ArcCosh[a*x])^n) + (2^(-4-n)*ArcCosh[a*x]^n*Gamma[1+n,-2*ArcCosh[a*x]]/(a^4*(-ArcCosh[a*x])^n) + (2^(-4-n)*Gamma[1+n,2*ArcCosh[a*x]]/a^4 + Gamma[1+n,4*ArcCosh[a*x]]/(2^(2*(3+n))*a^4)

Rule 2212

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:= Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]]*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 3389

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_) + (b_)*(x_)]^(p_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

& IGtQ[p, 0]

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x^3 \cosh^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cosh^3(x) \sinh(x) dx, x, \cosh^{-1}(ax)\right)}{a^4} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n \sinh(2x) + \frac{1}{8}x^n \sinh(4x)\right) dx, x, \cosh^{-1}(ax)\right)}{a^4} \\ &= \frac{\text{Subst}\left(\int x^n \sinh(4x) dx, x, \cosh^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int x^n \sinh(2x) dx, x, \cosh^{-1}(ax)\right)}{4a^4} \\ &= -\frac{\text{Subst}\left(\int e^{-4x} x^n dx, x, \cosh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int e^{4x} x^n dx, x, \cosh^{-1}(ax)\right)}{16a^4} - \frac{\text{Subst}\left(\int e^{2x} x^n dx, x, \cosh^{-1}(ax)\right)}{8a^4} \\ &= \frac{4^{-3-n} (-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -4 \cosh^{-1}(ax))}{a^4} + \frac{2^{-4-n} (-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -2 \cosh^{-1}(ax))}{a^4} - \frac{2^{-3-n} (-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, 2 \cosh^{-1}(ax))}{a^4} \end{aligned}$$

Mathematica [A]

time = 0.08, size = 97, normalized size = 0.83

$$\frac{4^{-3-n} (-\cosh^{-1}(ax))^{-n} (\cosh^{-1}(ax))^n \Gamma(1+n, -4 \cosh^{-1}(ax)) + 2^{2+n} \cosh^{-1}(ax)^n \Gamma(1+n, -2 \cosh^{-1}(ax)) + (-\cosh^{-1}(ax))^n (2^{2+n} \Gamma(1+n, 2 \cosh^{-1}(ax)) + \Gamma(1+n, 4 \cosh^{-1}(ax)))}{a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCosh[a*x]^n,x]

[Out] (4^(-3 - n)*(ArcCosh[a*x]^n*Gamma[1 + n, -4*ArcCosh[a*x]] + 2^(2 + n)*ArcCosh[a*x]^n*Gamma[1 + n, -2*ArcCosh[a*x]] + (-ArcCosh[a*x])^n*(2^(2 + n)*Gamma[1 + n, 2*ArcCosh[a*x]] + Gamma[1 + n, 4*ArcCosh[a*x]])))/(a^4*(-ArcCosh[a*x])^n)

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.

time = 4.07, size = 80, normalized size = 0.68

method	result
default	$\frac{\text{arccosh}(ax)^{n+2} \text{hypergeom}\left(\left[1+\frac{n}{2}\right], \left[\frac{3}{2}, 2+\frac{n}{2}\right], \text{arccosh}(ax)^2\right)}{2a^4(n+2)} + \frac{\text{arccosh}(ax)^{n+2} \text{hypergeom}\left(\left[1+\frac{n}{2}\right], \left[\frac{3}{2}, 2+\frac{n}{2}\right], 4\text{arccosh}(ax)^2\right)}{2a^4(n+2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^4/(n+2)*arccosh(a*x)^(n+2)*hypergeom([1+1/2*n],[3/2,2+1/2*n],arccosh(a*x)^2)+1/2/a^4/(n+2)*arccosh(a*x)^(n+2)*hypergeom([1+1/2*n],[3/2,2+1/2*n],4*arccosh(a*x)^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arccosh(a*x)^n, x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccosh(a*x)^n, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acosh}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**n,x)
```

```
[Out] Integral(x**3*acosh(a*x)**n, x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a*x)^n,x)
```

```
[Out] int(x^3*acosh(a*x)^n, x)
```

3.129 $\int x^2 \cosh^{-1}(ax)^n dx$

Optimal. Leaf size=113

$$\frac{3^{-1-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -3 \cosh^{-1}(ax))}{8a^3} + \frac{(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax))}{8a^3}$$

[Out] $1/8*3^{(-1-n)}*\operatorname{arccosh}(a*x)^n*\operatorname{GAMMA}(1+n, -3*\operatorname{arccosh}(a*x))/a^3/((- \operatorname{arccosh}(a*x))^n)+1/8*\operatorname{arccosh}(a*x)^n*\operatorname{GAMMA}(1+n, -\operatorname{arccosh}(a*x))/a^3/((- \operatorname{arccosh}(a*x))^n)+1/8*\operatorname{GAMMA}(1+n, \operatorname{arccosh}(a*x))/a^3+1/8*3^{(-1-n)}*\operatorname{GAMMA}(1+n, 3*\operatorname{arccosh}(a*x))/a^3$

Rubi [A]

time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5887, 5556, 3389, 2212}

$$\frac{3^{-n-1} \cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \operatorname{Gamma}(n+1, -3 \cosh^{-1}(ax))}{8a^3} + \frac{\cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \operatorname{Gamma}(n+1, -\cosh^{-1}(ax))}{8a^3} + \frac{\operatorname{Gamma}(n+1, \cosh^{-1}(ax))}{8a^3} + \frac{3^{-n-1} \operatorname{Gamma}(n+1, 3 \cosh^{-1}(ax))}{8a^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{ArcCosh}[a*x]^n, x]$

[Out] $(3^{(-1-n)}*\operatorname{ArcCosh}[a*x]^n*\operatorname{Gamma}[1+n, -3*\operatorname{ArcCosh}[a*x]])/(8*a^3*(-\operatorname{ArcCosh}[a*x])^n) + (\operatorname{ArcCosh}[a*x]^n*\operatorname{Gamma}[1+n, -\operatorname{ArcCosh}[a*x]])/(8*a^3*(-\operatorname{ArcCosh}[a*x])^n) + \operatorname{Gamma}[1+n, \operatorname{ArcCosh}[a*x]]/(8*a^3) + (3^{(-1-n)}*\operatorname{Gamma}[1+n, 3*\operatorname{ArcCosh}[a*x]])/(8*a^3)$

Rule 2212

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d)))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol]
:> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] & IGtQ[p, 0]
```

Rule 5887

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int x^2 \cosh^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cosh^2(x) \sinh(x) dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n \sinh(x) + \frac{1}{4}x^n \sinh(3x)\right) dx, x, \cosh^{-1}(ax)\right)}{a^3} \\
 &= \frac{\text{Subst}\left(\int x^n \sinh(x) dx, x, \cosh^{-1}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int x^n \sinh(3x) dx, x, \cosh^{-1}(ax)\right)}{4a^3} \\
 &= -\frac{\text{Subst}\left(\int e^{-3x} x^n dx, x, \cosh^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int e^{-x} x^n dx, x, \cosh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int e^x x^n dx, x, \cosh^{-1}(ax)\right)}{8a^3} \\
 &= \frac{3^{-1-n} (-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -3 \cosh^{-1}(ax))}{8a^3} + \frac{(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax))}{8a^3} + \frac{3^{-1-n} \Gamma(1+n, 3 \cosh^{-1}(ax))}{8a^3}
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 95, normalized size = 0.84

$$\frac{3^{-1-n} (-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -3 \cosh^{-1}(ax)) + (-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax)) + \Gamma(1+n, \cosh^{-1}(ax)) + 3^{-1-n} \Gamma(1+n, 3 \cosh^{-1}(ax))}{8a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcCosh[a*x]^n,x]

[Out] ((3^(-1 - n)*ArcCosh[a*x]^n*Gamma[1 + n, -3*ArcCosh[a*x]])/(-ArcCosh[a*x])^n + (ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(-ArcCosh[a*x])^n + Gamma[1 + n, ArcCosh[a*x]] + 3^(-1 - n)*Gamma[1 + n, 3*ArcCosh[a*x]])/(8*a^3)

Maple [F]

time = 1.49, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arccosh(a*x)^n,x)

[Out] int(x^2*arccosh(a*x)^n,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^n,x, algorithm="maxima")

[Out] integrate(x^2*arccosh(a*x)^n, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^n,x, algorithm="fricas")

[Out] integral(x^2*arccosh(a*x)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acosh}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*acosh(a*x)**n,x)

[Out] Integral(x**2*acosh(a*x)**n, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(a*x)^n,x, algorithm="giac")

[Out] integrate(x^2*arccosh(a*x)^n, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*acosh(a*x)^n,x)

[Out] int(x^2*acosh(a*x)^n, x)

3.130 $\int x \cosh^{-1}(ax)^n dx$

Optimal. Leaf size=59

$$\frac{2^{-3-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -2 \cosh^{-1}(ax))}{a^2} + \frac{2^{-3-n} \Gamma(1+n, 2 \cosh^{-1}(ax))}{a^2}$$

[Out] $2^{(-3-n)*\operatorname{arccosh}(a*x)^n*\Gamma(1+n,-2*\operatorname{arccosh}(a*x))/a^2/((- \operatorname{arccosh}(a*x))^n)+2^{(-3-n)*\Gamma(1+n,2*\operatorname{arccosh}(a*x))/a^2}$

Rubi [A]

time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5887, 5556, 12, 3389, 2212}

$$\frac{2^{-n-3} \cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \Gamma(n+1, -2 \cosh^{-1}(ax))}{a^2} + \frac{2^{-n-3} \Gamma(n+1, 2 \cosh^{-1}(ax))}{a^2}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCosh[a*x]^n,x]`

[Out] $(2^{(-3-n)*\operatorname{ArcCosh}[a*x]^n*\Gamma[1+n,-2*\operatorname{ArcCosh}[a*x]])/(a^2*(-\operatorname{ArcCosh}[a*x])^n) + (2^{(-3-n)*\Gamma[1+n,2*\operatorname{ArcCosh}[a*x]])/a^2$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 2212

`Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]`

Rule 3389

`Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]`

Rule 5556

`Int[Cosh[(a_) + (b_)*(x_)]^(p_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +`

$b*x]^n*\text{Cosh}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&$
 $\& \text{IGtQ}[p, 0]$

Rule 5887

$\text{Int}[(a_.) + \text{ArcCosh}[c_.)*(x_.)]*(b_.))^n*(x_.)^m, x_Symbol] :> \text{Dist}[$
 $1/(b*c^{(m + 1)}), \text{Subst}[\text{Int}[x^n*\text{Cosh}[-a/b + x/b]^m*\text{Sinh}[-a/b + x/b], x], x,$
 $a + b*\text{ArcCosh}[c*x]], x] /; \text{FreeQ}\{a, b, c, n\}, x\} \&\& \text{IGtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int x \cosh^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cosh(x) \sinh(x) dx, x, \cosh^{-1}(ax)\right)}{a^2} \\ &= \frac{\text{Subst}\left(\int \frac{1}{2}x^n \sinh(2x) dx, x, \cosh^{-1}(ax)\right)}{a^2} \\ &= \frac{\text{Subst}\left(\int x^n \sinh(2x) dx, x, \cosh^{-1}(ax)\right)}{2a^2} \\ &= -\frac{\text{Subst}\left(\int e^{-2x}x^n dx, x, \cosh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int e^{2x}x^n dx, x, \cosh^{-1}(ax)\right)}{4a^2} \\ &= \frac{2^{-3-n}(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -2 \cosh^{-1}(ax))}{a^2} + \frac{2^{-3-n} \Gamma(1+n, 2 \cosh^{-1}(ax))}{a^2} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 58, normalized size = 0.98

$$\frac{2^{-3-n}(-\cosh^{-1}(ax))^{-n} (\cosh^{-1}(ax)^n \Gamma(1+n, -2 \cosh^{-1}(ax)) + (-\cosh^{-1}(ax))^n \Gamma(1+n, 2 \cosh^{-1}(ax)))}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcCosh[a*x]^n,x]

[Out] $(2^{(-3 - n)}*(\text{ArcCosh}[a*x]^n*\text{Gamma}[1 + n, -2*\text{ArcCosh}[a*x]] + (-\text{ArcCosh}[a*x])^n*\text{Gamma}[1 + n, 2*\text{ArcCosh}[a*x]]))/(a^2*(-\text{ArcCosh}[a*x])^n)$

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.

time = 2.38, size = 38, normalized size = 0.64

method	result	size
default	$\frac{\text{arccosh}(ax)^{n+2} \text{hypergeom}\left(\left[1+\frac{n}{2}\right], \left[\frac{3}{2}, 2+\frac{n}{2}\right], \text{arccosh}(ax)^2\right)}{a^2(n+2)}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccosh(a*x)^n,x,method=_RETURNVERBOSE)`

[Out] $1/a^{2/(n+2)} \operatorname{arccosh}(a*x)^{(n+2)} \operatorname{hypergeom}([1+1/2*n], [3/2, 2+1/2*n], \operatorname{arccosh}(a*x)^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^n,x, algorithm="maxima")`

[Out] `integrate(x*arccosh(a*x)^n, x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^n,x, algorithm="fricas")`

[Out] `integral(x*arccosh(a*x)^n, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acosh}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acosh(a*x)**n,x)`

[Out] `Integral(x*acosh(a*x)**n, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccosh(a*x)^n,x, algorithm="giac")`

[Out] integrate(x*arccosh(a*x)^n, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*acosh(a*x)^n,x)

[Out] int(x*acosh(a*x)^n, x)

3.131 $\int \cosh^{-1}(ax)^n dx$

Optimal. Leaf size=49

$$\frac{(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax))}{2a} + \frac{\Gamma(1+n, \cosh^{-1}(ax))}{2a}$$

[Out] $1/2*\operatorname{arccosh}(a*x)^n*\operatorname{GAMMA}(1+n, -\operatorname{arccosh}(a*x))/a/((- \operatorname{arccosh}(a*x))^n)+1/2*\operatorname{GAMMA}(1+n, \operatorname{arccosh}(a*x))/a$

Rubi [A]

time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5881, 3389, 2212}

$$\frac{\cosh^{-1}(ax)^n (-\cosh^{-1}(ax))^{-n} \operatorname{Gamma}(n+1, -\cosh^{-1}(ax))}{2a} + \frac{\operatorname{Gamma}(n+1, \cosh^{-1}(ax))}{2a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcCosh}[a*x]^n, x]$

[Out] $(\operatorname{ArcCosh}[a*x]^n*\operatorname{Gamma}[1+n, -\operatorname{ArcCosh}[a*x]])/(2*a*(-\operatorname{ArcCosh}[a*x])^n) + \operatorname{Gamma}[1+n, \operatorname{ArcCosh}[a*x]]/(2*a)$

Rule 2212

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1))*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m])*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 3389

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5881

```
Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_), x_Symbol]
:> Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \sinh(x) dx, x, \cosh^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int e^{-x} x^n dx, x, \cosh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int e^x x^n dx, x, \cosh^{-1}(ax)\right)}{2a} \\
&= \frac{(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax))}{2a} + \frac{\Gamma(1+n, \cosh^{-1}(ax))}{2a}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 0.88

$$\frac{(-\cosh^{-1}(ax))^{-n} \cosh^{-1}(ax)^n \Gamma(1+n, -\cosh^{-1}(ax)) + \Gamma(1+n, \cosh^{-1}(ax))}{2a}$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCosh[a*x]^n, x]``[Out] ((ArcCosh[a*x]^n*Gamma[1 + n, -ArcCosh[a*x]])/(-ArcCosh[a*x])^n + Gamma[1 + n, ArcCosh[a*x]])/(2*a)`**Maple [C]** Result contains higher order function than in optimal. Order 5 vs. order 4.

time = 2.33, size = 40, normalized size = 0.82

method	result	size
default	$\frac{\text{arccosh}(ax)^{n+2} \text{hypergeom}\left(\left[1+\frac{n}{2}\right], \left[\frac{3}{2}, 2+\frac{n}{2}\right], \frac{\text{arccosh}(ax)^2}{4}\right)}{a^{(n+2)}}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccosh(a*x)^n, x, method=_RETURNVERBOSE)``[Out] 1/a/(n+2)*arccosh(a*x)^(n+2)*hypergeom([1+1/2*n], [3/2, 2+1/2*n], 1/4*arccosh(a*x)^2)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccosh(a*x)^n, x, algorithm="maxima")``[Out] integrate(arccosh(a*x)^n, x)`

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^n,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**n,x)

[Out] Integral(acosh(a*x)**n, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^n,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^n, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acosh}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)^n,x)

[Out] int(acosh(a*x)^n, x)

$$3.132 \quad \int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\cosh^{-1}(ax)^n}{x}, x\right)$$

[Out] Unintegrable(arccosh(a*x)^n/x, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Verification is not applicable to the result.

[In] Int[ArcCosh[a*x]^n/x, x]

[Out] Defer[Int][ArcCosh[a*x]^n/x, x]

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^n}{x} dx = \int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Mathematica [A]

time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^n}{x} dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCosh[a*x]^n/x, x]

[Out] Integrate[ArcCosh[a*x]^n/x, x]

Maple [A]

time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\text{arccosh}(ax)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)^n/x,x)

[Out] int(arccosh(a*x)^n/x,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^n/x,x, algorithm="maxima")

[Out] integrate(arccosh(a*x)^n/x, x)

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^n/x,x, algorithm="fricas")

[Out] integral(arccosh(a*x)^n/x, x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^n(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)**n/x,x)

[Out] Integral(acosh(a*x)**n/x, x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)^n/x,x, algorithm="giac")

[Out] integrate(arccosh(a*x)^n/x, x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\operatorname{acosh}(ax)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^n/x, x)
```

```
[Out] int(acosh(a*x)^n/x, x)
```

3.133 $\int x^3 (a + b \cosh^{-1}(cx)) dx$

Optimal. Leaf size=84

$$-\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}}{32c^3} - \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{16c} - \frac{3b\cosh^{-1}(cx)}{32c^4} + \frac{1}{4}x^4(a + b\cosh^{-1}(cx))$$

[Out] $-3/32*b*arccosh(c*x)/c^4+1/4*x^4*(a+b*arccosh(c*x))-3/32*b*x*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/c^3-1/16*b*x^3*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/c$

Rubi [A]

time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5883, 102, 12, 92, 54}

$$\frac{1}{4}x^4(a + b\cosh^{-1}(cx)) - \frac{3b\cosh^{-1}(cx)}{32c^4} - \frac{3bx\sqrt{cx-1}\sqrt{cx+1}}{32c^3} - \frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{16c}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*ArcCosh[c*x]),x]`

[Out] $(-3*b*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(32*c^3) - (b*x^3*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(16*c) - (3*b*\text{ArcCosh}[c*x])/(32*c^4) + (x^4*(a + b*\text{ArcCosh}[c*x]))/4$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 54

`Int[1/(Sqrt[(a_) + (b_.)*(x_)])*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]`

Rule 92

`Int[((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]`

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int x^3 (a + b \cosh^{-1}(cx)) dx &= \frac{1}{4}x^4 (a + b \cosh^{-1}(cx)) - \frac{1}{4}(bc) \int \frac{x^4}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= -\frac{bx^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{16c} + \frac{1}{4}x^4 (a + b \cosh^{-1}(cx)) - \frac{b \int \frac{3x^2}{\sqrt{-1 + cx} \sqrt{1 + cx}}}{16c} \\
 &= -\frac{bx^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{16c} + \frac{1}{4}x^4 (a + b \cosh^{-1}(cx)) - \frac{(3b) \int \frac{x^2}{\sqrt{-1 + cx} \sqrt{1 + cx}}}{16c} \\
 &= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx}}{32c^3} - \frac{bx^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{16c} + \frac{1}{4}x^4 (a + b \cosh^{-1}(cx)) \\
 &= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx}}{32c^3} - \frac{bx^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{16c} - \frac{3b \cosh^{-1}(cx)}{32c^4} + \frac{1}{4}x^4
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 105, normalized size = 1.25

$$\frac{ax^4}{4} - \frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx}}{32c^3} - \frac{bx^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{16c} + \frac{1}{4}bx^4 \cosh^{-1}(cx) - \frac{3b \tanh^{-1}\left(\frac{\sqrt{-1 + cx}}{\sqrt{1 + cx}}\right)}{16c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*x^4)/4 - (3*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) + (b*x^4*ArcCosh[c*x])/4 - (3*b*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(16*c^4)
```

Maple [A]

time = 1.97, size = 114, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{c^4 x^4 a}{4} + \frac{b c^4 x^4 \operatorname{arccosh}(cx)}{4} - \frac{b \sqrt{cx-1} \sqrt{cx+1}}{16} c^3 x^3 - \frac{3b \sqrt{cx-1} \sqrt{cx+1}}{32} c x - \frac{3b \sqrt{cx-1} \sqrt{cx+1}}{32 \sqrt{c^2}}}{c^4}$
default	$\frac{\frac{c^4 x^4 a}{4} + \frac{b c^4 x^4 \operatorname{arccosh}(cx)}{4} - \frac{b \sqrt{cx-1} \sqrt{cx+1}}{16} c^3 x^3 - \frac{3b \sqrt{cx-1} \sqrt{cx+1}}{32} c x - \frac{3b \sqrt{cx-1} \sqrt{cx+1}}{32 \sqrt{c^2}}}{c^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{c^4} * (\frac{1}{4} * c^4 * x^4 * a + \frac{1}{4} * b * c^4 * x^4 * \operatorname{arccosh}(c * x) - \frac{1}{16} * b * (c * x - 1)^{(1/2)} * (c * x + 1)^{(1/2)} * c^3 * x^3 - \frac{3}{32} * b * (c * x - 1)^{(1/2)} * (c * x + 1)^{(1/2)} * c * x - \frac{3}{32} * b * (c * x - 1)^{(1/2)} * (c * x + 1)^{(1/2)} / (c^2 * x^2 - 1)^{(1/2)} * \ln(c * x + (c^2 * x^2 - 1)^{(1/2)}))$

Maxima [A]

time = 0.27, size = 87, normalized size = 1.04

$$\frac{1}{4} a x^4 + \frac{1}{32} \left(8 x^4 \operatorname{arccosh}(cx) - \left(\frac{2 \sqrt{c^2 x^2 - 1} x^3}{c^2} + \frac{3 \sqrt{c^2 x^2 - 1} x}{c^4} + \frac{3 \log(2 c^2 x + 2 \sqrt{c^2 x^2 - 1} c)}{c^5} \right) c \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccosh(c*x)),x, algorithm="maxima")`

[Out] $\frac{1}{4} * a * x^4 + \frac{1}{32} * (8 * x^4 * \operatorname{arccosh}(c * x) - (2 * \sqrt{c^2 * x^2 - 1} * x^3 / c^2 + 3 * \sqrt{c^2 * x^2 - 1} * x / c^4 + 3 * \log(2 * c^2 * x + 2 * \sqrt{c^2 * x^2 - 1} * c) / c^5) * c) * b$

Fricas [A]

time = 0.35, size = 73, normalized size = 0.87

$$\frac{8 a c^4 x^4 + (8 b c^4 x^4 - 3 b) \log \left(c x + \sqrt{c^2 x^2 - 1} \right) - (2 b c^3 x^3 + 3 b c x) \sqrt{c^2 x^2 - 1}}{32 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccosh(c*x)),x, algorithm="fricas")`

[Out] $\frac{1}{32} * (8 * a * c^4 * x^4 + (8 * b * c^4 * x^4 - 3 * b) * \log(c * x + \sqrt{c^2 * x^2 - 1}) - (2 * b * c^3 * x^3 + 3 * b * c * x) * \sqrt{c^2 * x^2 - 1}) / c^4$

Sympy [C] Result contains complex when optimal does not.

time = 0.22, size = 87, normalized size = 1.04

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{acosh}(cx)}{4} - \frac{bx^3 \sqrt{c^2x^2 - 1}}{16c} - \frac{3bx \sqrt{c^2x^2 - 1}}{32c^3} - \frac{3b \operatorname{acosh}(cx)}{32c^4} & \text{for } c \neq 0 \\ \frac{x^4 \left(a + \frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*acosh(c*x)),x)

[Out] Piecewise((a*x**4/4 + b*x**4*acosh(c*x)/4 - b*x**3*sqrt(c**2*x**2 - 1)/(16*c) - 3*b*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*b*acosh(c*x)/(32*c**4), Ne(c, 0)), (x**4*(a + I*pi*b/2)/4, True))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (a + b \operatorname{acosh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b*acosh(c*x)),x)

[Out] int(x^3*(a + b*acosh(c*x)), x)

3.134 $\int x^2 (a + b \cosh^{-1}(cx)) dx$

Optimal. Leaf size=71

$$-\frac{2b\sqrt{-1+cx}\sqrt{1+cx}}{9c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{9c} + \frac{1}{3}x^3(a + b \cosh^{-1}(cx))$$

[Out] 1/3*x^3*(a+b*arccosh(c*x))-2/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c

Rubi [A]

time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5883, 102, 12, 75}

$$\frac{1}{3}x^3(a + b \cosh^{-1}(cx)) - \frac{2b\sqrt{cx-1}\sqrt{cx+1}}{9c^3} - \frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*ArcCosh[c*x]),x]

[Out] (-2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c) + (x^3*(a + b*ArcCosh[c*x]))/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 75

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 102

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int x^2(a + b \cosh^{-1}(cx)) dx &= \frac{1}{3}x^3(a + b \cosh^{-1}(cx)) - \frac{1}{3}(bc) \int \frac{x^3}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= -\frac{bx^2\sqrt{-1 + cx} \sqrt{1 + cx}}{9c} + \frac{1}{3}x^3(a + b \cosh^{-1}(cx)) - \frac{b \int \frac{2x}{\sqrt{-1 + cx} \sqrt{1 + cx}}}{9c} \\
 &= -\frac{bx^2\sqrt{-1 + cx} \sqrt{1 + cx}}{9c} + \frac{1}{3}x^3(a + b \cosh^{-1}(cx)) - \frac{(2b) \int \frac{x}{\sqrt{-1 + cx} \sqrt{1 + cx}}}{9c} \\
 &= -\frac{2b\sqrt{-1 + cx} \sqrt{1 + cx}}{9c^3} - \frac{bx^2\sqrt{-1 + cx} \sqrt{1 + cx}}{9c} + \frac{1}{3}x^3(a + b \cosh^{-1}(cx))
 \end{aligned}$$

Mathematica [A]

time = 0.04, size = 54, normalized size = 0.76

$$\frac{1}{9} \left(3ax^3 - \frac{b\sqrt{-1 + cx} \sqrt{1 + cx} (2 + c^2x^2)}{c^3} + 3bx^3 \cosh^{-1}(cx) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*a*x^3 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c^2*x^2))/c^3 + 3*b*x^3*Arc
Cosh[c*x])/9
```

Maple [A]

time = 2.70, size = 55, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\frac{c^3x^3a}{3} + b \left(\frac{c^3x^3 \operatorname{arccosh}(cx)}{3} - \frac{\sqrt{cx-1} \sqrt{cx+1} (c^2x^2+2)}{9} \right)}{c^3}$	55
default	$\frac{\frac{c^3x^3a}{3} + b \left(\frac{c^3x^3 \operatorname{arccosh}(cx)}{3} - \frac{\sqrt{cx-1} \sqrt{cx+1} (c^2x^2+2)}{9} \right)}{c^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)`

[Out] $1/c^3*(1/3*c^3*x^3*a+b*(1/3*c^3*x^3*arccosh(c*x)-1/9*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}*(c^2*x^2+2)))$

Maxima [A]

time = 0.27, size = 58, normalized size = 0.82

$$\frac{1}{3} ax^3 + \frac{1}{9} \left(3x^3 \operatorname{arccosh}(cx) - c \left(\frac{\sqrt{c^2x^2 - 1} x^2}{c^2} + \frac{2\sqrt{c^2x^2 - 1}}{c^4} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccosh(c*x)),x, algorithm="maxima")`

[Out] $1/3*a*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b$

Fricas [A]

time = 0.37, size = 65, normalized size = 0.92

$$\frac{3bc^3x^3 \log\left(cx + \sqrt{c^2x^2 - 1}\right) + 3ac^3x^3 - (bc^2x^2 + 2b)\sqrt{c^2x^2 - 1}}{9c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccosh(c*x)),x, algorithm="fricas")`

[Out] $1/9*(3*b*c^3*x^3*\log(c*x + \sqrt{c^2*x^2 - 1}) + 3*a*c^3*x^3 - (b*c^2*x^2 + 2*b)*\sqrt{c^2*x^2 - 1})/c^3$

Sympy [C] Result contains complex when optimal does not.

time = 0.14, size = 71, normalized size = 1.00

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{acosh}(cx)}{3} - \frac{bx^2 \sqrt{c^2x^2 - 1}}{9c} - \frac{2b\sqrt{c^2x^2 - 1}}{9c^3} & \text{for } c \neq 0 \\ \frac{x^3 \left(a + \frac{i\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acosh(c*x)),x)`

[Out] `Piecewise((a*x**3/3 + b*x**3*acosh(c*x)/3 - b*x**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*sqrt(c**2*x**2 - 1)/(9*c**3), Ne(c, 0)), (x**3*(a + I*pi*b/2)/3, True))`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
 eur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + b \operatorname{acosh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*acosh(c*x)),x)

[Out] int(x^2*(a + b*acosh(c*x)), x)

3.135 $\int x(a + b \cosh^{-1}(cx)) dx$

Optimal. Leaf size=55

$$-\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{b \cosh^{-1}(cx)}{4c^2} + \frac{1}{2}x^2(a + b \cosh^{-1}(cx))$$

[Out] $-1/4*b*arccosh(c*x)/c^2+1/2*x^2*(a+b*arccosh(c*x))-1/4*b*x*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/c$

Rubi [A]

time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5883, 92, 54}

$$\frac{1}{2}x^2(a + b \cosh^{-1}(cx)) - \frac{b \cosh^{-1}(cx)}{4c^2} - \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*ArcCosh[c*x]),x]

[Out] $-1/4*(b*x*sqrt[-1 + c*x]*sqrt[1 + c*x])/c - (b*ArcCosh[c*x])/(4*c^2) + (x^2*(a + b*ArcCosh[c*x]))/2$

Rule 54

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 92

Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int x(a + b \cosh^{-1}(cx)) dx &= \frac{1}{2}x^2(a + b \cosh^{-1}(cx)) - \frac{1}{2}(bc) \int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}} dx \\
&= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c} + \frac{1}{2}x^2(a + b \cosh^{-1}(cx)) - \frac{b \int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}}{4c} \\
&= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{b \cosh^{-1}(cx)}{4c^2} + \frac{1}{2}x^2(a + b \cosh^{-1}(cx))
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 76, normalized size = 1.38

$$\frac{ax^2}{2} - \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4c} + \frac{1}{2}bx^2 \cosh^{-1}(cx) - \frac{b \tanh^{-1}\left(\frac{\sqrt{-1+cx}}{\sqrt{1+cx}}\right)}{2c^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x*(a + b*ArcCosh[c*x]),x]`

```
[Out] (a*x^2)/2 - (b*x*sqrt[-1 + c*x]*sqrt[1 + c*x])/(4*c) + (b*x^2*ArcCosh[c*x])
/2 - (b*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(2*c^2)
```

Maple [A]

time = 2.12, size = 91, normalized size = 1.65

method	result	size
derivativedivides	$ \frac{\frac{a c^2 x^2}{2} + \frac{b c^2 x^2 \operatorname{arccosh}(cx)}{2} - \frac{b \sqrt{cx-1} \sqrt{cx+1}}{4} cx - \frac{b \sqrt{cx-1} \sqrt{cx+1} \ln\left(\frac{cx + \sqrt{c^2 x^2 - 1}}{c}\right)}{4 \sqrt{c^2 x^2 - 1}}}{c^2} $	91
default	$ \frac{\frac{a c^2 x^2}{2} + \frac{b c^2 x^2 \operatorname{arccosh}(cx)}{2} - \frac{b \sqrt{cx-1} \sqrt{cx+1}}{4} cx - \frac{b \sqrt{cx-1} \sqrt{cx+1} \ln\left(\frac{cx + \sqrt{c^2 x^2 - 1}}{c}\right)}{4 \sqrt{c^2 x^2 - 1}}}{c^2} $	91

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)`

```
[Out] 1/c^2*(1/2*a*c^2*x^2+1/2*b*c^2*x^2*arccosh(c*x)-1/4*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*c*x-1/4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*
x^2-1)^(1/2)))
```

Maxima [A]

time = 0.26, size = 66, normalized size = 1.20

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2 \operatorname{arccosh}(cx) - c\left(\frac{\sqrt{c^2x^2 - 1}x}{c^2} + \frac{\log\left(2c^2x + 2\sqrt{c^2x^2 - 1}c\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(a+b*arccosh(c*x)),x, algorithm="maxima")`

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b
```

Fricas [A]

time = 0.38, size = 61, normalized size = 1.11

$$\frac{2ac^2x^2 - \sqrt{c^2x^2 - 1}bcx + (2bc^2x^2 - b)\log\left(cx + \sqrt{c^2x^2 - 1}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(a+b*arccosh(c*x)),x, algorithm="fricas")`

```
[Out] 1/4*(2*a*c^2*x^2 - sqrt(c^2*x^2 - 1)*b*c*x + (2*b*c^2*x^2 - b)*log(c*x + sqrt(c^2*x^2 - 1)))/c^2
```

Sympy [C] Result contains complex when optimal does not.

time = 0.10, size = 61, normalized size = 1.11

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{acosh}(cx)}{2} - \frac{bx\sqrt{c^2x^2 - 1}}{4c} - \frac{b \operatorname{acosh}(cx)}{4c^2} & \text{for } c \neq 0 \\ \frac{x^2\left(a + \frac{i\pi b}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(a+b*acosh(c*x)),x)`

```
[Out] Piecewise((a*x**2/2 + b*x**2*acosh(c*x)/2 - b*x*sqrt(c**2*x**2 - 1)/(4*c) - b*acosh(c*x)/(4*c**2), Ne(c, 0)), (x**2*(a + I*pi*b/2)/2, True))
```

Giac [A]

time = 0.48, size = 80, normalized size = 1.45

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2 \log\left(cx + \sqrt{c^2x^2 - 1}\right) - c\left(\frac{\sqrt{c^2x^2 - 1}x}{c^2} - \frac{\log\left(\left|-x|c| + \sqrt{c^2x^2 - 1}\right|\right)}{c^2|c|}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] $\frac{1}{2}ax^2 + \frac{1}{4}(2x^2 \log(cx + \sqrt{c^2x^2 - 1}) - c(\sqrt{c^2x^2 - 1})x/c^2 - \log(\text{abs}(-x\text{abs}(c) + \sqrt{c^2x^2 - 1}))) / (c^2\text{abs}(c))) * b$

Mupad [B]

time = 0.41, size = 47, normalized size = 0.85

$$\frac{ax^2}{2} + bx \operatorname{acosh}(cx) \left(\frac{x}{2} - \frac{1}{4c^2x} \right) - \frac{bx \sqrt{cx-1} \sqrt{cx+1}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*acosh(c*x)),x)

[Out] $(a*x^2)/2 + b*x*acosh(c*x)*(x/2 - 1/(4*c^2*x)) - (b*x*(c*x - 1)^{(1/2)}*(c*x + 1)^{(1/2)})/(4*c)$

3.136 $\int (a + b \cosh^{-1}(cx)) dx$

Optimal. Leaf size=35

$$ax - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c} + bx \cosh^{-1}(cx)$$

[Out] a*x+b*x*arccosh(c*x)-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5879, 75}

$$ax - \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c} + bx \cosh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b*ArcCosh[c*x], x]

[Out] a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]

Rule 75

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int (a + b \cosh^{-1}(cx)) dx &= ax + b \int \cosh^{-1}(cx) dx \\ &= ax + bx \cosh^{-1}(cx) - (bc) \int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}} dx \\ &= ax - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c} + bx \cosh^{-1}(cx) \end{aligned}$$

Mathematica [A]

time = 0.06, size = 35, normalized size = 1.00

$$ax - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c} + bx \cosh^{-1}(cx)$$

Antiderivative was successfully verified.

`[In] Integrate[a + b*ArcCosh[c*x], x]``[Out] a*x - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + b*x*ArcCosh[c*x]`**Maple [A]**

time = 0.70, size = 34, normalized size = 0.97

method	result	size
default	$ax + \frac{b(cx \operatorname{arccosh}(cx) - \sqrt{cx-1}\sqrt{cx+1})}{c}$	34
derivativedivides	$\frac{acx + b(cx \operatorname{arccosh}(cx) - \sqrt{cx-1}\sqrt{cx+1})}{c}$	36

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(a+b*arccosh(c*x), x, method=_RETURNVERBOSE)``[Out] a*x+b/c*(c*x*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2))`**Maxima [A]**

time = 0.26, size = 30, normalized size = 0.86

$$ax + \frac{(cx \operatorname{arccosh}(cx) - \sqrt{c^2x^2 - 1})b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(a+b*arccosh(c*x), x, algorithm="maxima")``[Out] a*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b/c`**Fricas [A]**

time = 0.35, size = 43, normalized size = 1.23

$$\frac{bcx \log\left(cx + \sqrt{c^2x^2 - 1}\right) + acx - \sqrt{c^2x^2 - 1}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(a+b*arccosh(c*x), x, algorithm="fricas")`

[Out] $(b*c*x*\log(c*x + \sqrt{c^2*x^2 - 1}) + a*c*x - \sqrt{c^2*x^2 - 1}*b)/c$

Sympy [A]

time = 0.06, size = 31, normalized size = 0.89

$$ax + b \left(\begin{cases} x \operatorname{acosh}(cx) - \frac{\sqrt{c^2x^2 - 1}}{c} & \text{for } c \neq 0 \\ \frac{i\pi x}{2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*acosh(c*x),x)`

[Out] $a*x + b*\operatorname{Piecewise}((x*\operatorname{acosh}(c*x) - \sqrt{c**2*x**2 - 1})/c, \operatorname{Ne}(c, 0)), (I*\pi*x/2, \operatorname{True}))$

Giac [A]

time = 0.41, size = 41, normalized size = 1.17

$$\left(x \log \left(cx + \sqrt{c^2x^2 - 1} \right) - \frac{\sqrt{c^2x^2 - 1}}{c} \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arccosh(c*x),x, algorithm="giac")`

[Out] $(x*\log(c*x + \sqrt{c^2*x^2 - 1}) - \sqrt{c^2*x^2 - 1}/c)*b + a*x$

Mupad [B]

time = 0.45, size = 31, normalized size = 0.89

$$ax + bx \operatorname{acosh}(cx) - \frac{b \sqrt{cx - 1} \sqrt{cx + 1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + b*acosh(c*x),x)`

[Out] $a*x + b*x*\operatorname{acosh}(c*x) - (b*(c*x - 1)^{(1/2)}*(c*x + 1)^{(1/2)})/c$

3.137 $\int \frac{a+b \cosh^{-1}(cx)}{x} dx$

Optimal. Leaf size=55

$$\frac{(a + b \cosh^{-1}(cx))^2}{2b} + (a + b \cosh^{-1}(cx)) \log \left(1 + e^{-2 \cosh^{-1}(cx)} \right) - \frac{1}{2} b \text{PolyLog} \left(2, -e^{-2 \cosh^{-1}(cx)} \right)$$

[Out] 1/2*(a+b*arccosh(c*x))^2/b+(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))^2)-1/2*b*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))^2)

Rubi [A]

time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5882, 3799, 2221, 2317, 2438}

$$\frac{(a + b \cosh^{-1}(cx))^2}{2b} + \log \left(e^{-2 \cosh^{-1}(cx)} + 1 \right) (a + b \cosh^{-1}(cx)) - \frac{1}{2} b \text{Li}_2 \left(-e^{-2 \cosh^{-1}(cx)} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])/x,x]

[Out] (a + b*ArcCosh[c*x])^2/(2*b) + (a + b*ArcCosh[c*x])*Log[1 + E^(-2*ArcCosh[c*x])] - (b*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x
_Symbol] := Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```

c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x))))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

Rule 5882

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/(x_), x_Symbol] := Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
b, c}, x] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \cosh^{-1}(cx)}{x} dx &= \text{Subst} \left(\int (a + bx) \tanh(x) dx, x, \cosh^{-1}(cx) \right) \\
&= -\frac{(a + b \cosh^{-1}(cx))^2}{2b} + 2 \text{Subst} \left(\int \frac{e^{2x}(a + bx)}{1 + e^{2x}} dx, x, \cosh^{-1}(cx) \right) \\
&= -\frac{(a + b \cosh^{-1}(cx))^2}{2b} + (a + b \cosh^{-1}(cx)) \log \left(1 + e^{2 \cosh^{-1}(cx)} \right) - b \text{Subst} \left(\int \log \right) \\
&= -\frac{(a + b \cosh^{-1}(cx))^2}{2b} + (a + b \cosh^{-1}(cx)) \log \left(1 + e^{2 \cosh^{-1}(cx)} \right) - \frac{1}{2} b \text{Subst} \left(\int \right) \\
&= -\frac{(a + b \cosh^{-1}(cx))^2}{2b} + (a + b \cosh^{-1}(cx)) \log \left(1 + e^{2 \cosh^{-1}(cx)} \right) + \frac{1}{2} b \text{Li}_2 \left(-e^{2 \cosh^{-1}(cx)} \right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 48, normalized size = 0.87

$$a \log(x) + \frac{1}{2} b \left(\cosh^{-1}(cx) \left(\cosh^{-1}(cx) + 2 \log \left(1 + e^{-2 \cosh^{-1}(cx)} \right) \right) - \text{PolyLog} \left(2, -e^{-2 \cosh^{-1}(cx)} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x, x]
```

```
[Out] a*Log[x] + (b*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])])
- PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

Maple [A]

time = 2.53, size = 75, normalized size = 1.36

method	result
derivativedivides	$a \ln(cx) - \frac{b \text{arccosh}(cx)^2}{2} + b \text{arccosh}(cx) \ln \left(1 + (cx + \sqrt{cx - 1} \sqrt{cx + 1})^2 \right) + \frac{b \text{polylog} \left(\right)}{2}$

default	$a \ln(cx) - \frac{b \operatorname{arccosh}(cx)^2}{2} + b \operatorname{arccosh}(cx) \ln\left(1 + (cx + \sqrt{cx-1} \sqrt{cx+1})^2\right) + \frac{b \operatorname{polylog}\left(2, -(cx + \sqrt{cx-1} \sqrt{cx+1})^2\right)}{2}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccosh(c*x))/x,x,method=_RETURNVERBOSE)`

[Out] $a \ln(cx) - 1/2 * b * \operatorname{arccosh}(cx)^2 + b * \operatorname{arccosh}(cx) * \ln(1 + (cx + \sqrt{cx-1}) * \sqrt{cx+1}) + 1/2 * b * \operatorname{polylog}(2, -(cx + \sqrt{cx-1}) * \sqrt{cx+1})^2$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))/x,x, algorithm="maxima")`

[Out] $b * \int \frac{\log(cx + \sqrt{cx+1}) * \sqrt{cx-1}}{x} dx + a * \log(x)$

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))/x,x, algorithm="fricas")`

[Out] $\int (b * \operatorname{arccosh}(cx) + a) / x dx$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acosh(c*x))/x,x)`

[Out] $\int (a + b * \operatorname{acosh}(cx)) / x dx$

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{acosh}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/x,x)
```

```
[Out] int((a + b*acosh(c*x))/x, x)
```

$$3.138 \quad \int \frac{a+b \cosh^{-1}(cx)}{x^2} dx$$

Optimal. Leaf size=37

$$-\frac{a+b \cosh^{-1}(cx)}{x} + bc \operatorname{ArcTan}\left(\sqrt{-1+cx} \sqrt{1+cx}\right)$$

[Out] $(-a-b*\operatorname{arccosh}(c*x))/x+b*c*\operatorname{arctan}((c*x-1)^{(1/2)}*(c*x+1)^{(1/2)})$

Rubi [A]

time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5883, 94, 211}

$$bc \operatorname{ArcTan}\left(\sqrt{cx-1} \sqrt{cx+1}\right) - \frac{a+b \cosh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcCosh}[c*x])/x^2, x]$

[Out] $-((a + b*\operatorname{ArcCosh}[c*x])/x) + b*c*\operatorname{ArcTan}[\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x]]$

Rule 94

$\operatorname{Int}[1/(\operatorname{Sqrt}[(a_.) + (b_.)*(x_.)]*\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] \rightarrow \operatorname{Dist}[b*f, \operatorname{Subst}[\operatorname{Int}[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, \operatorname{Sqrt}[a + b*x]*\operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[2*b*d*e - f*(b*c + a*d), 0]$

Rule 211

$\operatorname{Int}(((a_.) + (b_.)*(x_.)^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b]$

Rule 5883

$\operatorname{Int}(((a_.) + \operatorname{ArcCosh}[(c_.)*(x_.)]*(b_.))^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^n/(d*(m+1))), x] - \operatorname{Dist}[b*c*(n/(d*(m+1))), \operatorname{Int}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^{(n-1)})/(\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[-1 + c*x])], x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{a + b \cosh^{-1}(cx)}{x^2} dx &= -\frac{a + b \cosh^{-1}(cx)}{x} + (bc) \int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}} dx \\
&= -\frac{a + b \cosh^{-1}(cx)}{x} + (bc^2) \text{Subst}\left(\int \frac{1}{c+cx^2} dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right) \\
&= -\frac{a + b \cosh^{-1}(cx)}{x} + bc \tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 65, normalized size = 1.76

$$-\frac{a}{x} - \frac{b \cosh^{-1}(cx)}{x} + \frac{bc\sqrt{-1+c^2x^2} \text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*ArcCosh[c*x])/x^2,x]``[Out] -(a/x) - (b*ArcCosh[c*x])/x + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])`**Maple [A]**

time = 1.94, size = 66, normalized size = 1.78

method	result	size
derivativedivides	$c \left(-\frac{a}{cx} - \frac{b \operatorname{arccosh}(cx)}{cx} - \frac{b\sqrt{cx-1}\sqrt{cx+1} \operatorname{arctan}\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} \right)$	66
default	$c \left(-\frac{a}{cx} - \frac{b \operatorname{arccosh}(cx)}{cx} - \frac{b\sqrt{cx-1}\sqrt{cx+1} \operatorname{arctan}\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} \right)$	66

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a+b*arccosh(c*x))/x^2,x,method=_RETURNVERBOSE)``[Out] c*(-a/c/x-b/c/x*arccosh(c*x)-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))`**Maxima [A]**

time = 0.46, size = 30, normalized size = 0.81

$$-\left(c \arcsin\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^2,x, algorithm="maxima")

[Out] -(c*arcsin(1/(c*abs(x)))) + arccosh(c*x)/x)*b - a/x

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(33) = 66.

time = 0.37, size = 74, normalized size = 2.00

$$\frac{2bcx \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + bx \log\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bx - b) \log\left(cx + \sqrt{c^2x^2 - 1}\right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^2,x, algorithm="fricas")

[Out] (2*b*c*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + b*x*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*x - b)*log(c*x + sqrt(c^2*x^2 - 1)) - a)/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))/x**2,x)

[Out] Integral((a + b*acosh(c*x))/x**2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^2,x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)/x^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))/x^2,x)

[Out] int((a + b*acosh(c*x))/x^2, x)

$$3.139 \quad \int \frac{a+b \cosh^{-1}(cx)}{x^3} dx$$

Optimal. Leaf size=43

$$\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{a+b \cosh^{-1}(cx)}{2x^2}$$

[Out] $1/2*(-a-b*\operatorname{arccosh}(c*x))/x^2+1/2*b*c*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/x$

Rubi [A]

time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5883, 97}

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2x} - \frac{a+b \cosh^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcCosh}[c*x])/x^3, x]$

[Out] $(b*c*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(2*x) - (a + b*\operatorname{ArcCosh}[c*x])/(2*x^2)$

Rule 97

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[b*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}*((e + f*x)^{(p+1)})/((m+1)*(b*c - a*d)*(b*e - a*f)), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& \operatorname{EqQ}[\operatorname{Simplify}[m + n + p + 3], 0] \&\& \operatorname{EqQ}[a*d*f*(m+1) + b*c*f*(n+1) + b*d*e*(p+1), 0] \&\& \operatorname{NeQ}[m, -1]$

Rule 5883

$\operatorname{Int}[(a_. + \operatorname{ArcCosh}[(c_.)*(x_.)]*(b_.))^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^n/(d*(m+1))), x] - \operatorname{Dist}[b*c*(n/(d*(m+1))), \operatorname{Int}[(d*x)^{(m+1)}*((a + b*\operatorname{ArcCosh}[c*x])^{(n-1)})/(\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[-1 + c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{a+b \cosh^{-1}(cx)}{x^3} dx &= -\frac{a+b \cosh^{-1}(cx)}{2x^2} + \frac{1}{2}(bc) \int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}} dx \\ &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{a+b \cosh^{-1}(cx)}{2x^2} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 48, normalized size = 1.12

$$-\frac{a}{2x^2} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2x} - \frac{b\cosh^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*ArcCosh[c*x])/x^3,x]``[Out] -1/2*a/x^2 + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*ArcCosh[c*x])/(2*x^2)`**Maple [A]**

time = 1.91, size = 52, normalized size = 1.21

method	result	size
derivativedivides	$c^2 \left(-\frac{a}{2c^2x^2} + b \left(-\frac{\operatorname{arccosh}(cx)}{2c^2x^2} + \frac{\sqrt{cx-1}\sqrt{cx+1}}{2cx} \right) \right)$	52
default	$c^2 \left(-\frac{a}{2c^2x^2} + b \left(-\frac{\operatorname{arccosh}(cx)}{2c^2x^2} + \frac{\sqrt{cx-1}\sqrt{cx+1}}{2cx} \right) \right)$	52

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a+b*arccosh(c*x))/x^3,x,method=_RETURNVERBOSE)``[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arccosh(c*x)+1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/x))`**Maxima [A]**

time = 0.47, size = 36, normalized size = 0.84

$$\frac{1}{2}b \left(\frac{\sqrt{c^2x^2-1}c}{x} - \frac{\operatorname{arccosh}(cx)}{x^2} \right) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*arccosh(c*x))/x^3,x, algorithm="maxima")``[Out] 1/2*b*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a/x^2`**Fricas [A]**

time = 0.34, size = 48, normalized size = 1.12

$$\frac{\sqrt{c^2x^2-1}bcx + ax^2 - b\log\left(cx + \sqrt{c^2x^2-1}\right) - a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^3,x, algorithm="fricas")

[Out] 1/2*(sqrt(c^2*x^2 - 1)*b*c*x + a*x^2 - b*log(c*x + sqrt(c^2*x^2 - 1)) - a)/x^2

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))/x**3,x)

[Out] Integral((a + b*acosh(c*x))/x**3, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^3,x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)/x^3, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))/x^3,x)

[Out] int((a + b*acosh(c*x))/x^3, x)

$$3.140 \quad \int \frac{a+b \cosh^{-1}(cx)}{x^4} dx$$

Optimal. Leaf size=71

$$\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{a+b \cosh^{-1}(cx)}{3x^3} + \frac{1}{6}bc^3 \text{ArcTan}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

[Out] 1/3*(-a-b*arccosh(c*x))/x^3+1/6*b*c^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2

Rubi [A]

time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5883, 105, 12, 94, 211}

$$-\frac{a+b \cosh^{-1}(cx)}{3x^3} + \frac{1}{6}bc^3 \text{ArcTan}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])/x^4,x]

[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (a + b*ArcCosh[c*x])/(3*x^3) + (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/6

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 94

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 105

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 5883

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*((d_)*(x_)^(m_)), x_Symbol] := Simp[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \cosh^{-1}(cx)}{x^4} dx &= -\frac{a + b \cosh^{-1}(cx)}{3x^3} + \frac{1}{3}(bc) \int \frac{1}{x^3 \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2} - \frac{a + b \cosh^{-1}(cx)}{3x^3} + \frac{1}{6}(bc) \int \frac{c^2}{x \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2} - \frac{a + b \cosh^{-1}(cx)}{3x^3} + \frac{1}{6}(bc^3) \int \frac{1}{x \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2} - \frac{a + b \cosh^{-1}(cx)}{3x^3} + \frac{1}{6}(bc^4) \text{Subst} \left(\int \frac{1}{c + cx^2} dx, x, \sqrt{-1 + cx} \right) \\
 &= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2} - \frac{a + b \cosh^{-1}(cx)}{3x^3} + \frac{1}{6} bc^3 \tan^{-1} \left(\frac{\sqrt{-1 + cx} \sqrt{1 + cx}}{c} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 101, normalized size = 1.42

$$-\frac{a}{3x^3} + \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2} - \frac{b \cosh^{-1}(cx)}{3x^3} + \frac{bc^3 \sqrt{-1 + c^2 x^2} \text{ArcTan} \left(\frac{\sqrt{-1 + c^2 x^2}}{\sqrt{-1 + cx} \sqrt{1 + cx}} \right)}{6 \sqrt{-1 + cx} \sqrt{1 + cx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcCosh[c*x])/x^4, x]

[Out] -1/3*a/x^3 + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (b*ArcCosh[c*x])/(3*x^3) + (b*c^3*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Maple [A]

time = 1.95, size = 91, normalized size = 1.28

method	result
derivativedivides	$c^3 \left(-\frac{a}{3c^3x^3} - \frac{b \operatorname{arccosh}(cx)}{3c^3x^3} - \frac{b\sqrt{cx-1} \sqrt{cx+1} \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{6\sqrt{c^2x^2-1}} + \frac{b\sqrt{cx-1} \sqrt{cx+1}}{6c^2x^2} \right)$
default	$c^3 \left(-\frac{a}{3c^3x^3} - \frac{b \operatorname{arccosh}(cx)}{3c^3x^3} - \frac{b\sqrt{cx-1} \sqrt{cx+1} \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{6\sqrt{c^2x^2-1}} + \frac{b\sqrt{cx-1} \sqrt{cx+1}}{6c^2x^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccosh(c*x))/x^4,x,method=_RETURNVERBOSE)`

[Out] $c^3*(-1/3*a/c^3/x^3-1/3*b/c^3/x^3*\operatorname{arccosh}(c*x)-1/6*b*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/(c^2*x^2-1)^{(1/2)}*\arctan(1/(c^2*x^2-1)^{(1/2)})+1/6*b*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/c^2/x^2)$

Maxima [A]

time = 0.48, size = 52, normalized size = 0.73

$$-\frac{1}{6} \left(\left(c^2 \arcsin\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x^2-1}}{x^2} \right) c + \frac{2 \operatorname{arccosh}(cx)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))/x^4,x, algorithm="maxima")`

[Out] $-1/6*((c^2*\arcsin(1/(c*\operatorname{abs}(x)))) - \operatorname{sqrt}(c^2*x^2 - 1)/x^2)*c + 2*\operatorname{arccosh}(c*x)/x^3)*b - 1/3*a/x^3$

Fricas [A]

time = 0.39, size = 100, normalized size = 1.41

$$\frac{2bc^3x^3 \arctan(-cx + \sqrt{c^2x^2-1}) + 2bx^3 \log(-cx + \sqrt{c^2x^2-1}) + \sqrt{c^2x^2-1}bcx + 2(bx^3 - b) \log(cx + \sqrt{c^2x^2-1}) - 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))/x^4,x, algorithm="fricas")`

[Out] $1/6*(2*b*c^3*x^3*\arctan(-c*x + \operatorname{sqrt}(c^2*x^2 - 1)) + 2*b*x^3*\log(-c*x + \operatorname{sqrt}(c^2*x^2 - 1)) + \operatorname{sqrt}(c^2*x^2 - 1)*b*c*x + 2*(b*x^3 - b)*\log(c*x + \operatorname{sqrt}(c^2*x^2 - 1)) - 2*a)/x^3$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))/x**4,x)

[Out] Integral((a + b*acosh(c*x))/x**4, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/x^4,x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)/x^4, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))/x^4,x)

[Out] int((a + b*acosh(c*x))/x^4, x)

$$3.141 \quad \int \frac{a+b \cosh^{-1}(cx)}{x^5} dx$$

Optimal. Leaf size=72

$$\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{12x^3} + \frac{bc^3\sqrt{-1+cx}\sqrt{1+cx}}{6x} - \frac{a+b \cosh^{-1}(cx)}{4x^4}$$

[Out] 1/4*(-a-b*arccosh(c*x))/x^4+1/12*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^3+1/6*b*c^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x

Rubi [A]

time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5883, 105, 12, 97}

$$-\frac{a+b \cosh^{-1}(cx)}{4x^4} + \frac{bc^3\sqrt{cx-1}\sqrt{cx+1}}{6x} + \frac{bc\sqrt{cx-1}\sqrt{cx+1}}{12x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])/x^5,x]

[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*x^3) + (b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x) - (a + b*ArcCosh[c*x])/(4*x^4)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 97

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]

Rule 105

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \cosh^{-1}(cx)}{x^5} dx &= -\frac{a + b \cosh^{-1}(cx)}{4x^4} + \frac{1}{4}(bc) \int \frac{1}{x^4 \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{12x^3} - \frac{a + b \cosh^{-1}(cx)}{4x^4} + \frac{1}{12}(bc) \int \frac{2c^2}{x^2 \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{12x^3} - \frac{a + b \cosh^{-1}(cx)}{4x^4} + \frac{1}{6}(bc^3) \int \frac{1}{x^2 \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{12x^3} + \frac{bc^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{6x} - \frac{a + b \cosh^{-1}(cx)}{4x^4}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 50, normalized size = 0.69

$$\frac{-3a + bcx \sqrt{-1 + cx} \sqrt{1 + cx} (1 + 2c^2 x^2) - 3b \cosh^{-1}(cx)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/x^5, x]
```

```
[Out] (-3*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 + 2*c^2*x^2) - 3*b*ArcCosh[c*
x])/(12*x^4)
```

Maple [A]

time = 1.96, size = 62, normalized size = 0.86

method	result	size
derivativedivides	$c^4 \left(-\frac{a}{4c^4 x^4} + b \left(-\frac{\operatorname{arccosh}(cx)}{4c^4 x^4} + \frac{\sqrt{cx-1} \sqrt{cx+1} (2c^2 x^2 + 1)}{12c^3 x^3} \right) \right)$	62
default	$c^4 \left(-\frac{a}{4c^4 x^4} + b \left(-\frac{\operatorname{arccosh}(cx)}{4c^4 x^4} + \frac{\sqrt{cx-1} \sqrt{cx+1} (2c^2 x^2 + 1)}{12c^3 x^3} \right) \right)$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccosh(c*x))/x^5,x,method=_RETURNVERBOSE)`

[Out] $c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arccosh(c*x)+1/12*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}*(2*c^2*x^2+1)/c^3/x^3))$

Maxima [A]

time = 0.47, size = 57, normalized size = 0.79

$$\frac{1}{12} \left(\left(\frac{2\sqrt{c^2x^2-1}c^2}{x} + \frac{\sqrt{c^2x^2-1}}{x^3} \right) c - \frac{3 \operatorname{arccosh}(cx)}{x^4} \right) b - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))/x^5,x, algorithm="maxima")`

[Out] $1/12*((2*\sqrt{c^2*x^2-1}*c^2/x + \sqrt{c^2*x^2-1}/x^3)*c - 3*arccosh(c*x)/x^4)*b - 1/4*a/x^4$

Fricas [A]

time = 0.36, size = 60, normalized size = 0.83

$$\frac{3ax^4 - 3b \log(cx + \sqrt{c^2x^2 - 1}) + (2bc^3x^3 + bcx)\sqrt{c^2x^2 - 1} - 3a}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))/x^5,x, algorithm="fricas")`

[Out] $1/12*(3*a*x^4 - 3*b*\log(c*x + \sqrt{c^2*x^2 - 1})) + (2*b*c^3*x^3 + b*c*x)*\sqrt{c^2*x^2 - 1} - 3*a)/x^4$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acosh(c*x))/x**5,x)`

[Out] `Integral((a + b*acosh(c*x))/x**5, x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/x^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/x^5,x)
```

```
[Out] int((a + b*acosh(c*x))/x^5, x)
```

3.142 $\int x^2 \sqrt{a + b \cosh^{-1}(cx)} dx$

Optimal. Leaf size=213

$$\frac{1}{3}x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{\sqrt{b} e^{a/b} \sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} - \frac{\sqrt{b} e^{\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}$$

[Out] $-1/144*\exp(3*a/b)*\operatorname{erf}(3^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^3-1/144*\operatorname{erfi}(3^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^3/\exp(3*a/b)-1/16*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^3-1/16*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^3/\exp(a/b)+1/3*x^3*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.50, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5884, 5953, 3393, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi} \sqrt{b} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} - \frac{\sqrt{\frac{\pi}{3}} \sqrt{b} e^{\frac{3a}{b}} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3} - \frac{\sqrt{\pi} \sqrt{b} e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} - \frac{\sqrt{\frac{\pi}{3}} \sqrt{b} e^{-\frac{3a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3} + \frac{1}{3}x^3 \sqrt{a + b \cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]], x]$

[Out] $(x^3*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/3 - (\operatorname{Sqrt}[b]*E^{(a/b)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(16*c^3) - (\operatorname{Sqrt}[b]*E^{((3*a)/b)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(48*c^3) - (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(16*c^3*E^{(a/b)}) - (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(48*c^3*E^{((3*a)/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_)*(e_) + (f_)*(x_))}/\operatorname{Sqrt}[(c_) + (d_)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\amp; \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_) + (b_)*((c_) + (d_)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\amp; \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)](n_.), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)m, Sin[e + f*x]n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))(n_.)(x_)(m_.), x_Symbol] := Simp[
x(m + 1)*((a + b*ArcCosh[c*x])n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x(m + 1)*((a + b*ArcCosh[c*x])(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))(n_.)(x_)(m_.)*((d1_.) + (e1_.)*(x
_))(p_.)*((d2_.) + (e2_.)*(x_))(p_.), x_Symbol] := Dist[(1/(b*c(m + 1)))*
Simp[(d1 + e1*x)p/(1 + c*x)p]*Simp[(d2 + e2*x)p/(-1 + c*x)p], Subst[Int
[xn*Cosh[-a/b + x/b]m*Sinh[-a/b + x/b](2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{a + b \cosh^{-1}(cx)} dx &= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{1}{6} (bc) \int \frac{x^3}{\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}} dx \\
&= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \frac{\cosh^3(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{6c^3} \\
&= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \left(\frac{3 \cosh(x)}{4\sqrt{a + bx}} + \frac{\cosh(3x)}{4\sqrt{a + bx}} \right) dx, x, \cosh^{-1}(cx) \right)}{6c^3} \\
&= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \frac{\cosh(3x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{24c^3} - \frac{b \operatorname{Subst} \left(\int \frac{\cosh(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{48c^3} \\
&= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \frac{e^{-3x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{48c^3} - \frac{b \operatorname{Subst} \left(\int \frac{e^{-x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{48c^3} \\
&= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{\operatorname{Subst} \left(\int e^{\frac{3a}{b} - \frac{3x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)} \right)}{24c^3} - \frac{\operatorname{Subst} \left(\int e^{\frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)} \right)}{24c^3} \\
&= \frac{1}{3} x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{\sqrt{b} e^{a/b} \sqrt{\pi} \operatorname{erf} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{16c^3} - \frac{\sqrt{b} e^{\frac{3a}{b}} \sqrt{\pi} \operatorname{erf} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{16c^3}
\end{aligned}$$

Mathematica [A]

time = 0.35, size = 214, normalized size = 1.00

$$\frac{e^{-\frac{3a}{b}} \sqrt{a + b \cosh^{-1}(cx)} \left(9e^{\frac{3a}{b}} \sqrt{-\frac{a + b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{3}{2}, \frac{3}{b} + \cosh^{-1}(cx)\right) + \sqrt{3} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{3}{2}, -\frac{3(a + b \cosh^{-1}(cx))}{b}\right) + 9e^{\frac{a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{3}{2}, -\frac{a + b \cosh^{-1}(cx)}{b}\right) + \sqrt{3} e^{\frac{a}{b}} \sqrt{-\frac{a + b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{3}{2}, \frac{3(a + b \cosh^{-1}(cx))}{b}\right) \right)}{72c^3 \sqrt{-\frac{(a + b \cosh^{-1}(cx))^2}{b^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a + b*ArcCosh[c*x]], x]

[Out] (Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x])/b] + 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c*x])/b)))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \operatorname{arccosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)*x^2, x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:  integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*acosh(c*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)*x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*acosh(c*x))^(1/2),x)`

[Out] `int(x^2*(a + b*acosh(c*x))^(1/2), x)`

3.143 $\int x \sqrt{a + b \cosh^{-1}(cx)} dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{a + b \cosh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{\sqrt{b} e^{\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2} - \frac{\sqrt{b} e^{-\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}$$

[Out] $-1/32*\exp(2*a/b)*\operatorname{erf}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^2-1/32*\operatorname{erfi}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^2/\exp(2*a/b)-1/4*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/c^2+1/2*x^2*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.40, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5884, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} e^{\frac{2a}{b}} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} e^{-\frac{2a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2} - \frac{\sqrt{a + b \cosh^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2 \sqrt{a + b \cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]], x]$

[Out] $-1/4*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/c^2 + (x^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/2 - (\operatorname{Sqrt}[b]*E^{((2*a)/b)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(16*c^2) - (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(16*c^2*E^{((2*a)/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\amp; !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2))}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]))], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \&\amp; \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2))}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]))], x] /; \operatorname{Fr}$

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 3393

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(q_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^q/(-1 + c*x)^q], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int x \sqrt{a + b \cosh^{-1}(cx)} dx &= \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{1}{4} (bc) \int \frac{x^2}{\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}} \\
&= \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \frac{\cosh^2(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{4c^2} \\
&= \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \left(\frac{1}{2\sqrt{a + bx}} + \frac{\cosh(2x)}{2\sqrt{a + bx}} \right) dx, x, \cosh^{-1}(cx) \right)}{4c^2} \\
&= -\frac{\sqrt{a + b \cosh^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \frac{\cosh(2x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{8c^2} \\
&= -\frac{\sqrt{a + b \cosh^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst} \left(\int \frac{e^{-2x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{16c^2} \\
&= -\frac{\sqrt{a + b \cosh^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{\operatorname{Subst} \left(\int e^{\frac{2a}{b} - \frac{2x^2}{b}} dx, x, \cosh^{-1}(cx) \right)}{8c^2} \\
&= -\frac{\sqrt{a + b \cosh^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{\sqrt{b} e^{\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{erf} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{16c^2}
\end{aligned}$$

Mathematica [A]

time = 0.29, size = 136, normalized size = 0.94

$$\frac{8\sqrt{a + b \cosh^{-1}(cx)} \cosh(2 \cosh^{-1}(cx)) + \sqrt{b} \sqrt{2\pi} \operatorname{Erfi} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right) (-\cosh(\frac{2a}{b}) + \sinh(\frac{2a}{b})) - \sqrt{b} \sqrt{2\pi} \operatorname{Erf} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right) (\cosh(\frac{2a}{b}) + \sinh(\frac{2a}{b}))}{32c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a + b*ArcCosh[c*x]], x]

[Out] (8*Sqrt[a + b*ArcCosh[c*x]]*Cosh[2*ArcCosh[c*x]] + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b]) - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]))/(32*c^2)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a + b \operatorname{arccosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)*x, x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:  integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*acosh(c*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)*x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*acosh(c*x))^(1/2), x)`

[Out] `int(x*(a + b*acosh(c*x))^(1/2), x)`

3.144 $\int \sqrt{a + b \cosh^{-1}(cx)} dx$

Optimal. Leaf size=102

$$x\sqrt{a + b \cosh^{-1}(cx)} - \frac{\sqrt{b} e^{a/b} \sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b} e^{-\frac{a}{b}} \sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

[Out] $-1/4*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}/c-1/4*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}/c/\exp(a/b)+x*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.28, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5879, 5953, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi} \sqrt{b} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{\pi} \sqrt{b} e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} + x\sqrt{a + b \cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*ArcCosh[c*x]], x]`

[Out] $x*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]] - (\operatorname{Sqrt}[b]*E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(4*c) - (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(4*c*E^{(a/b)})$

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`
`> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /;` `FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=` `Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /;` `FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=` `Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /;` `FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3388

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \sqrt{a + b \cosh^{-1}(cx)} \, dx &= x \sqrt{a + b \cosh^{-1}(cx)} - \frac{1}{2}(bc) \int \frac{x}{\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}} \, dx \\
 &= x \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a + bx}} \, dx, x, \cosh^{-1}(cx)\right)}{2c} \\
 &= x \sqrt{a + b \cosh^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \frac{e^{-x}}{\sqrt{a + bx}} \, dx, x, \cosh^{-1}(cx)\right)}{4c} - \frac{b \operatorname{Subst}\left(\int \frac{e^x}{\sqrt{a + bx}} \, dx, x, \cosh^{-1}(cx)\right)}{4c} \\
 &= x \sqrt{a + b \cosh^{-1}(cx)} - \frac{\operatorname{Subst}\left(\int e^{\frac{a}{b} - \frac{x^2}{b}} \, dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{2c} - \frac{\operatorname{Subst}\left(\int e^{\frac{a}{b} + \frac{x^2}{b}} \, dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{2c} \\
 &= x \sqrt{a + b \cosh^{-1}(cx)} - \frac{\sqrt{b} e^{a/b} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b} e^{-a/b} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}
 \end{aligned}$$

Mathematica [A]

time = 0.16, size = 100, normalized size = 0.98

$$\frac{e^{-\frac{a}{b}} \sqrt{a + b \cosh^{-1}(cx)} \left(\frac{e^{\frac{2a}{b}} \Gamma\left(\frac{3}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b} + \cosh^{-1}(cx)}} + \frac{\Gamma\left(\frac{3}{2}, -\frac{a + b \cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a + b \cosh^{-1}(cx)}{b}}}\right)}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*ArcCosh[c*x]], x]

[Out] (Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/Sqrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a + b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b))

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{arccosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))^(1/2), x)

[Out] int((a+b*arccosh(c*x))^(1/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*arccosh(c*x) + a), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))**(1/2),x)

[Out] Integral(sqrt(a + b*acosh(c*x)), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*arccosh(c*x) + a), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))^(1/2),x)

[Out] int((a + b*acosh(c*x))^(1/2), x)

3.145 $\int x^2 (a + b \cosh^{-1}(cx))^{3/2} dx$

Optimal. Leaf size=292

$$\frac{b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{6c} + \frac{1}{3}x^3(a+b\cosh^{-1}(cx))^{3/2}$$

[Out] $\frac{1}{3}x^3(a+b\operatorname{arccosh}(cx))^{3/2} - \frac{1}{288}b^{3/2}\exp(3a/b)\operatorname{erf}(3^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2})\operatorname{erfi}(3^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) - \frac{1}{288}b^{3/2}\operatorname{erfi}(3^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2})\operatorname{erf}(3^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) - \frac{1}{32}b^{3/2}\exp(a/b)\operatorname{erf}((a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2})\operatorname{erfi}((a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) - \frac{1}{32}b^{3/2}\operatorname{erfi}((a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2})\operatorname{erf}((a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) - \frac{1}{3}b^{3/2}(cx-1)^{1/2}(cx+1)^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2} - \frac{1}{6}b^{3/2}x^2(cx-1)^{1/2}(cx+1)^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2} - \frac{1}{6}b^{3/2}x^2(cx-1)^{1/2}(cx+1)^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2} - \frac{1}{6}b^{3/2}x^2(cx-1)^{1/2}(cx+1)^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}$

Rubi [A]

time = 0.78, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5884, 5939, 5915, 5881, 3389, 2211, 2236, 2235, 5887, 5556}

$$\frac{3\sqrt{\pi}b^{3/2}e^{3a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3} - \frac{\sqrt{\pi}b^{3/2}e^{3a/b}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3} + \frac{3\sqrt{\pi}b^{3/2}e^{3a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{\sqrt{\pi}b^{3/2}e^{3a/b}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3} - \frac{b\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{3c^3} + \frac{1}{3}x^3(a+b\cosh^{-1}(cx))^{3/2} - \frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{6c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2(a + b\operatorname{ArcCosh}[c*x])^{3/2}, x]$

[Out] $-\frac{1}{3}(b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\operatorname{ArcCosh}[c*x]})/c^3 - (b^2x^2\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\operatorname{ArcCosh}[c*x]})/(6c) + (x^3(a+b\operatorname{ArcCosh}[c*x])^{3/2})/3 - (3b^{3/2}E^{(3a/b)}\sqrt{\pi}\operatorname{Erf}[\sqrt{a+b\operatorname{ArcCosh}[c*x]}/\sqrt{b}])/(32c^3) - (b^{3/2}E^{(3a/b)}\sqrt{\pi/3}\operatorname{Erf}[(\sqrt{3}\sqrt{a+b\operatorname{ArcCosh}[c*x]})/\sqrt{b}])/(96c^3) + (3b^{3/2}\sqrt{\pi}\operatorname{Erfi}[\sqrt{a+b\operatorname{ArcCosh}[c*x]}/\sqrt{b}])/(32c^3E^{(3a/b)}) + (b^{3/2}\sqrt{\pi/3}\operatorname{Erfi}[(\sqrt{3}\sqrt{a+b\operatorname{ArcCosh}[c*x]})/\sqrt{b}])/(96c^3E^{(3a/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_.)*(x_)))/\sqrt{(c_.)+(d_.)*(x_)}}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d)}), x], x, \sqrt{c+dx}], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[UseGamma]

Rule 2235

$\operatorname{Int}[(F_)^{((a_.)+(b_.)*((c_.)+(d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a\sqrt{\pi}\operatorname{Erfi}[(c+dx)\operatorname{Rt}[b\operatorname{Log}[F], 2]]/(2*d\operatorname{Rt}[b\operatorname{Log}[F], 2])], x] /;$ FreeQ[

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_) + (b_)*(x_)]^(p_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5881

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 5884

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5887

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5915

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)*((d1_) + (e1_)*(x_))^(p_)*((d2_) + (e2_)*(x_))^(p_), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], x] /; FreeQ[{a, b, c, d1, d2, e1, e2, n}, x] && IGtQ[p, 0]

p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d1_) + (e1_.)*(x_.))^(p_.)*((d2_) + (e2_.)*(x_.))^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \cosh^{-1}(cx))^{3/2} dx &= \frac{1}{3} x^3 (a + b \cosh^{-1}(cx))^{3/2} - \frac{1}{2} (bc) \int \frac{x^3 \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= -\frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} + \frac{1}{3} x^3 (a + b \cosh^{-1}(cx))^{3/2} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c} \\
&= -\frac{b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{3c^3} - \frac{bx^2 \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{6c}
\end{aligned}$$

Mathematica [A]

time = 1.62, size = 540, normalized size = 1.85

Warning: Unable to verify antiderivative.

`[In] Integrate[x^2*(a + b*ArcCosh[c*x])^(3/2), x]`

```

[Out] (a*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*
Gamma[3/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2
, (-3*(a + b*ArcCosh[c*x])/b] + 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gam
ma[3/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcC

```

```
osh[c*x])/b))*Gamma[3/2, (3*(a + b*ArcCosh[c*x])/b)]/(72*c^3*E^((3*a)/b)*
Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]) + (Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcCosh[
c*x]*Sqrt[a + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a
)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]*(2*
ArcCosh[c*x]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(288*c^3)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^2(a + b \operatorname{arccosh}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arccosh(c*x))^(3/2),x)

[Out] int(x^2*(a+b*arccosh(c*x))^(3/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")

[Out] integrate((b*arccosh(c*x) + a)^(3/2)*x^2, x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b \operatorname{acosh}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**(3/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \operatorname{acosh}(cx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^(3/2), x)
```

3.146 $\int x (a + b \cosh^{-1}(cx))^{3/2} dx$

Optimal. Leaf size=184

$$\frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c} - \frac{(a+b\cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\cosh^{-1}(cx))^{3/2} - \frac{3b^{3/2}e^{\frac{2a}{b}}}{\dots}$$

[Out] $-1/4*(a+b*\operatorname{arccosh}(c*x))^{3/2}/c^2+1/2*x^2*(a+b*\operatorname{arccosh}(c*x))^{3/2}-3/128*b^{3/2}*\exp(2*a/b)*\operatorname{erf}(2^{1/2}*(a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*2^{1/2}*Pi^{1/2}/c^2+3/128*b^{3/2}*\operatorname{erfi}(2^{1/2}*(a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*2^{1/2}*Pi^{1/2}/c^2/\exp(2*a/b)-3/8*b*x*(c*x-1)^{1/2}*(c*x+1)^{1/2}*(a+b*\operatorname{arccosh}(c*x))^{1/2}/c$

Rubi [A]

time = 0.50, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5884, 5939, 5893, 5887, 5556, 12, 3389, 2211, 2236, 2235}

$$\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2} + \frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2} - \frac{(a+b\cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\cosh^{-1}(cx))^{3/2} - \frac{3bx\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{8c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*(a + b*\operatorname{ArcCosh}[c*x])^{3/2}, x]$

[Out] $(-3*b*x*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/(8*c) - (a + b*\operatorname{ArcCosh}[c*x])^{3/2}/(4*c^2) + (x^2*(a + b*\operatorname{ArcCosh}[c*x])^{3/2})/2 - (3*b^{3/2}*E^{((2*a)/b)}*\operatorname{Sqrt}[Pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(64*c^2) + (3*b^{3/2}*\operatorname{Sqrt}[Pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(64*c^2*E^{((2*a)/b)})$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[Pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_) + (b_)*(x_)]^(p_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]ⁿ*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5884

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((x_))^(m_), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^{n/(m + 1)}), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5887

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((x_))^(m_), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[xⁿ*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5893

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)/(Sqrt[(d1_) + (e1_)*(x_)]*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*((d1_) + (e1_)*(x_))^(p_)*((d2_) + (e2_)*(x_))^(p_), x_Symbol] := Simp[f*(f*x)^{(m -}

```

1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int x(a + b \cosh^{-1}(cx))^{3/2} dx &= \frac{1}{2}x^2(a + b \cosh^{-1}(cx))^{3/2} - \frac{1}{4}(3bc) \int \frac{x^2 \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} + \frac{1}{2}x^2(a + b \cosh^{-1}(cx))^{3/2} + \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} - \frac{(a + b \cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2 \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} - \frac{(a + b \cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2 \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} - \frac{(a + b \cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2 \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} - \frac{(a + b \cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2 \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} - \frac{(a + b \cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2 \\
&= -\frac{3bx \sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{8c} - \frac{(a + b \cosh^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2
\end{aligned}$$

Mathematica [A]

time = 0.76, size = 165, normalized size = 0.90

$$\frac{3b^{3/2}\sqrt{2\pi}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)-3b^{3/2}\sqrt{2\pi}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+8\sqrt{a+b\cosh^{-1}(cx)}\left(4a\cosh\left(2\cosh^{-1}(cx)\right)+4b\cosh^{-1}(cx)\cosh\left(2\cosh^{-1}(cx)\right)-3b\sinh\left(2\cosh^{-1}(cx)\right)\right)}{128c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (3*b^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 3*b^(3/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[a + b*ArcCosh[c*x]]*(4*a*Cosh[2*ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] - 3*b*Sinh[2*ArcCosh[c*x]]))/(128*c^2)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{arccosh}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)*x, x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{acosh}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*acosh(c*x))**(3/2),x)

[Out] Integral(x*(a + b*acosh(c*x))**(3/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)^(3/2)*x, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x (a + b \operatorname{acosh}(cx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*acosh(c*x))^(3/2),x)

[Out] int(x*(a + b*acosh(c*x))^(3/2), x)

3.147 $\int (a + b \cosh^{-1}(cx))^{3/2} dx$

Optimal. Leaf size=140

$$\frac{3b\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x(a+b\cosh^{-1}(cx))^{3/2} - \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}$$

[Out] $x*(a+b*\operatorname{arccosh}(c*x))^{3/2}-3/8*b^{3/2}*exp(a/b)*erf((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/c+3/8*b^{3/2}*erfi((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/c/exp(a/b)-3/2*b*(c*x-1)^{1/2}*(c*x+1)^{1/2}*(a+b*\operatorname{arccosh}(c*x))^{1/2}/c$

Rubi [A]

time = 0.26, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5879, 5915, 5881, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c} + \frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c} - \frac{3b\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}}{2c} + x(a+b\cosh^{-1}(cx))^{3/2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcCosh}[c*x])^{3/2}, x]$

[Out] $(-3*b*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/(2*c) + x*(a + b*\operatorname{ArcCosh}[c*x])^{3/2} - (3*b^{3/2}*E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(8*c) + (3*b^{3/2}*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(8*c*E^{(a/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /;$ Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (a + b \cosh^{-1}(cx))^{3/2} dx &= x(a + b \cosh^{-1}(cx))^{3/2} - \frac{1}{2}(3bc) \int \frac{x \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= -\frac{3b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{2c} + x(a + b \cosh^{-1}(cx))^{3/2} + \frac{1}{4} \dots \\
&= -\frac{3b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{2c} + x(a + b \cosh^{-1}(cx))^{3/2} - \dots \\
&= -\frac{3b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{2c} + x(a + b \cosh^{-1}(cx))^{3/2} - \dots \\
&= -\frac{3b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{2c} + x(a + b \cosh^{-1}(cx))^{3/2} - \dots \\
&= -\frac{3b\sqrt{-1 + cx} \sqrt{1 + cx} \sqrt{a + b \cosh^{-1}(cx)}}{2c} + x(a + b \cosh^{-1}(cx))^{3/2} - \dots
\end{aligned}$$

Mathematica [A]

time = 0.47, size = 269, normalized size = 1.92

$$\frac{ae^{-t} \sqrt{a + b \cosh^{-1}(cx)} \left(\frac{x \operatorname{Erfi} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right) + \frac{\operatorname{Erfi} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{\sqrt{a + b \cosh^{-1}(cx)}}}{\sqrt{a + b \cosh^{-1}(cx)}} \right) + b \left(-12 \sqrt{\frac{-1 + cx}{1 + cx}} (1 + cx) \sqrt{a + b \cosh^{-1}(cx)} + 8cx \cosh^{-1}(cx) \sqrt{a + b \cosh^{-1}(cx)} + \frac{(2a+3b)\sqrt{\pi} \operatorname{Erfi} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right) (\cosh(\frac{a}{b}) - \sinh(\frac{a}{b}))}{\sqrt{b}} + \frac{(2a-3b)\sqrt{\pi} \operatorname{Erfi} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right) (\cosh(\frac{a}{b}) + \sinh(\frac{a}{b}))}{\sqrt{b}} \right)}{2c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcCosh[c*x])^(3/2), x]

[Out] (a*sqrt[a + b*ArcCosh[c*x]]*(E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/sqrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -(a + b*ArcCosh[c*x])/b])/sqrt[-((a + b*ArcCosh[c*x])/b))]/(2*c*E^(a/b)) + (b*(-12*sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*sqrt[a + b*ArcCosh[c*x]] + 8*c*x*ArcCosh[c*x]*sqrt[a + b*ArcCosh[c*x]] + ((2*a + 3*b)*sqrt[Pi]*Erfi[sqrt[a + b*ArcCosh[c*x]]/sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/sqrt[b] + ((2*a - 3*b)*sqrt[Pi]*Erf[sqrt[a + b*ArcCosh[c*x]]/sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/sqrt[b]))/(8*c)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(cx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccosh(c*x))^(3/2),x)`

[Out] `int((a+b*arccosh(c*x))^(3/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*arccosh(c*x) + a)^(3/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acosh}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acosh(c*x))**(3/2),x)`

[Out] `Integral((a + b*acosh(c*x))**(3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccosh(c*x) + a)^(3/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(cx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))^(3/2), x)

[Out] int((a + b*acosh(c*x))^(3/2), x)

3.148 $\int x^2 (a + b \cosh^{-1}(cx))^{5/2} dx$

Optimal. Leaf size=337

$$\frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{a+b\cosh^{-1}(cx)}}{6c^2}$$

[Out] $\frac{1}{3}x^3(a+b\operatorname{arccosh}(cx))^{5/2} - \frac{5}{1728}b^{5/2}\exp(3a/b)\operatorname{erf}(3^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) * 3^{1/2}\operatorname{Pi}^{1/2}/c^3 - \frac{5}{1728}b^{5/2}\operatorname{erfi}(3^{1/2}(a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) * 3^{1/2}\operatorname{Pi}^{1/2}/c^3/\exp(3a/b) - \frac{15}{64}b^{5/2}\exp(a/b)\operatorname{erf}((a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) * \operatorname{Pi}^{1/2}/c^3 - \frac{15}{64}b^{5/2}\operatorname{erfi}((a+b\operatorname{arccosh}(cx))^{1/2}/b^{1/2}) * \operatorname{Pi}^{1/2}/c^3/\exp(a/b) - \frac{5}{9}b^{5/2}(a+b\operatorname{arccosh}(cx))^{3/2}(cx-1)^{1/2}(cx+1)^{1/2}/c^3 - \frac{5}{18}b^2x^2(a+b\operatorname{arccosh}(cx))^{3/2}(cx-1)^{1/2}(cx+1)^{1/2}/c^3 + \frac{5}{6}b^2x(a+b\operatorname{arccosh}(cx))^{3/2}(cx-1)^{1/2}(cx+1)^{1/2}/c^2 + \frac{5}{36}b^2x^3(a+b\operatorname{arccosh}(cx))^{3/2}$

Rubi [A]

time = 1.23, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5884, 5939, 5915, 5879, 5953, 3388, 2211, 2236, 2235, 3393}

$$\frac{15\sqrt{3}b^{5/2}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3} - \frac{5\sqrt{3}b^{5/2}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3} - \frac{15\sqrt{3}b^{5/2}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3} - \frac{5\sqrt{3}b^{5/2}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3} + \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2} + \frac{5b^2x^3\sqrt{a+b\cosh^{-1}(cx)}}{36} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))^{3/2}}{9c^3} + \frac{5b^2x^2\sqrt{a+b\cosh^{-1}(cx)}}{36} - \frac{5b^2x\sqrt{a+b\cosh^{-1}(cx)}}{6c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2(a + b\operatorname{ArcCosh}[c*x])^{5/2}, x]$

[Out] $\frac{5b^2x\sqrt{a+b\operatorname{ArcCosh}[c*x]}}{(6c^2)} + \frac{5b^2x^3\sqrt{a+b\operatorname{ArcCosh}[c*x]}}{36} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{ArcCosh}[c*x])^{3/2}}{(9c^3)} - \frac{5b^2x^2\sqrt{-1+cx}\sqrt{1+cx}(a+b\operatorname{ArcCosh}[c*x])^{3/2}}{(18c)} + \frac{x^3(a+b\operatorname{ArcCosh}[c*x])^{5/2}}{3} - \frac{15b^{5/2}E^{(a/b)}\sqrt{\operatorname{Pi}}\operatorname{Erf}\left[\frac{\sqrt{a+b\operatorname{ArcCosh}[c*x]}}{\sqrt{b}}\right]}{(64c^3)} - \frac{5b^{5/2}E^{((3a)/b)}\sqrt{\operatorname{Pi}/3}\operatorname{Erf}\left[\frac{\sqrt{3}\sqrt{a+b\operatorname{ArcCosh}[c*x]}}{\sqrt{b}}\right]}{(576c^3)} - \frac{15b^{5/2}\sqrt{\operatorname{Pi}}\operatorname{Erfi}\left[\frac{\sqrt{a+b\operatorname{ArcCosh}[c*x]}}{\sqrt{b}}\right]}{(64c^3E^{(a/b)})} - \frac{5b^{5/2}\sqrt{\operatorname{Pi}/3}\operatorname{Erfi}\left[\frac{\sqrt{3}\sqrt{a+b\operatorname{ArcCosh}[c*x]}}{\sqrt{b}}\right]}{(576c^3E^{((3a)/b)})}$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_)))} / \sqrt{(c_.) + (d_.) * (x_)}], x_Symbol] : > \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \sqrt{c + dx}], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\amp; \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 3393

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]ⁿ, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])ⁿ, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*x^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])ⁿ/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*x*((d1_.) + (e1_.)*(x_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])ⁿ/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5939

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d1_.) + (e
1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]

```

Rule 5953

```

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x
_))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \cosh^{-1}(cx))^{5/2} dx &= \frac{1}{3} x^3 (a + b \cosh^{-1}(cx))^{5/2} - \frac{1}{6} (5bc) \int \frac{x^3 (a + b \cosh^{-1}(cx))^{3/2}}{\sqrt{-1+cx} \sqrt{1+cx}} dx \\
&= -\frac{5bx^2 \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{18c} + \frac{1}{3} x^3 (a + b \cosh^{-1}(cx))^{5/2} \\
&= \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3} \\
&= \frac{5b^2 x \sqrt{a + b \cosh^{-1}(cx)}}{6c^2} + \frac{5}{36} b^2 x^3 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b \sqrt{-1+cx} \sqrt{1+cx} (a + b \cosh^{-1}(cx))^{3/2}}{9c^3}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 924 vs. 2(337) = 674.

time = 9.23, size = 924, normalized size = 2.74

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*(a + b*ArcCosh[c*x])^(5/2), x]

[Out] (a^2*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3

/2, (-3*(a + b*ArcCosh[c*x]))/b] + 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b]]/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]) + (a*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]*(2*ArcCosh[c*x]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(144*c^3) - (27*(-4*b*Sqrt[a + b*ArcCosh[c*x]]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a - 5*b*ArcCosh[c*x]) + b*c*x*(15 + 4*ArcCosh[c*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(12*a^2 + 12*a*b + 5*b^2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) - 12*b*Sqrt[a + b*ArcCosh[c*x]]*(b*(5 + 12*ArcCosh[c*x]^2)*Cosh[3*ArcCosh[c*x]] + 2*(a - 5*b*ArcCosh[c*x])*Sinh[3*ArcCosh[c*x]])))/(1728*c^3)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x^2(a + b \operatorname{arccosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arccosh(c*x))^(5/2),x)

[Out] int(x^2*(a+b*arccosh(c*x))^(5/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")

[Out] integrate((b*arccosh(c*x) + a)^(5/2)*x^2, x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^2 (a + b \operatorname{acosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**(5/2), x)
```

Giac [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int x^2 (a + b \operatorname{acosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^(5/2), x)
```

3.149 $\int x (a + b \cosh^{-1}(cx))^{5/2} dx$

Optimal. Leaf size=228

$$-\frac{15b^2 \sqrt{a + b \cosh^{-1}(cx)}}{64c^2} + \frac{15}{32} b^2 x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx \sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} - (a + b \cosh^{-1}(cx))^{5/2}$$

[Out] $-1/4*(a+b*\operatorname{arccosh}(c*x))^{(5/2)}/c^2+1/2*x^2*(a+b*\operatorname{arccosh}(c*x))^{(5/2)}-15/512*b^{(5/2)}*\exp(2*a/b)*\operatorname{erf}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^2-15/512*b^{(5/2)}*\operatorname{erfi}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^2/\exp(2*a/b)-5/8*b*x*(a+b*\operatorname{arccosh}(c*x))^{(3/2)}*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/c-15/64*b^2*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/c^2+15/32*b^2*x^2*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.82, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$,

Rules used = {5884, 5939, 5893, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{2a/b}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2} - \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-2a/b}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2} - \frac{15b^2\sqrt{a+b\cosh^{-1}(cx)}}{64c^2} + \frac{15}{32}b^2x^2\sqrt{a+b\cosh^{-1}(cx)} - \frac{(a+b\cosh^{-1}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\cosh^{-1}(cx))^{5/2} - \frac{5bx\sqrt{cx-1}\sqrt{cx+1}(a+b\cosh^{-1}(cx))^{3/2}}{8c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*(a + b*\operatorname{ArcCosh}[c*x])^{(5/2)}, x]$

[Out] $(-15*b^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/(64*c^2) + (15*b^2*x^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/32 - (5*b*x*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x]*(a + b*\operatorname{ArcCosh}[c*x])^{(3/2)})/(8*c) - (a + b*\operatorname{ArcCosh}[c*x])^{(5/2)}/(4*c^2) + (x^2*(a + b*\operatorname{ArcCosh}[c*x])^{(5/2)})/2 - (15*b^{(5/2)}*E^{((2*a)/b)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(256*c^2) - (15*b^{(5/2)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(256*c^2*E^{((2*a)/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_)^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 3393

Int[((c_.) + (d_.)*(x_)^(m_.)*sin[(e_.) + (f_.)*(x_)^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 5884

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 5893

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5939

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x_)^(p_.)*((d2_) + (e2_.)*(x_)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x_)^(p_.))*((d2_.) + (e2_.)*(x_)^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a + b \cosh^{-1}(cx))^{5/2} dx &= \frac{1}{2}x^2(a + b \cosh^{-1}(cx))^{5/2} - \frac{1}{4}(5bc) \int \frac{x^2(a + b \cosh^{-1}(cx))^{3/2}}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= -\frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} + \frac{1}{2}x^2(a + b \cosh^{-1}(cx))^{5/2} \\
&= \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} \\
&= \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} \\
&= \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} \\
&= -\frac{15b^2 \sqrt{a + b \cosh^{-1}(cx)}}{64c^2} + \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} \\
&= -\frac{15b^2 \sqrt{a + b \cosh^{-1}(cx)}}{64c^2} + \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} \\
&= -\frac{15b^2 \sqrt{a + b \cosh^{-1}(cx)}}{64c^2} + \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c} \\
&= -\frac{15b^2 \sqrt{a + b \cosh^{-1}(cx)}}{64c^2} + \frac{15}{32}b^2x^2 \sqrt{a + b \cosh^{-1}(cx)} - \frac{5bx\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{8c}
\end{aligned}$$

Mathematica [A]

time = 1.34, size = 207, normalized size = 0.91

$$\frac{-15b^{5/2}\sqrt{2\pi}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)-15b^{5/2}\sqrt{2\pi}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+8\sqrt{a+b\cosh^{-1}(cx)}\left((16a^2+15b^2)\cosh(2\operatorname{ArcCosh}(cx))+16b^2\cosh^{-1}(cx)^2\cosh(2\operatorname{ArcCosh}(cx))-20ab\sinh(2\operatorname{ArcCosh}(cx))+4b\cosh^{-1}(cx)(8a\cosh(2\operatorname{ArcCosh}(cx))-5b\sinh(2\operatorname{ArcCosh}(cx)))\right)}{512c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*ArcCosh[c*x])^(5/2),x]

[Out] $(-15b^{5/2}\sqrt{2\pi}\operatorname{Erfi}[(\sqrt{2}\sqrt{a+b\operatorname{ArcCosh}[c*x]})/\sqrt{b}])*(\operatorname{Cosh}[(2*a)/b]-\operatorname{Sinh}[(2*a)/b])-15b^{5/2}\sqrt{2\pi}\operatorname{Erf}[(\sqrt{2}\sqrt{a+b\operatorname{ArcCosh}[c*x]})/\sqrt{b}]*(\operatorname{Cosh}[(2*a)/b]+\operatorname{Sinh}[(2*a)/b])+8\sqrt{a+b\operatorname{ArcCosh}[c*x]}*((16*a^2+15*b^2)*\operatorname{Cosh}[2*\operatorname{ArcCosh}[c*x]]+16*b^2*\operatorname{ArcCosh}[c*x]^2*\operatorname{Cosh}[2*\operatorname{ArcCosh}[c*x]]-20*a*b*\operatorname{Sinh}[2*\operatorname{ArcCosh}[c*x]]+4*b*\operatorname{ArcCosh}[c*x]*(8*a*\operatorname{Cosh}[2*\operatorname{ArcCosh}[c*x]]-5*b*\operatorname{Sinh}[2*\operatorname{ArcCosh}[c*x]])))/(512*c^2)$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{arccosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arccosh(c*x))^(5/2),x)

[Out] int(x*(a+b*arccosh(c*x))^(5/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")

[Out] integrate((b*arccosh(c*x) + a)^(5/2)*x, x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{acosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*acosh(c*x))**(5/2),x)

[Out] Integral(x*(a + b*acosh(c*x))**(5/2), x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x(a + b \operatorname{acosh}(cx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*acosh(c*x))^(5/2),x)

[Out] int(x*(a + b*acosh(c*x))^(5/2), x)

3.150 $\int (a + b \cosh^{-1}(cx))^{5/2} dx$

Optimal. Leaf size=160

$$\frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))^{3/2}}{2c} + x(a+b\cosh^{-1}(cx))^{5/2} - \frac{15b^{5/2}e^a}{\dots}$$

[Out] $x*(a+b*\operatorname{arccosh}(c*x))^{5/2} - 15/16*b^{5/2}*exp(a/b)*erf((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/c - 15/16*b^{5/2}*erfi((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/c/exp(a/b) - 5/2*b*(a+b*\operatorname{arccosh}(c*x))^{3/2}*(c*x-1)^{1/2}*(c*x+1)^{1/2}/c + 15/4*b^2*x*(a+b*\operatorname{arccosh}(c*x))^{1/2}$

Rubi [A]

time = 0.47, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5879, 5915, 5953, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c} - \frac{15\sqrt{\pi}b^{5/2}e^{-a/b}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c} + \frac{15}{4}b^2x\sqrt{a+b\cosh^{-1}(cx)} - \frac{5b\sqrt{cx-1}\sqrt{cx+1}(a+b\cosh^{-1}(cx))^{3/2}}{2c} + x(a+b\cosh^{-1}(cx))^{5/2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcCosh}[c*x])^{5/2}, x]$

[Out] $(15*b^2*x*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/4 - (5*b*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x]*(a + b*\operatorname{ArcCosh}[c*x])^{3/2})/(2*c) + x*(a + b*\operatorname{ArcCosh}[c*x])^{5/2} - (15*b^{5/2}*E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(16*c) - (15*b^{5/2}*E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(16*c*E^{(a/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

Rule 2236

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /; \operatorname{Fr}$

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5879

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 5915

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Simp[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]

Rule 5953

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.)*((d1_.) + (e1_.)*(x_.))^(p_.)*((d2_.) + (e2_.)*(x_.))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \cosh^{-1}(cx))^{5/2} dx &= x(a + b \cosh^{-1}(cx))^{5/2} - \frac{1}{2}(5bc) \int \frac{x(a + b \cosh^{-1}(cx))^{3/2}}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
&= -\frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c} + x(a + b \cosh^{-1}(cx))^{5/2} + \frac{1}{4} \\
&= \frac{15}{4}b^2x \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c} + \\
&= \frac{15}{4}b^2x \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c} + \\
&= \frac{15}{4}b^2x \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c} + \\
&= \frac{15}{4}b^2x \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c} + \\
&= \frac{15}{4}b^2x \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c} +
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 452 vs. 2(160) = 320.

time = 1.64, size = 452, normalized size = 2.82

$$\frac{b\sqrt{a + b \cosh^{-1}(cx)} \left(\frac{5cx}{4} (1 + cx)(a - b \cosh^{-1}(cx)) + b\sqrt{a + b \cosh^{-1}(cx)} \right) + b^2x \sqrt{a + b \cosh^{-1}(cx)} - \frac{5b\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{3/2}}{2c}}{\sqrt{-1 + cx} \sqrt{1 + cx}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcCosh[c*x])^(5/2), x]

[Out] (4*b*Sqrt[a + b*ArcCosh[c*x]]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a - 5*b*ArcCosh[c*x]) + b*c*x*(15 + 4*ArcCosh[c*x]^2)) + (8*a^2*Sqrt[a + b*ArcCosh[c*x]]*(E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]]/Sqrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -(a + b*ArcCosh[c*x])/b])/Sqrt[-((a + b*ArcCosh[c*x])/b)))/E^(a/b) - Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 4*a*b*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b])))/Sqrt[b] + ((2

$$\frac{(a - 3b) \sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{a + b \operatorname{ArcCosh}[c x]}}{\sqrt{b}}\right) (\cosh[a/b] + \sinh[a/b])}{\sqrt{b}}}{16c}$$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccosh(c*x))^(5/2),x)`

[Out] `int((a+b*arccosh(c*x))^(5/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*arccosh(c*x) + a)^(5/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acosh}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acosh(c*x))**(5/2),x)`

[Out] `Integral((a + b*acosh(c*x))**(5/2), x)`

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^(5/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
 eur & l) Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(cx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))^(5/2),x)

[Out] int((a + b*acosh(c*x))^(5/2), x)

$$3.151 \quad \int \frac{x^2}{\sqrt{a + b \cosh^{-1}(cx)}} dx$$

Optimal. Leaf size=194

$$\frac{e^{a/b} \sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3} - \frac{e^{\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3} + \frac{e^{-\frac{a}{b}} \sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3}$$

[Out] $-1/24*\exp(3*a/b)*\operatorname{erf}(3^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^3/b^{(1/2)}+1/24*\operatorname{erfi}(3^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^3/\exp(3*a/b)/b^{(1/2)}-1/8*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*\operatorname{Pi}^{(1/2)}/c^3/b^{(1/2)}+1/8*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*\operatorname{Pi}^{(1/2)}/c^3/\exp(a/b)/b^{(1/2)}$

Rubi [A]

time = 0.20, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3} - \frac{\sqrt{\frac{\pi}{3}} e^{\frac{3a}{b}} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3} + \frac{\sqrt{\pi} e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3} + \frac{\sqrt{\frac{\pi}{3}} e^{-\frac{3a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]], x]$

[Out] $-1/8*(E^{(a/b)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(\operatorname{Sqrt}[b]*c^3) - (E^{((3*a)/b)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(8*\operatorname{Sqrt}[b]*c^3) + (\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(8*\operatorname{Sqrt}[b]*c^3*E^{(a/b)}) + (\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(8*\operatorname{Sqrt}[b]*c^3*E^{((3*a)/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \&\amp; \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, x\} \&\amp; \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_)^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_)^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a + b \cosh^{-1}(cx)}} dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x) \sinh(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a + bx}} + \frac{\sinh(3x)}{4\sqrt{a + bx}}\right) dx, x, \cosh^{-1}(cx)\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{4c^3} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{4c^3} \\
&= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{8c^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{8c^3} \\
&= \frac{\text{Subst}\left(\int e^{\frac{3a}{b} - \frac{3x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{4bc^3} - \frac{\text{Subst}\left(\int e^{\frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{4bc^3} \\
&= \frac{e^{a/b} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3} - \frac{e^{\frac{3a}{b}} \sqrt{\frac{\pi}{3}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b} c^3}
\end{aligned}$$

Mathematica [A]

time = 0.26, size = 195, normalized size = 1.01

$$\frac{e^{-\frac{3a}{b}} \left(3e^{\frac{3a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right) + \sqrt{3} \sqrt{\frac{a + b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{1}{2}, -\frac{3(a + b \cosh^{-1}(cx))}{b}\right) + 3e^{\frac{3a}{b}} \sqrt{\frac{-a + b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{1}{2}, -\frac{a + b \cosh^{-1}(cx)}{b}\right) + \sqrt{3} e^{\frac{3a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{1}{2}, \frac{3(a + b \cosh^{-1}(cx))}{b}\right) \right)}{24c^3 \sqrt{a + b \cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/Sqrt[a + b*ArcCosh[c*x]], x]`

```
[Out] (3*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 3*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x])/b)]/(24*c^3*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + b \operatorname{arccosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arccosh(c*x))^(1/2),x)
```

Maxima [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arccosh(c*x) + a), x)
```

Fricas [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^2}{\sqrt{a + b \operatorname{acosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*acosh(c*x)), x)
```

Giac [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*arccosh(c*x) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a + b \operatorname{acosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*acosh(c*x))^(1/2),x)

[Out] int(x^2/(a + b*acosh(c*x))^(1/2), x)

$$3.152 \quad \int \frac{x}{\sqrt{a + b \cosh^{-1}(cx)}} dx$$

Optimal. Leaf size=107

$$\frac{e^{\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{Erf} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{4\sqrt{b} c^2} + \frac{e^{-\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{Erfi} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{4\sqrt{b} c^2}$$

[Out] $-1/8*\exp(2*a/b)*\operatorname{erf}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^2/b^{(1/2)}+1/8*\operatorname{erfi}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/c^2/\exp(2*a/b)/b^{(1/2)}$

Rubi [A]

time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5887, 5556, 12, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\frac{\pi}{2}} e^{-\frac{2a}{b}} \operatorname{Erfi} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{4\sqrt{b} c^2} - \frac{\sqrt{\frac{\pi}{2}} e^{\frac{2a}{b}} \operatorname{Erf} \left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{4\sqrt{b} c^2}$$

Antiderivative was successfully verified.

[In] `Int[x/Sqrt[a + b*ArcCosh[c*x]],x]`

[Out] $-1/4*(E^{((2*a)/b)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(\operatorname{Sqrt}[b]*c^2) + (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(4*\operatorname{Sqrt}[b]*c^2*E^{((2*a)/b)})$

Rule 12

`Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 2211

`Int[(F_)^((g_)*((e_) + (f_)*(x_)))/Sqrt[(c_) + (d_)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{`

$F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

Rule 2236

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x_Symbol] \rightarrow \text{Simp}[F^a*\text{Sqrt}[\text{Pi}]*(\text{Erf}[(c + d*x)*\text{Rt}[(-b)*\text{Log}[F], 2]]/(2*d*\text{Rt}[(-b)*\text{Log}[F], 2])), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{NegQ}[b]$

Rule 3389

$\text{Int}(((c_.) + (d_.)*(x_))^{(m_.)*\sin[(e_.) + (f_.)*(x_)]}, x_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/E^{I*(e + f*x)}, x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{I*(e + f*x)}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x]$

Rule 5556

$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_)]^{(p_.)*((c_.) + (d_.)*(x_))^{(m_.)*\text{Sinh}[(a_.) + (b_.)*(x_)]^{(n_.)}}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sinh}[a + b*x]^{n*}\text{Cosh}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \& \& \text{IGtQ}[p, 0]$

Rule 5887

$\text{Int}(((a_.) + \text{ArcCosh}[(c_.)*(x_)]*(b_.))^{(n_.)*(x_)^{(m_.)}}, x_Symbol] \rightarrow \text{Dist}[1/(b*c^{(m + 1)}), \text{Subst}[\text{Int}[x^n*\text{Cosh}[-a/b + x/b]^m*\text{Sinh}[-a/b + x/b], x], x, a + b*\text{ArcCosh}[c*x]], x] /; \text{FreeQ}\{a, b, c, n\}, x] \&\& \text{IGtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a + b \cosh^{-1}(cx)}} dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x) \sinh(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{2c^2} \\
&= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{4c^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx)\right)}{4c^2} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b} - \frac{2x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{2bc^2} + \frac{\text{Subst}\left(\int e^{-\frac{2a}{b} + \frac{2x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{2bc^2} \\
&= -\frac{e^{\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b} c^2} + \frac{e^{-\frac{2a}{b}} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b} c^2}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 104, normalized size = 0.97

$$\frac{\sqrt{\frac{\pi}{2}} \left(\operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right) (-\cosh(\frac{2a}{b}) + \sinh(\frac{2a}{b})) + \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right) (\cosh(\frac{2a}{b}) + \sinh(\frac{2a}{b})) \right)}{4\sqrt{b} c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a + b*ArcCosh[c*x]], x]

[Out] $-1/4*(\text{Sqrt}[\text{Pi}/2]*(\text{Erfi}[(\text{Sqrt}[2]*\text{Sqrt}[a + b*\text{ArcCosh}[c*x]])/\text{Sqrt}[b]])/\text{Sqrt}[b])*(-\text{Cosh}[(2*a)/b] + \text{Sinh}[(2*a)/b]) + \text{Erf}[(\text{Sqrt}[2]*\text{Sqrt}[a + b*\text{ArcCosh}[c*x]])/\text{Sqrt}[b]]*(\text{Cosh}[(2*a)/b] + \text{Sinh}[(2*a)/b])))/(\text{Sqrt}[b]*c^2)$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \operatorname{arccosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*arccosh(c*x))^(1/2),x)`

[Out] `int(x/(a+b*arccosh(c*x))^(1/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(x/sqrt(b*arccosh(c*x) + a), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \operatorname{acosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*acosh(c*x))**(1/2),x)`

[Out] `Integral(x/sqrt(a + b*acosh(c*x)), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")`

[Out] `integrate(x/sqrt(b*arccosh(c*x) + a), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + b \operatorname{acosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*acosh(c*x))^(1/2), x)

[Out] int(x/(a + b*acosh(c*x))^(1/2), x)

$$3.153 \quad \int \frac{1}{\sqrt{a + b \cosh^{-1}(cx)}} dx$$

Optimal. Leaf size=88

$$\frac{e^{a/b} \sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c} + \frac{e^{-\frac{a}{b}} \sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c}$$

[Out] $-1/2*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/c/b^{1/2}+1/2*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/c/\exp(a/b)/b^{1/2}$

Rubi [A]

time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5881, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi} e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c} - \frac{\sqrt{\pi} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[a + b*ArcCosh[c*x]],x]`

[Out] $-1/2*(E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(\operatorname{Sqrt}[b]*c) + (\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(2*\operatorname{Sqrt}[b]*c*E^{(a/b)})$

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`
`> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /;` `FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=` `Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /;` `FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=` `Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /;` `FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3389

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5881

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + b \cosh^{-1}(cx)}} dx &= \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b} - \frac{x}{b}\right)}{\sqrt{x}} dx, x, a + b \cosh^{-1}(cx)\right)}{bc} \\ &= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b} - \frac{ix}{b}\right)}}{\sqrt{x}} dx, x, a + b \cosh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b} - \frac{ix}{b}\right)}}{\sqrt{x}} dx, x, a + b \cosh^{-1}(cx)\right)}{2bc} \\ &= -\frac{\text{Subst}\left(\int e^{\frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)}\right)}{bc} \\ &= -\frac{e^{a/b} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c} + \frac{e^{-a/b} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 100, normalized size = 1.14

$$\frac{e^{-\frac{a}{b}} \left(e^{\frac{2a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right) + \sqrt{-\frac{a + b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{1}{2}, -\frac{a + b \cosh^{-1}(cx)}{b}\right) \right)}{2c \sqrt{a + b \cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*ArcCosh[c*x]], x]

[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(2*c*E^(a/b)*Sqrt[a + b*ArcCosh[c*x]])

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{arccosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+b*arccosh(c*x))^(1/2),x)``[Out] int(1/(a+b*arccosh(c*x))^(1/2),x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")``[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)`**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")``[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*acosh(c*x))**(1/2),x)``[Out] Integral(1/sqrt(a + b*acosh(c*x)), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*acosh(c*x))^(1/2),x)

[Out] int(1/(a + b*acosh(c*x))^(1/2), x)

$$3.154 \quad \int \frac{x^2}{(a+b \cosh^{-1}(cx))^{3/2}} dx$$

Optimal. Leaf size=231

$$\frac{2x^2 \sqrt{-1+cx} \sqrt{1+cx}}{bc \sqrt{a+b \cosh^{-1}(cx)}} + \frac{e^{a/b} \sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{e^{\frac{3a}{b}} \sqrt{3\pi} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \dots$$

[Out] $1/4 \cdot \exp(a/b) \cdot \operatorname{erf}((a+b \cdot \operatorname{arccosh}(c \cdot x))^{1/2}/b^{1/2}) \cdot \pi^{1/2}/b^{3/2}/c^3 + 1/4 \cdot \operatorname{erfi}((a+b \cdot \operatorname{arccosh}(c \cdot x))^{1/2}/b^{1/2}) \cdot \pi^{1/2}/b^{3/2}/c^3 \cdot \exp(a/b) + 1/4 \cdot \exp(3a/b) \cdot \operatorname{erf}(3^{1/2} \cdot (a+b \cdot \operatorname{arccosh}(c \cdot x))^{1/2}/b^{1/2}) \cdot 3^{1/2} \cdot \pi^{1/2}/b^{3/2}/c^3 + 1/4 \cdot \operatorname{erfi}(3^{1/2} \cdot (a+b \cdot \operatorname{arccosh}(c \cdot x))^{1/2}/b^{1/2}) \cdot 3^{1/2} \cdot \pi^{1/2}/b^{3/2}/c^3 \cdot \exp(3a/b) - 2x^2 \cdot (c \cdot x - 1)^{1/2} \cdot (c \cdot x + 1)^{1/2}/b/c/(a+b \cdot \operatorname{arccosh}(c \cdot x))^{1/2}$

Rubi [A]

time = 0.21, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{\sqrt{3\pi} e^{\frac{3a}{b}} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{\sqrt{\pi} e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{\sqrt{3\pi} e^{-\frac{3a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{2x^2 \sqrt{cx-1} \sqrt{cx+1}}{bc \sqrt{a+b \cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/(a + b \cdot \operatorname{ArcCosh}[c \cdot x])^{3/2}, x]$

[Out] $(-2x^2 \sqrt{-1+cx} \sqrt{1+cx})/(bc \sqrt{a+b \cdot \operatorname{ArcCosh}[c \cdot x]}) + (E^{(a/b)} \sqrt{\pi} \operatorname{Erf}[\sqrt{a+b \cdot \operatorname{ArcCosh}[c \cdot x]}/\sqrt{b}])/(4b^{3/2}c^3) + (E^{(3a/b)} \sqrt{3\pi} \operatorname{Erf}[(\sqrt{3} \sqrt{a+b \cdot \operatorname{ArcCosh}[c \cdot x]})/\sqrt{b}])/(4b^{3/2}c^3) + (\sqrt{\pi} \operatorname{Erfi}[\sqrt{a+b \cdot \operatorname{ArcCosh}[c \cdot x]}/\sqrt{b}])/(4b^{3/2}c^3) + (\sqrt{3\pi} \operatorname{Erfi}[(\sqrt{3} \sqrt{a+b \cdot \operatorname{ArcCosh}[c \cdot x]})/\sqrt{b}])/(4b^{3/2}c^3) \cdot E^{(a/b)} + (\sqrt{3\pi} \operatorname{Erfi}[(\sqrt{3} \sqrt{a+b \cdot \operatorname{ArcCosh}[c \cdot x]})/\sqrt{b}])/(4b^{3/2}c^3) \cdot E^{(3a/b)}$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.) \cdot ((e_.) + (f_.) \cdot (x_)))}/\sqrt{(c_.) + (d_.) \cdot (x_)}], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g \cdot (e - c \cdot (f/d)) + f \cdot g \cdot (x^2/d))}, x], x, \sqrt{c + d \cdot x}], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g, x\} \&\amp; \operatorname{!TrueQ}\{\$UseGamma\}$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.) \cdot ((c_.) + (d_.) \cdot (x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^a \sqrt{\pi} \operatorname{Erfi}[(c + d \cdot x) \cdot \operatorname{Rt}[b \cdot \operatorname{Log}[F], 2]]/(2 \cdot d \cdot \operatorname{Rt}[b \cdot \operatorname{Log}[F], 2])], x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^(n_)*(x_)^(m_.), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + b \cosh^{-1}(cx))^{3/2}} dx &= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{bc \sqrt{a + b \cosh^{-1}(cx)}} - \frac{2 \operatorname{Subst} \left(\int \left(-\frac{\cosh(x)}{4\sqrt{a + bx}} - \frac{3 \cosh(3x)}{4\sqrt{a + bx}} \right) dx, x, c \right)}{bc^3} \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{bc \sqrt{a + b \cosh^{-1}(cx)}} + \frac{\operatorname{Subst} \left(\int \frac{\cosh(x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{2bc^3} + \frac{3 \operatorname{Subst} \left(\int \frac{\cosh(3x)}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{2bc^3} \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{bc \sqrt{a + b \cosh^{-1}(cx)}} + \frac{\operatorname{Subst} \left(\int \frac{e^{-x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{4bc^3} + \frac{\operatorname{Subst} \left(\int \frac{e^{3x}}{\sqrt{a + bx}} dx, x, \cosh^{-1}(cx) \right)}{4bc^3} \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{bc \sqrt{a + b \cosh^{-1}(cx)}} + \frac{\operatorname{Subst} \left(\int e^{\frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)} \right)}{2b^2 c^3} + \frac{\operatorname{Subst} \left(\int e^{\frac{a}{b} - \frac{9x^2}{b}} dx, x, \sqrt{a + b \cosh^{-1}(cx)} \right)}{2b^2 c^3} \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{bc \sqrt{a + b \cosh^{-1}(cx)}} + \frac{e^{a/b} \sqrt{\pi} \operatorname{erf} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{4b^{3/2} c^3} + \frac{e^{\frac{3a}{b}} \sqrt{3\pi} \operatorname{erf} \left(\frac{\sqrt{3(a + b \cosh^{-1}(cx))}}{\sqrt{b}} \right)}{4b^{3/2} c^3}
\end{aligned}$$

Mathematica [A]

time = 0.48, size = 247, normalized size = 1.07

$$\frac{e^{-\frac{3a}{b}} \left(-2e^{\frac{a}{b}} \sqrt{\frac{-1+cx}{1+cx}} (1+cx) - e^{\frac{a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right) + \sqrt{3} \sqrt{\frac{a+b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{1}{2}, -\frac{3(a+b \cosh^{-1}(cx))}{b}\right) + e^{\frac{a}{b}} \sqrt{\frac{a+b \cosh^{-1}(cx)}{b}} \Gamma\left(\frac{1}{2}, -\frac{a+b \cosh^{-1}(cx)}{b}\right) - \sqrt{3} e^{\frac{a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{1}{2}, \frac{3(a+b \cosh^{-1}(cx))}{b}\right) - 2e^{\frac{3a}{b}} \sinh(3 \cosh^{-1}(cx)) \right)}{4bc^2 \sqrt{a + b \cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(a + b*ArcCosh[c*x])^(3/2), x]

[Out] $(-2 * E^{((3 * a) / b)} * \operatorname{Sqrt}[-(1 + c * x) / (1 + c * x)] * (1 + c * x) - E^{((4 * a) / b)} * \operatorname{Sqrt}[a / b + \operatorname{ArcCosh}[c * x]] * \operatorname{Gamma}[1 / 2, a / b + \operatorname{ArcCosh}[c * x]] + \operatorname{Sqrt}[3] * \operatorname{Sqrt}[-((a + b * \operatorname{ArcCosh}[c * x]) / b)] * \operatorname{Gamma}[1 / 2, (-3 * (a + b * \operatorname{ArcCosh}[c * x])) / b] + E^{((2 * a) / b)} * \operatorname{Sqrt}[-((a + b * \operatorname{ArcCosh}[c * x]) / b)] * \operatorname{Gamma}[1 / 2, -((a + b * \operatorname{ArcCosh}[c * x]) / b)] - \operatorname{Sqrt}[3] * E^{((6 * a) / b)} * \operatorname{Sqrt}[a / b + \operatorname{ArcCosh}[c * x]] * \operatorname{Gamma}[1 / 2, (3 * (a + b * \operatorname{ArcCosh}[c * x])) / b] - 2 * E^{((3 * a) / b)} * \operatorname{Sinh}[3 * \operatorname{ArcCosh}[c * x]]) / (4 * b * c^{3/2} * E^{((3 * a) / b)} * \operatorname{Sqrt}[a + b * \operatorname{ArcCosh}[c * x]])$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{arccosh}(cx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a+b*arccosh(c*x))^(3/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*acosh(c*x))**(3/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*acosh(c*x))^(3/2),x)

[Out] int(x^2/(a + b*acosh(c*x))^(3/2), x)

$$3.155 \quad \int \frac{x}{(a+b \cosh^{-1}(cx))^{3/2}} dx$$

Optimal. Leaf size=140

$$\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}$$

[Out] $1/2*\exp(2*a/b)*\operatorname{erf}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/b^{(3/2)}/c^2+1/2*\operatorname{erfi}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/b^{(3/2)}/c^2/\exp(2*a/b)-2*x*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b/c/(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} - \frac{2x\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x/(a + b*\operatorname{ArcCosh}[c*x])^{(3/2)}, x]$

[Out] $(-2*x*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(b*c*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]) + (E^{((2*a)/b)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(b^{(3/2)}*c^2) + (\operatorname{Sqrt}[\pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(b^{(3/2)}*c^2)*E^{((2*a)/b)}$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\amp; \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \&\amp; \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)*(x_)(m_.), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b2*c(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x(n +
1), Cosh[-a/b + x/b](m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]2), x], x], x, a
+ b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(a + b \cosh^{-1}(cx))^{3/2}} dx &= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}} dx, x, \cosh^{-1}(cx)\right)}{bc^2} \\ &= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}} dx, x, \cosh^{-1}(cx)\right)}{bc^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}} dx, x, \cosh^{-1}(cx)\right)}{bc^2} \\ &= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}} dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c^2} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} \end{aligned}$$

Mathematica [A]

time = 0.87, size = 135, normalized size = 0.96

$$\frac{\sqrt{2\pi} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right) \left(\cosh\left(\frac{2a}{b}\right) - \sinh\left(\frac{2a}{b}\right)\right) + \sqrt{2\pi} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right) \left(\cosh\left(\frac{2a}{b}\right) + \sinh\left(\frac{2a}{b}\right)\right) - \frac{2\sqrt{b} \sinh(2\cosh^{-1}(cx))}{\sqrt{a+b\cosh^{-1}(cx)}}}{2b^{3/2}c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*ArcCosh[c*x])^(3/2), x]

[Out] (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*Sinh[2*ArcCosh[c*x]])/Sqrt[a + b*ArcCosh[c*x]])/(2*b^(3/2)*c^2)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arccosh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*arccosh(c*x))^(3/2), x)

[Out] int(x/(a+b*arccosh(c*x))^(3/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arccosh(c*x))^(3/2), x, algorithm="maxima")

[Out] integrate(x/(b*arccosh(c*x) + a)^(3/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arccosh(c*x))^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(a+b*acosh(c*x))**(3/2),x)``[Out] Integral(x/(a + b*acosh(c*x))**(3/2), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")``[Out] integrate(x/(b*arccosh(c*x) + a)^(3/2), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \operatorname{acosh}(cx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(a + b*acosh(c*x))^(3/2),x)``[Out] int(x/(a + b*acosh(c*x))^(3/2), x)`

$$3.156 \quad \int \frac{1}{(a+b \cosh^{-1}(cx))^{3/2}} dx$$

Optimal. Leaf size=120

$$\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

[Out] exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))^(1/2)

Rubi [A]

time = 0.26, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5880, 5953, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{\sqrt{\pi} e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])^(-3/2), x]

[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))

Rule 2211

Int[(F_)^((g_)*((e_) + (f_)*(x_)))/Sqrt[(c_) + (d_)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_), x_Symbol] := Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d1_.) + (e1_.)*(x
_))^(p_.)*((d2_.) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cosh^{-1}(cx))^{3/2}} dx &= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2c) \int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}(cx)}} dx}{b} \\
&= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}} dx, x, \cosh^{-1}(cx)\right)}{bc} \\
&= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}} dx, x, \cosh^{-1}(cx)\right)}{bc} + \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}} dx, x, \cosh^{-1}(cx)\right)}{bc} \\
&= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c} + \frac{2\text{Subst}\left(\int e^{\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c} \\
&= -\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 132, normalized size = 1.10

$$\frac{e^{-\frac{a}{b}} \left(-2e^{a/b} \sqrt{\frac{-1+cx}{1+cx}} (1+cx) - e^{\frac{2a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right) + \sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}} \Gamma\left(\frac{1}{2}, -\frac{a+b\cosh^{-1}(cx)}{b}\right) \right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[(a + b*ArcCosh[c*x])^(-3/2), x]`

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(b*c*E^(a/b)*Sqrt[a + b*ArcCosh[c*x]])
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arccosh}(cx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*arccosh(c*x))^(3/2),x)`

[Out] `int(1/(a+b*arccosh(c*x))^(3/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*arccosh(c*x) + a)^(-3/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*acosh(c*x))**(3/2),x)`

[Out] `Integral((a + b*acosh(c*x))**(-3/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccosh(c*x) + a)^(-3/2), x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*acosh(c*x))^(3/2), x)

[Out] int(1/(a + b*acosh(c*x))^(3/2), x)

$$3.157 \quad \int \frac{x^2}{(a+b \cosh^{-1}(cx))^{5/2}} dx$$

Optimal. Leaf size=276

$$\frac{2x^2 \sqrt{-1+cx} \sqrt{1+cx}}{3bc (a+b \cosh^{-1}(cx))^{3/2}} + \frac{8x}{3b^2 c^2 \sqrt{a+b \cosh^{-1}(cx)}} - \frac{4x^3}{b^2 \sqrt{a+b \cosh^{-1}(cx)}} - \frac{e^{a/b} \sqrt{\pi} \operatorname{Erf} \left(\frac{\sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{6b^{5/2} c^3}$$

[Out] $-1/6*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/b^{5/2}/c^3+1/6*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/b^{5/2}/c^3/\exp(a/b)-1/2*\exp(3*a/b)*\operatorname{erf}(3^{1/2}*(a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*3^{1/2}*\pi^{1/2}/b^{5/2}/c^3+1/2*\operatorname{erfi}(3^{1/2}*(a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*3^{1/2}*\pi^{1/2}/b^{5/2}/c^3/\exp(3*a/b)-2/3*x^2*(c*x-1)^{1/2}*(c*x+1)^{1/2}/b/c/(a+b*\operatorname{arccosh}(c*x))^{3/2}+8/3*x/b^2/c^2/(a+b*\operatorname{arccosh}(c*x))^{1/2}-4*x^3/b^2/(a+b*\operatorname{arccosh}(c*x))^{1/2}$

Rubi [A]

time = 0.86, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5886, 5951, 5887, 5556, 3389, 2211, 2236, 2235, 5881}

$$\frac{\sqrt{\pi} e^{a/b} \operatorname{Erf} \left(\frac{\sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{6b^{5/2} c^3} - \frac{\sqrt{3\pi} e^{3a/b} \operatorname{Erf} \left(\frac{\sqrt{3} \sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{2b^{5/2} c^3} + \frac{\sqrt{\pi} e^{-a/b} \operatorname{Erfi} \left(\frac{\sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{6b^{5/2} c^3} + \frac{\sqrt{3\pi} e^{-3a/b} \operatorname{Erfi} \left(\frac{\sqrt{3} \sqrt{a+b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{2b^{5/2} c^3} + \frac{8x}{3b^2 c^2 \sqrt{a+b \cosh^{-1}(cx)}} - \frac{4x^3}{b^2 \sqrt{a+b \cosh^{-1}(cx)}} - \frac{2x^2 \sqrt{cx-1} \sqrt{cx+1}}{3bc (a+b \cosh^{-1}(cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/(a + b*\operatorname{ArcCosh}[c*x])^{5/2}, x]$

[Out] $(-2*x^2*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(3*b*c*(a + b*\operatorname{ArcCosh}[c*x])^{3/2}) + (8*x)/(3*b^2*c^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]) - (4*x^3)/(b^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]) - (E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(6*b^{5/2}*c^3) - (E^{((3*a)/b)}*\operatorname{Sqrt}[3*\pi]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(2*b^{5/2}*c^3) + (\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(6*b^{5/2}*c^3*E^{(a/b)}) + (\operatorname{Sqrt}[3*\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(2*b^{5/2}*c^3*E^{((3*a)/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!TrueQ}\{ \$UseGamma \}$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] :> \operatorname{Simp}[F^{a*}\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3389

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 5556

Int[Cosh[(a_) + (b_)*(x_)]^(p_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5881

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 5886

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5887

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_), x_Symbol] := Dist[1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5951

Int[(((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_))/(Sqrt[(d1_) + (e1_)*(x_)]*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a

Mathematica [A]

time = 1.43, size = 340, normalized size = 1.23

$$\frac{e^{-3\sqrt{a+bx}} \left(4a^2 e^{3\sqrt{a+bx}} \sqrt{\frac{a+bx}{a+3b\cosh(cx)}} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) + \cosh^2(cx) - 4\sqrt{a+bx} e^{-3\sqrt{a+bx}} \left(-\frac{3\sqrt{a+bx}}{2} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) - \frac{3\sqrt{a+bx}}{2} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \right) - 24a^2 e^{-3\sqrt{a+bx}} \left(-\frac{3\sqrt{a+bx}}{2} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \right) + 4^{\frac{1}{2}} \left((1 + e^{2\sqrt{a+bx}}) \left(a \left(e^{-4\sqrt{a+bx}} + 4e^{4\sqrt{a+bx}} \right) + 4 \left(-1 + 4\cosh^2(cx) - 4e^{2\sqrt{a+bx}} \cosh^2(cx) + e^{4\sqrt{a+bx}} (1 + 4\cosh^2(cx)) \right) \right) \right) + 4\sqrt{a+bx} e^{-3\sqrt{a+bx}} \sqrt{\frac{a+bx}{a+3b\cosh(cx)}} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \right)}{12b^2 (a+3b\cosh(cx))^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*ArcCosh[c*x])^(5/2), x]

[Out] (2*E^((4*a)/b + 3*ArcCosh[c*x])*Sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCosh[c*x]))*Gamma[1/2, a/b + ArcCosh[c*x]] - 6*Sqrt[3]*b*E^(3*ArcCosh[c*x])*(-(a + b*ArcCosh[c*x])/b)^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 2*b*E^((2*a)/b + 3*ArcCosh[c*x])*(-(a + b*ArcCosh[c*x])/b)^(3/2)*Gamma[1/2, -(a + b*ArcCosh[c*x])/b] + E^((3*a)/b)*(-(1 + E^(2*ArcCosh[c*x]))*(a*(6 - 4*E^(2*ArcCosh[c*x]) + 6*E^(4*ArcCosh[c*x])) + b*(-1 + 6*ArcCosh[c*x] - 4*E^(2*ArcCosh[c*x])*ArcCosh[c*x] + E^(4*ArcCosh[c*x])*(1 + 6*ArcCosh[c*x]))) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c*x]))*Sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCosh[c*x])*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b])/((12*b^2*c^3*E^(3*(a/b + ArcCosh[c*x]))*(a + b*ArcCosh[c*x]))^(3/2))

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{arccosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*arccosh(c*x))^(5/2), x)**[Out]** int(x^2/(a+b*arccosh(c*x))^(5/2), x)**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*arccosh(c*x))^(5/2), x, algorithm="maxima")**[Out]** integrate(x^2/(b*arccosh(c*x) + a)^(5/2), x)**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*acosh(c*x))**(5/2),x)

[Out] Integral(x**2/(a + b*acosh(c*x))**(5/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")

[Out] integrate(x^2/(b*arccosh(c*x) + a)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*acosh(c*x))^(5/2),x)

[Out] int(x^2/(a + b*acosh(c*x))^(5/2), x)

$$3.158 \quad \int \frac{x}{(a+b \cosh^{-1}(cx))^{5/2}} dx$$

Optimal. Leaf size=188

$$-\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}$$

[Out] $-2/3*\exp(2*a/b)*\operatorname{erf}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(5/2)}/c^2+2/3*\operatorname{erfi}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(5/2)}/c^2/\exp(2*a/b)-2/3*x*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b/c/(a+b*\operatorname{arccosh}(c*x))^{(3/2)}+4/3/b^2/c^2/(a+b*\operatorname{arccosh}(c*x))^{(1/2)}-8/3*x^2/b^2/(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.56, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5886, 5951, 5887, 5556, 12, 3389, 2211, 2236, 2235, 5893}

$$-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} + \frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2x\sqrt{cx-1}\sqrt{cx+1}}{3bc(a+b\cosh^{-1}(cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x/(a+b*\operatorname{ArcCosh}[c*x])^{(5/2)},x]$

[Out] $(-2*x*\operatorname{Sqrt}[-1+c*x]*\operatorname{Sqrt}[1+c*x])/(3*b*c*(a+b*\operatorname{ArcCosh}[c*x])^{(3/2)}) + 4/(3*b^2*c^2*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]]) - (8*x^2)/(3*b^2*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]]) - (2*E^{((2*a)/b)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(3*b^{(5/2)}*c^2) + (2*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(3*b^{(5/2)}*c^2*E^{((2*a)/b)})$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2211

$\operatorname{Int}[(F_*)^{((g_*)*((e_*)+(f_*)(x_)))/\operatorname{Sqrt}[(c_*)+(d_*)(x_)]}, x_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+dx]], x] /;$ FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)](p_.)*((c_.) + (d_.)*(x_))(m_.)*Sinh[(a_.) +
(b_.)*(x_)](n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)m, Sinh[a +
b*x]n*Cosh[a + b*x]p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)(x_)(m_.), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x(m + 1)*((a + b*ArcCosh[c*x]
)(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x(m - 1)*((a + b*ArcCosh[c*x])(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)(x_)(m_.), x_Symbol] := Dist[
1/(b*c(m + 1)), Subst[Int[xn*Cosh[-a/b + x/b]m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5951

```

Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.))*((f_.)*(x_.))^(m_.)]/(Sqrt[(d1
_) + (e1_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] :> Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \cosh^{-1}(cx))^{5/2}} dx &= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))^{3/2}} dx}{3bc} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} + \frac{4}{3b^2c^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} + \dots
\end{aligned}$$

Mathematica [A]

time = 1.08, size = 157, normalized size = 0.84

$$\frac{2\sqrt{2\pi} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right) \left(\cosh\left(\frac{2a}{b}\right) - \sinh\left(\frac{2a}{b}\right)\right) - 2\sqrt{2\pi} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right) \left(\cosh\left(\frac{2a}{b}\right) + \sinh\left(\frac{2a}{b}\right)\right) - \frac{\sqrt{b} (4(a+b\cosh^{-1}(cx)) \cosh(2\cosh^{-1}(cx)) + b \sinh(2\cosh^{-1}(cx)))}{(a+b\cosh^{-1}(cx))^{3/2}}}{3b^{5/2}c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*ArcCosh[c*x])^(5/2), x]

```
[Out] (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 2*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (Sqrt[b]*(4*(a + b*ArcCosh[c*x])*Cosh[2*ArcCosh[c*x]] + b*Sinh[2*ArcCosh[c*x]]))/(a + b*ArcCosh[c*x])^(3/2))/(3*b^(5/2)*c^2)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arccosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^(5/2),x)
```

```
[Out] int(x/(a+b*arccosh(c*x))^(5/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccosh(c*x) + a)^(5/2), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{acosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**(5/2),x)
```

[Out] Integral(x/(a + b*acosh(c*x))**(5/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")

[Out] integrate(x/(b*arccosh(c*x) + a)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \operatorname{acosh}(cx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*acosh(c*x))^(5/2),x)

[Out] int(x/(a + b*acosh(c*x))^(5/2), x)

$$3.159 \quad \int \frac{1}{(a+b \cosh^{-1}(cx))^{5/2}} dx$$

Optimal. Leaf size=148

$$\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{3bc(a+b\cosh^{-1}(cx))^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2e^{a/b}\sqrt{\pi}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}$$

[Out] $-2/3*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*\pi^{(1/2)}/b^{(5/2)}/c+2/3*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*\pi^{(1/2)}/b^{(5/2)}/c/\exp(a/b)-2/3*(c*x-1)^{(1/2)*(c*x+1)^{(1/2)}/b/c/(a+b*\operatorname{arccosh}(c*x))^{(3/2)}-4/3*x/b^2/(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.30, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5880, 5951, 5881, 3389, 2211, 2236, 2235}

$$-\frac{2\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4x}{3b^2\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2\sqrt{cx-1}\sqrt{cx+1}}{3bc(a+b\cosh^{-1}(cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcCosh}[c*x])^{(-5/2)}, x]$

[Out] $(-2*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(3*b*c*(a + b*\operatorname{ArcCosh}[c*x])^{(3/2)}) - (4*x)/(3*b^2*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]) - (2*E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(3*b^{(5/2)}*c) + (2*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(3*b^{(5/2)}*c*E^{(a/b)})$

Rule 2211

$\operatorname{Int}[(F_{.})^{((g_{.})*(e_{.}) + (f_{.})*(x_{.}))}/\operatorname{Sqrt}[(c_{.}) + (d_{.})*(x_{.})], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_{.})^{((a_{.}) + (b_{.})*((c_{.}) + (d_{.})*(x_{.}))^2)}, x_Symbol] :> \operatorname{Simp}[F^{a*\operatorname{Sqrt}[\pi]}*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_)^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_)^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)])*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cosh^{-1}(cx))^{5/2}} dx &= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{3bc (a + b \cosh^{-1}(cx))^{3/2}} + \frac{(2c) \int \frac{x}{\sqrt{-1+cx} \sqrt{1+cx} (a+b \cosh^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{3bc (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x}{3b^2 \sqrt{a + b \cosh^{-1}(cx)}} + \frac{4 \int \frac{1}{\sqrt{a + b \cosh^{-1}(cx)}}}{3b^2} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{3bc (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x}{3b^2 \sqrt{a + b \cosh^{-1}(cx)}} - \frac{4 \operatorname{Subst} \left(\int \frac{\sinh(\frac{a}{b} - \frac{x}{b})}{\sqrt{x}} \right)}{3b^2} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{3bc (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x}{3b^2 \sqrt{a + b \cosh^{-1}(cx)}} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{-i(\frac{ia}{b} - \frac{ix}{b})}}{\sqrt{x}} \right)}{3b^2} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{3bc (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x}{3b^2 \sqrt{a + b \cosh^{-1}(cx)}} - \frac{4 \operatorname{Subst} \left(\int e^{\frac{a}{b} - \frac{x^2}{b}} dx \right)}{3b^2} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{3bc (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x}{3b^2 \sqrt{a + b \cosh^{-1}(cx)}} - \frac{2e^{a/b} \sqrt{\pi} \operatorname{erf} \left(\frac{\sqrt{a + b \cosh^{-1}(cx)}}{\sqrt{b}} \right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.72, size = 192, normalized size = 1.30

$$\frac{e^{-\frac{a+b \cosh^{-1}(cx)}{b}} \left(2e^{\frac{a}{b} + \cosh^{-1}(cx)} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} (a + b \cosh^{-1}(cx)) \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right) - 2 \left(e^{a/b} \left(b e^{\cosh^{-1}(cx)} \sqrt{\frac{-1+cx}{1+cx}} (1+cx) + (1 + e^{2 \cosh^{-1}(cx)}) (a + b \cosh^{-1}(cx)) \right) + b e^{\cosh^{-1}(cx)} \left(-\frac{a+b \cosh^{-1}(cx)}{b} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{a+b \cosh^{-1}(cx)}{b}\right) \right) \right)}{3b^2 c (a + b \cosh^{-1}(cx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcCosh[c*x])^(-5/2), x]

[Out] (2*E^((2*a)/b + ArcCosh[c*x])*Sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCosh[c*x]) *Gamma[1/2, a/b + ArcCosh[c*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c*x]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + (1 + E^(2*ArcCosh[c*x]))*(a + b*ArcCosh[c*x])) + b*E^ArcCosh[c*x]*(-(a + b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, -(a + b*ArcCosh[c*x])/b]))/(3*b^2*c*E^((a + b*ArcCosh[c*x])/b)*(a + b*ArcCosh[c*x])^(3/2))

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arccosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*arccosh(c*x))^(5/2),x)`

[Out] `int(1/(a+b*arccosh(c*x))^(5/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccosh(c*x))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*arccosh(c*x) + a)^(-5/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arccosh(c*x))^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*acosh(c*x))**(5/2),x)`

[Out] `Integral((a + b*acosh(c*x))**(-5/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arccosh(c*x))^(5/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)^(-5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*acosh(c*x))^(5/2),x)

[Out] int(1/(a + b*acosh(c*x))^(5/2), x)

$$3.160 \quad \int \frac{x^2}{(a+b \cosh^{-1}(cx))^{7/2}} dx$$

Optimal. Leaf size=361

$$-\frac{2x^2\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2(a+b\cosh^{-1}(cx))^{3/2}} + \frac{16\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c^3\sqrt{a+b\cosh^{-1}(cx)}}$$

[Out] $8/15*x/b^2/c^2/(a+b*\operatorname{arccosh}(c*x))^{(3/2)} - 4/5*x^3/b^2/(a+b*\operatorname{arccosh}(c*x))^{(3/2)}$
 $+ 1/15*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*Pi^{(1/2)}/b^{(7/2)}/c^3 +$
 $1/15*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*Pi^{(1/2)}/b^{(7/2)}/c^3/\exp(a/b) + 3$
 $/5*\exp(3*a/b)*\operatorname{erf}(3^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*3^{(1/2)}*Pi^{(1/2)}$
 $/b^{(7/2)}/c^3 + 3/5*\operatorname{erfi}(3^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*3^{(1/2)}*Pi^{(1/2)}$
 $/b^{(7/2)}/c^3/\exp(3*a/b) - 2/5*x^2*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b/c/(a+b*$
 $\operatorname{arccosh}(c*x))^{(5/2)} + 16/15*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b^3/c^3/(a+b*\operatorname{arccosh}($
 $c*x))^{(1/2)} - 24/5*x^2*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b^3/c/(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 1.08, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5886, 5951, 5885, 3388, 2211, 2236, 2235, 5880, 5953}

$$\frac{\sqrt{\pi} e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3} + \frac{3\sqrt{3\pi} e^{3a/b} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3} + \frac{\sqrt{\pi} e^{-a/b} \operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3} + \frac{3\sqrt{3\pi} e^{-3a/b} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3} + \frac{16\sqrt{cx-1}\sqrt{cx+1}}{15b^3c^3\sqrt{a+b\cosh^{-1}(cx)}} - \frac{24x^2\sqrt{cx-1}\sqrt{cx+1}}{5b^3c^3\sqrt{a+b\cosh^{-1}(cx)}} + \frac{8x}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{15b^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{2x^2\sqrt{cx-1}\sqrt{cx+1}}{5bc(a+b\cosh^{-1}(cx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/(a + b*\operatorname{ArcCosh}[c*x])^{(7/2)}, x]$

[Out] $(-2*x^2*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(5*b*c*(a + b*\operatorname{ArcCosh}[c*x])^{(5/2)}) +$
 $(8*x)/(15*b^2*c^2*(a + b*\operatorname{ArcCosh}[c*x])^{(3/2)}) - (4*x^3)/(5*b^2*(a + b*\operatorname{ArcCo}$
 $\operatorname{sh}[c*x])^{(3/2)}) + (16*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(15*b^3*c^3*\operatorname{Sqrt}[a + b*$
 $\operatorname{ArcCosh}[c*x]) - (24*x^2*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(5*b^3*c^3*\operatorname{Sqrt}[a + b*$
 $\operatorname{ArcCosh}[c*x]) + (E^{(a/b)}*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/($
 $15*b^{(7/2)}*c^3) + (3*E^{((3*a)/b)}*\operatorname{Sqrt}[3*Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}$
 $[c*x]]/\operatorname{Sqrt}[b]])/(5*b^{(7/2)}*c^3) + (\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]$
 $/\operatorname{Sqrt}[b]])/(15*b^{(7/2)}*c^3*E^{(a/b)}) + (3*\operatorname{Sqrt}[3*Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[a +$
 $b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(5*b^{(7/2)}*c^3*E^{((3*a)/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x_Symbol] :$
 $> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*$
 $x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\amp; \operatorname{!TrueQ}\{ \$UseGamma\}$

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[Fa*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[Fa*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E(I*k*Pi)*E(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E(I*k*Pi)*E(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))(n_), x_Symbol] := Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))(n_)*(x_)(m_), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b2*c(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x(n +
1), Cosh[-a/b + x/b](m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]2), x], x], x, a
+ b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))(n_)*(x_)(m_), x_Symbol] := Simp[
xm*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x(m + 1)*((a + b*ArcCosh[c*x]
)(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x(m - 1)*((a + b*ArcCosh[c*x])(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```

Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a
+ b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

```

Rule 5953

```

Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + b \cosh^{-1}(cx))^{7/2}} dx &= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4 \int \frac{x}{\sqrt{-1 + cx} \sqrt{1 + cx} (a + b \cosh^{-1}(cx))^{5/2}} dx}{5bc} + \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2 c^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2 (a + b \cosh^{-1}(cx))^{1/2}} + \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2 c^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2 (a + b \cosh^{-1}(cx))^{1/2}} + \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2 c^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2 (a + b \cosh^{-1}(cx))^{1/2}} + \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2 c^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2 (a + b \cosh^{-1}(cx))^{1/2}} + \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2 c^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2 (a + b \cosh^{-1}(cx))^{1/2}} + \\
&= -\frac{2x^2 \sqrt{-1 + cx} \sqrt{1 + cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{8x}{15b^2 c^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{4x^3}{5b^2 (a + b \cosh^{-1}(cx))^{1/2}} +
\end{aligned}$$

Mathematica [A]

time = 1.74, size = 394, normalized size = 1.09

$$-\frac{4x^3 \sqrt{-1+cx} \sqrt{1+cx}}{5b^2 (a+b \operatorname{ArcCosh}[cx])^{1/2}} + \frac{8x}{15b^2 c^2 (a+b \operatorname{ArcCosh}[cx])^{3/2}} - \frac{2x^2 \sqrt{-1+cx} \sqrt{1+cx}}{5bc (a+b \operatorname{ArcCosh}[cx])^{5/2}} + \frac{4 \int \frac{x}{\sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^{5/2}} dx}{5bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(a + b*ArcCosh[c*x])^(7/2), x]

[Out] $(-6*b^2*\sqrt{(-1 + c*x)/(1 + c*x)}*(1 + c*x) - (2*(a + b*\operatorname{ArcCosh}[c*x]))*(-2*a + b - 2*b*\operatorname{ArcCosh}[c*x] + 2*\operatorname{E}^{(a/b + \operatorname{ArcCosh}[c*x])}*\sqrt{a/b + \operatorname{ArcCosh}[c*x]})*(a + b*\operatorname{ArcCosh}[c*x])*Gamma[1/2, a/b + \operatorname{ArcCosh}[c*x]])/\operatorname{E}^{\operatorname{ArcCosh}[c*x]} - (2*(a + b*\operatorname{ArcCosh}[c*x])*(\operatorname{E}^{(a/b + \operatorname{ArcCosh}[c*x])}*(2*a + b + 2*b*\operatorname{ArcCosh}[c*x]) + 2*b*(-((a + b*\operatorname{ArcCosh}[c*x])/b))^{3/2})*Gamma[1/2, -((a + b*\operatorname{ArcCosh}[c*x])/b)$

$$\left. \right) \left. \right) / E^{(a/b)} - 3*(a + b*\text{ArcCosh}[c*x]) * ((12*\text{Sqrt}[3]*b*(-((a + b*\text{ArcCosh}[c*x]) / b))^{(3/2)} * \text{Gamma}[1/2, (-3*(a + b*\text{ArcCosh}[c*x]) / b)) / E^{((3*a)/b)} + (2*(b + 6*a*(-1 + E^{(6*\text{ArcCosh}[c*x])}) - 6*b*\text{ArcCosh}[c*x] + b * E^{(6*\text{ArcCosh}[c*x])}) * (1 + 6*\text{ArcCosh}[c*x]) + 6*\text{Sqrt}[3] * E^{(3*(a/b + \text{ArcCosh}[c*x]))} * \text{Sqrt}[a/b + \text{ArcCosh}[c*x]]) * (a + b*\text{ArcCosh}[c*x]) * \text{Gamma}[1/2, (3*(a + b*\text{ArcCosh}[c*x]) / b)) / E^{(3*\text{ArcCosh}[c*x])}) - 6*b^2 * \text{Sinh}[3*\text{ArcCosh}[c*x]]) / (60*b^3*c^3*(a + b*\text{ArcCosh}[c*x])^{(5/2)})$$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{arccosh}(cx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*arccosh(c*x))^(7/2),x)

[Out] int(x^2/(a+b*arccosh(c*x))^(7/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*arccosh(c*x))^(7/2),x, algorithm="maxima")

[Out] integrate(x^2/(b*arccosh(c*x) + a)^(7/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*arccosh(c*x))^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*acosh(c*x))**(7/2),x)

[Out] Integral(x**2/(a + b*acosh(c*x))**(7/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*arccosh(c*x))^(7/2),x, algorithm="giac")

[Out] integrate(x^2/(b*arccosh(c*x) + a)^(7/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b \operatorname{acosh}(cx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*acosh(c*x))^(7/2),x)

[Out] int(x^2/(a + b*acosh(c*x))^(7/2), x)

$$3.161 \quad \int \frac{x}{(a+b \cosh^{-1}(cx))^{7/2}} dx$$

Optimal. Leaf size=229

$$-\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{32x\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}}$$

[Out] $4/15/b^2/c^2/(a+b*\operatorname{arccosh}(c*x))^{(3/2)} - 8/15*x^2/b^2/(a+b*\operatorname{arccosh}(c*x))^{(3/2)} + 8/15*\exp(2*a/b)*\operatorname{erf}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(7/2)}/c^2 + 8/15*\operatorname{erfi}(2^{(1/2)}*(a+b*\operatorname{arccosh}(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(7/2)}/c^2/\exp(2*a/b) - 2/5*x*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b/c/(a+b*\operatorname{arccosh}(c*x))^{(5/2)} - 32/15*x*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}/b^3/c/(a+b*\operatorname{arccosh}(c*x))^{(1/2)}$

Rubi [A]

time = 0.57, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5886, 5951, 5885, 3388, 2211, 2236, 2235, 5893}

$$\frac{8\sqrt{2\pi} e^{\frac{2a}{b}} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2} + \frac{8\sqrt{2\pi} e^{-\frac{2a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2} - \frac{32x\sqrt{cx-1}\sqrt{cx+1}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{2x\sqrt{cx-1}\sqrt{cx+1}}{5bc(a+b\cosh^{-1}(cx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*ArcCosh[c*x])^(7/2), x]

[Out] $(-2*x*\operatorname{Sqrt}[-1+cx]*\operatorname{Sqrt}[1+cx])/(5*b*c*(a+b*\operatorname{ArcCosh}[c*x])^{(5/2)}) + 4/(15*b^2*c^2*(a+b*\operatorname{ArcCosh}[c*x])^{(3/2)}) - (8*x^2)/(15*b^2*(a+b*\operatorname{ArcCosh}[c*x])^{(3/2)}) - (32*x*\operatorname{Sqrt}[-1+cx]*\operatorname{Sqrt}[1+cx])/(15*b^3*c*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]]) + (8*E^{((2*a)/b)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(15*b^{(7/2)}*c^2) + (8*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+b*\operatorname{ArcCosh}[c*x]])/\operatorname{Sqrt}[b]])/(15*b^{(7/2)}*c^2*E^{((2*a)/b)})$

Rule 2211

Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e-c*(f/d))+f*g*(x^2/d)), x], x, Sqrt[c+d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F_)^((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c+d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3388

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 5885

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*(x_)^(m_), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rule 5886

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*(x_)^(m_), x_Symbol] := Simp[x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[c*(m + 1)/(b*(n + 1)), Int[x^(m + 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 5893

Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)/(Sqrt[(d1_) + (e1_)*(x_)])*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && NeQ[n, -1]

Rule 5951

Int((((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_))^(m_))/(Sqrt[(d1_) + (e1_)*(x_)])*Sqrt[(d2_) + (e2_)*(x_)]), x_Symbol] := Simp[(f*x)^m*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]

```

]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \cosh^{-1}(cx))^{7/2}} dx &= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} - \frac{2\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))^{5/2}} dx}{5bc} + \dots \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{5/2}} \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{5/2}} \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{5/2}} \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{5/2}} \\
&= -\frac{2x\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} + \frac{4}{15b^2c^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\cosh^{-1}(cx))^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 1.15, size = 175, normalized size = 0.76

$$\frac{8\sqrt{2\pi}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right) - \sinh\left(\frac{2a}{b}\right)\right) + 8\sqrt{2\pi}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right) + \sinh\left(\frac{2a}{b}\right)\right) + \frac{\sqrt{b}\left(-4b(a+b\cosh^{-1}(cx))\cosh(2\cosh^{-1}(cx)) - (3b^2+16(a+b\cosh^{-1}(cx))^2)\sinh(2\cosh^{-1}(cx))\right)}{(a+b\cosh^{-1}(cx))^{5/2}}}{15b^{7/2}c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*ArcCosh[c*x])^(7/2), x]

[Out] (8*sqrt[2]*pi)*Erfi[(sqrt[2]*sqrt[a + b*ArcCosh[c*x]])/sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + 8*sqrt[2]*pi)*Erf[(sqrt[2]*sqrt[a + b*ArcCosh[c*x]])/S

$\text{qrt}[b]] * (\text{Cosh}[(2*a)/b] + \text{Sinh}[(2*a)/b]) + (\text{Sqrt}[b] * (-4*b*(a + b*\text{ArcCosh}[c*x]) * \text{Cosh}[2*\text{ArcCosh}[c*x]] - (3*b^2 + 16*(a + b*\text{ArcCosh}[c*x])^2) * \text{Sinh}[2*\text{ArcCosh}[c*x]])) / (a + b*\text{ArcCosh}[c*x])^{(5/2)} / (15*b^{(7/2)} * c^2)$

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arccosh}(cx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*arccosh(c*x))^(7/2),x)`

[Out] `int(x/(a+b*arccosh(c*x))^(7/2),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arccosh(c*x))^(7/2),x, algorithm="maxima")`

[Out] `integrate(x/(b*arccosh(c*x) + a)^(7/2), x)`

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arccosh(c*x))^(7/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{acosh}(cx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*acosh(c*x))**(7/2),x)`

[Out] `Integral(x/(a + b*acosh(c*x))**(7/2), x)`

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arccosh(c*x))^(7/2),x, algorithm="giac")

[Out] integrate(x/(b*arccosh(c*x) + a)^(7/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a + b \operatorname{acosh}(cx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*acosh(c*x))^(7/2),x)

[Out] int(x/(a + b*acosh(c*x))^(7/2), x)

$$3.162 \quad \int \frac{1}{(a+b \cosh^{-1}(cx))^{7/2}} dx$$

Optimal. Leaf size=188

$$\frac{2\sqrt{-1+cx}\sqrt{1+cx}}{5bc(a+b\cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx}\sqrt{1+cx}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} + \frac{4e^{a/b}\sqrt{\pi}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}$$

[Out] $-4/15*x/b^2/(a+b*\operatorname{arccosh}(c*x))^{3/2}+4/15*\exp(a/b)*\operatorname{erf}((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/b^{7/2}/c+4/15*\operatorname{erfi}((a+b*\operatorname{arccosh}(c*x))^{1/2}/b^{1/2})*\pi^{1/2}/b^{7/2}/c/\exp(a/b)-2/5*(c*x-1)^{1/2}*(c*x+1)^{1/2}/b/c/(a+b*\operatorname{arccosh}(c*x))^{5/2}-8/15*(c*x-1)^{1/2}*(c*x+1)^{1/2}/b^3/c/(a+b*\operatorname{arccosh}(c*x))^{3/2}$

Rubi [A]

time = 0.48, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5880, 5951, 5953, 3388, 2211, 2236, 2235}

$$\frac{4\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c} + \frac{4\sqrt{\pi}e^{-a/b}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c} - \frac{8\sqrt{cx-1}\sqrt{cx+1}}{15b^3c\sqrt{a+b\cosh^{-1}(cx)}} - \frac{4x}{15b^2(a+b\cosh^{-1}(cx))^{3/2}} - \frac{2\sqrt{cx-1}\sqrt{cx+1}}{5bc(a+b\cosh^{-1}(cx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcCosh}[c*x])^{-7/2}, x]$

[Out] $(-2*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(5*b*c*(a + b*\operatorname{ArcCosh}[c*x])^{5/2}) - (4*x)/(15*b^2*(a + b*\operatorname{ArcCosh}[c*x])^{3/2}) - (8*\operatorname{Sqrt}[-1 + c*x]*\operatorname{Sqrt}[1 + c*x])/(15*b^3*c*\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]) + (4*E^{(a/b)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(15*b^{7/2}*c) + (4*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{ArcCosh}[c*x]]/\operatorname{Sqrt}[b]])/(15*b^{7/2}*c*E^{(a/b)})$

Rule 2211

$\operatorname{Int}[(F_)^{((g_)*(e_)+(f_)*(x_))}/\operatorname{Sqrt}[(c_)+(d_)*(x_)], x_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$ $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_)+(b_)*((c_)+(d_)*(x_))^2)}, x_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))m*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))n, x_Symbol] := Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*(a + b*ArcCosh[c*x])(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5951

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))n*((f_.)*(x_))m/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Simp[(f*x)m*((a
+ b*ArcCosh[c*x])(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)(m - 1)*(a + b*ArcCosh[c*x])(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))n*(x_)m*((d1_) + (e1_.)*(x
_))p*((d2_) + (e2_.)*(x_))p, x_Symbol] := Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)p/(1 + c*x)p]*Simp[(d2 + e2*x)p/(-1 + c*x)p], Subst[Int
[xn*Cosh[-a/b + x/b]m*Sinh[-a/b + x/b](2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cosh^{-1}(cx))^{7/2}} dx &= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} + \frac{(2c) \int \frac{x}{\sqrt{-1+cx} \sqrt{1+cx} (a+b \cosh^{-1}(cx))^{5/2}} dx}{5b} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} + \frac{4 \int \frac{1}{(a+b \cosh^{-1}(cx))}}{15b^2} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx} \sqrt{1+cx}}{15b^3 c \sqrt{a + b \cosh^{-1}(cx)}} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx} \sqrt{1+cx}}{15b^3 c \sqrt{a + b \cosh^{-1}(cx)}} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx} \sqrt{1+cx}}{15b^3 c \sqrt{a + b \cosh^{-1}(cx)}} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx} \sqrt{1+cx}}{15b^3 c \sqrt{a + b \cosh^{-1}(cx)}} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx} \sqrt{1+cx}}{15b^3 c \sqrt{a + b \cosh^{-1}(cx)}} \\
&= -\frac{2\sqrt{-1+cx} \sqrt{1+cx}}{5bc (a + b \cosh^{-1}(cx))^{5/2}} - \frac{4x}{15b^2 (a + b \cosh^{-1}(cx))^{3/2}} - \frac{8\sqrt{-1+cx} \sqrt{1+cx}}{15b^3 c \sqrt{a + b \cosh^{-1}(cx)}}
\end{aligned}$$

Mathematica [A]

time = 0.49, size = 214, normalized size = 1.14

$$\frac{-6\sqrt{\frac{-1+cx}{1+cx}}(1+cx) - \frac{2c^{-\cosh^{-1}(cx)}(a+b \cosh^{-1}(cx)) \left(-2a+b-2b \cosh^{-1}(cx) + 2c^{\frac{a}{b} + \cosh^{-1}(cx)} \sqrt{\frac{a}{b} + \cosh^{-1}(cx)} (a+b \cosh^{-1}(cx)) \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(cx)\right) \right)}{b^2} - \frac{2c^{-\frac{a}{b} + \cosh^{-1}(cx)} \left(c^{\frac{a}{b} + \cosh^{-1}(cx)} (2a+b+2b \cosh^{-1}(cx)) + 2b \left(-\frac{a+b \cosh^{-1}(cx)}{b} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{a+b \cosh^{-1}(cx)}{b}\right) \right)}{b^2}}{15bc (a + b \cosh^{-1}(cx))^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcCosh[c*x])^(-7/2), x]

[Out] (-6*sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - (2*(a + b*ArcCosh[c*x])*(-2*a + b - 2*b*ArcCosh[c*x] + 2*E^(a/b + ArcCosh[c*x])*sqrt[a/b + ArcCosh[c*x]]*(a + b*ArcCosh[c*x])*Gamma[1/2, a/b + ArcCosh[c*x]]))/(b^2*E^ArcCosh[c*x]) - (2*(a + b*ArcCosh[c*x])*(E^(a/b + ArcCosh[c*x])*(2*a + b + 2*b*ArcCosh[c*x]

) + 2*b*(-((a + b*ArcCosh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)]/((b^2*E^(a/b)))/(15*b*c*(a + b*ArcCosh[c*x])^(5/2))

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arccosh}(cx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*arccosh(c*x))^(7/2),x)

[Out] int(1/(a+b*arccosh(c*x))^(7/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arccosh(c*x))^(7/2),x, algorithm="maxima")

[Out] integrate((b*arccosh(c*x) + a)^(-7/2), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arccosh(c*x))^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*acosh(c*x))**(7/2),x)

[Out] Integral((a + b*acosh(c*x))**(-7/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*arccosh(c*x))^(7/2),x, algorithm="giac")``[Out] integrate((b*arccosh(c*x) + a)^(-7/2), x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a + b*acosh(c*x))^(7/2),x)``[Out] int(1/(a + b*acosh(c*x))^(7/2), x)`

3.163 $\int \sqrt{fx} (a + b \cosh^{-1}(cx))^2 dx$

Optimal. Leaf size=128

$$\frac{2(fx)^{3/2} (a + b \cosh^{-1}(cx))^2}{3f} - \frac{8bc(fx)^{5/2} \sqrt{1 - cx} (a + b \cosh^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2 x^2\right)}{15f^2 \sqrt{-1 + cx}} - \frac{16b^2 c^2 (fx)^{7/2} {}_3F_2\left(\frac{7}{4}, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; c^2 x^2\right)}{105f^3}$$

[Out] $2/3*(f*x)^{(3/2)}*(a+b*\operatorname{arccosh}(c*x))^{2/f}-16/105*b^2*c^2*(f*x)^{(7/2)}*\operatorname{hypergeom}([1, 7/4, 7/4], [9/4, 11/4], c^2*x^2)/f^3-8/15*b*c*(f*x)^{(5/2)}*(a+b*\operatorname{arccosh}(c*x))*\operatorname{hypergeom}([1/2, 5/4], [9/4], c^2*x^2)*(-c*x+1)^{(1/2)}/f^2/(c*x-1)^{(1/2)}$

Rubi [A]

time = 0.20, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5883, 5949}

$$-\frac{16b^2c^2(fx)^{7/2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}; c^2x^2\right)}{105f^3} - \frac{8bc\sqrt{1-cx} (fx)^{5/2} {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2x^2\right) (a + b \cosh^{-1}(cx))}{15f^2\sqrt{cx-1}} + \frac{2(fx)^{3/2} (a + b \cosh^{-1}(cx))^2}{3f}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[f*x]*(a + b*ArcCosh[c*x])^2,x]`

[Out] $(2*(f*x)^{(3/2)}*(a + b*\operatorname{ArcCosh}[c*x])^2)/(3*f) - (8*b*c*(f*x)^{(5/2)}*\operatorname{Sqrt}[1 - c*x]*(a + b*\operatorname{ArcCosh}[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(15*f^2*\operatorname{Sqrt}[-1 + c*x]) - (16*b^2*c^2*(f*x)^{(7/2)}*\operatorname{HypergeometricPFQ}[\{1, 7/4, 7/4\}, \{9/4, 11/4\}, c^2*x^2])/(105*f^3)$

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.))/(Sqrt[(d1_) + (
e1_.)*(x_.)]*Sqrt[(d2_) + (e2_.)*(x_.)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[\{1, 1 + m/2,
1 + m/2\}, \{3/2 + m/2, 2 + m/2\}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{fx} (a + b \cosh^{-1}(cx))^2 dx = \frac{2(fx)^{3/2} (a + b \cosh^{-1}(cx))^2}{3f} - \frac{(4bc) \int \frac{(fx)^{3/2} (a + b \cosh^{-1}(cx))}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx}{3f}$$

$$= \frac{2(fx)^{3/2} (a + b \cosh^{-1}(cx))^2}{3f} - \frac{8bc(fx)^{5/2} \sqrt{1 - c^2 x^2} (a + b \cosh^{-1}(cx))}{15f^2 \sqrt{-1 + cx} \sqrt{1 + cx}}$$

Mathematica [A]

time = 0.28, size = 118, normalized size = 0.92

$$\frac{2}{105} x \sqrt{fx} \left(35(a + b \cosh^{-1}(cx))^2 - 4bcx \left(\frac{7\sqrt{1 - c^2 x^2} (a + b \cosh^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2 x^2\right)}{\sqrt{-1 + cx} \sqrt{1 + cx}} + 2bc {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; c^2 x^2\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[f*x]*(a + b*ArcCosh[c*x])^2,x]

[Out] (2*x*Sqrt[f*x]*(35*(a + b*ArcCosh[c*x])^2 - 4*b*c*x*((7*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 2*b*c*x*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2]))) / 105

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(cx))^2 \sqrt{fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))^2*(f*x)^(1/2),x)

[Out] int((a+b*arccosh(c*x))^2*(f*x)^(1/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^2*(f*x)^(1/2),x, algorithm="maxima")

[Out] 2/3*b^2*sqrt(f)*x^(3/2)*log(c*x + sqrt(c*x + 1))*sqrt(c*x - 1)^2 + 2/3*(f*x)^(3/2)*a^2/f + integrate(2/3*((3*a*b*c^2*sqrt(f) - 2*b^2*c^2*sqrt(f))*x^2

```
- 3*a*b*sqrt(f))*sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(x) + ((3*a*b*c^3*sqrt(f)
- 2*b^2*c^3*sqrt(f))*x^3 - (3*a*b*c*sqrt(f) - 2*b^2*c*sqrt(f))*x)*sqrt(x))
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x +
1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(f*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(f*x), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{fx} (a + b \operatorname{acosh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(a + b*acosh(c*x))**2, x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(cx))^2 \sqrt{fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(f*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(f*x)^(1/2), x)
```


3.164 $\int (dx)^m (a + b \cosh^{-1}(cx))^2 dx$

Optimal. Leaf size=181

$$\frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))^2}{d(1+m)} - \frac{2bc(dx)^{2+m} \sqrt{1-cx} (a + b \cosh^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; c^2x^2\right)}{d^2(1+m)(2+m)\sqrt{-1+cx}} - \frac{2b^2c^2(dx)^{3+m}}{d^3(1+m)(2+m)(3+m)}$$

[Out] (d*x)^(1+m)*(a+b*arccosh(c*x))^2/d/(1+m)-2*b^2*c^2*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m], [2+1/2*m, 5/2+1/2*m], c^2*x^2)/d^3/(3+m)/(m^2+3*m+2)-2*b*c*(d*x)^(2+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)*(-c*x+1)^(1/2)/d^2/(1+m)/(2+m)/(c*x-1)^(1/2)

Rubi [A]

time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$,

Rules used = {5883, 5949}

$$\frac{2b^2c^2(dx)^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2x^2\right)}{d^3(m+1)(m+2)(m+3)} - \frac{2bc\sqrt{1-cx} (dx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; c^2x^2\right) (a + b \cosh^{-1}(cx))}{d^2(m+1)(m+2)\sqrt{cx-1}} + \frac{(dx)^{m+1} (a + b \cosh^{-1}(cx))^2}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*ArcCosh[c*x])^2,x]

[Out] ((d*x)^(1+m)*(a + b*ArcCosh[c*x])^2)/(d*(1+m)) - (2*b*c*(d*x)^(2+m)*Sqrt[1 - c*x]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(d^2*(1+m)*(2+m)*Sqrt[-1 + c*x]) - (2*b^2*c^2*(d*x)^(3+m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(d^3*(1+m)*(2+m)*(3+m))

Rule 5883

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol] :> Simp[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^n/(d*(m+1))), x] - Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCosh[c*x])^(n-1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 5949

Int[(((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.))/(Sqrt[(d1_.) + (e1_.)*(x_.)]*Sqrt[(d2_.) + (e2_.)*(x_.)]), x_Symbol] :> Simp[((f*x)^(m+1)/(f*(m+1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m+2)/(f^2*(m+1)*(m+2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2

, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && !IntegerQ[m]

Rubi steps

$$\int (dx)^m (a + b \cosh^{-1}(cx))^2 dx = \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))^2}{d(1+m)} - \frac{(2bc) \int \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))}{\sqrt{-1+cx} \sqrt{1+cx}} dx}{d(1+m)}$$

$$= \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))^2}{d(1+m)} - \frac{2bc(dx)^{2+m} \sqrt{1-c^2x^2} (a + b \cosh^{-1}(cx))}{d^2(1+m)(2+m)\sqrt{-1+cx}}$$

Mathematica [A]

time = 0.17, size = 164, normalized size = 0.91

$$\frac{x(dx)^m \left((a + b \cosh^{-1}(cx))^2 - \frac{2bcx\sqrt{1-c^2x^2} (a + b \cosh^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{(2+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2c^2x^2 {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}, c^2x^2\right)}{6+5m+m^2} \right)}{1+m}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*ArcCosh[c*x])^2,x]

[Out] (x*(d*x)^m*((a + b*ArcCosh[c*x])^2 - (2*b*c*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b^2*c^2*x^2*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2)))/(1 + m)

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arccosh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a+b*arccosh(c*x))^2,x)

[Out] int((d*x)^m*(a+b*arccosh(c*x))^2,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arccosh(c*x))^2,x, algorithm="maxima")

```
[Out] b^2*d^m*x*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)) + integrate(-2*((a*b*d^m*(m + 1) - (a*b*c^2*d^m*(m + 1) - b^2*c^2*d^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m - ((a*b*c^3*d^m*(m + 1) - b^2*c^3*d^m)*x^3 - (a*b*c*d^m*(m + 1) - b^2*c*d^m)*x)*x^m)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*(m + 1)*x^3 - c*(m + 1)*x + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*(d*x)^m, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acosh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*acosh(c*x))**2, x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(d*x)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(cx))^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d*x)^m, x)
```

3.165 $\int (dx)^m (a + b \cosh^{-1}(cx)) dx$

Optimal. Leaf size=106

$$\frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))}{d(1+m)} - \frac{bc(dx)^{2+m} \sqrt{1-c^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; c^2x^2\right)}{d^2(1+m)(2+m)\sqrt{-1+cx}\sqrt{1+cx}}$$

[Out] (d*x)^(1+m)*(a+b*arccosh(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^2/(1+m)/(2+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,

Rules used = {5883, 127, 372, 371}

$$\frac{(dx)^{m+1} (a + b \cosh^{-1}(cx))}{d(m+1)} - \frac{bc\sqrt{1-c^2x^2} (dx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; c^2x^2\right)}{d^2(m+1)(m+2)\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*ArcCosh[c*x]),x]

[Out] ((d*x)^(1+m)*(a + b*ArcCosh[c*x]))/(d*(1+m)) - (b*c*(d*x)^(2+m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(d^2*(1+m)*(2+m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Rule 127

Int[((f_)*(x_))^(p_)*((a_) + (b_)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]

Rule 371

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*((c*x)^(m+1)/(c*(m+1)))*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 372

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x]))], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \cosh^{-1}(cx)) dx &= \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))}{d(1+m)} - \frac{(bc) \int \frac{(dx)^{1+m}}{\sqrt{-1+cx} \sqrt{1+cx}} dx}{d(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))}{d(1+m)} - \frac{(bc\sqrt{-1+c^2x^2}) \int \frac{(dx)^{1+m}}{\sqrt{-1+c^2x^2}} dx}{d(1+m)\sqrt{-1+cx} \sqrt{1+cx}} \\ &= \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))}{d(1+m)} - \frac{(bc\sqrt{1-c^2x^2}) \int \frac{(dx)^{1+m}}{\sqrt{1-c^2x^2}} dx}{d(1+m)\sqrt{-1+cx} \sqrt{1+cx}} \\ &= \frac{(dx)^{1+m} (a + b \cosh^{-1}(cx))}{d(1+m)} - \frac{bc(dx)^{2+m} \sqrt{1-c^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; c^2x^2\right)}{d^2(1+m)(2+m)\sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

Mathematica [A]

time = 0.10, size = 87, normalized size = 0.82

$$\frac{x(dx)^m \left(a + b \cosh^{-1}(cx) - \frac{bcx\sqrt{1-c^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; c^2x^2\right)}{(2+m)\sqrt{-1+cx} \sqrt{1+cx}} \right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (x*(d*x)^m*(a + b*ArcCosh[c*x] - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1
[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)))/(1 + m)
```

Maple [F]

time = 9.59, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arccosh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccosh(c*x)),x)
[Out] int((d*x)^m*(a+b*arccosh(c*x)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] -(c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 - m - 1), x) - c*d^m*integrate
(x*x^m/(c^3*(m + 1)*x^3 - c*(m + 1)*x + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c*x
+ 1)*sqrt(c*x - 1)), x) - d^m*x*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/
(m + 1))*b + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acosh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acosh(c*x)),x)
[Out] Integral((d*x)**m*(a + b*acosh(c*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*(d*x)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(cx)) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*acosh(c*x))*(d*x)^m, x)`

[Out] `int((a + b*acosh(c*x))*(d*x)^m, x)`

$$3.166 \quad \int \frac{(dx)^m}{a+b \cosh^{-1}(cx)} dx$$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b \cosh^{-1}(cx)}, x\right)$$

[Out] Unintegrable((d*x)^m/(a+b*arccosh(c*x)), x)

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b \cosh^{-1}(cx)} dx$$

Verification is not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcCosh[c*x]), x]

[Out] Defer[Int] [(d*x)^m/(a + b*ArcCosh[c*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \cosh^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b \cosh^{-1}(cx)} dx$$

Mathematica [A]

time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b \cosh^{-1}(cx)} dx$$

Verification is not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcCosh[c*x]), x]

[Out] Integrate[(d*x)^m/(a + b*ArcCosh[c*x]), x]

Maple [A]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b \operatorname{arccosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m/(a+b*arccosh(c*x)),x)`

[Out] `int((d*x)^m/(a+b*arccosh(c*x)),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arccosh(c*x)),x, algorithm="maxima")`

[Out] `integrate((d*x)^m/(b*arccosh(c*x) + a), x)`

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arccosh(c*x)),x, algorithm="fricas")`

[Out] `integral((d*x)^m/(b*arccosh(c*x) + a), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m/(a+b*acosh(c*x)),x)`

[Out] `Integral((d*x)**m/(a + b*acosh(c*x)), x)`

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arccosh(c*x)),x, algorithm="giac")`

[Out] `integrate((d*x)^m/(b*arccosh(c*x) + a), x)`

Mupad [A]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^m}{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*acosh(c*x)),x)
```

```
[Out] int((d*x)^m/(a + b*acosh(c*x)), x)
```

Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```

```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```

(*9 = unknown function*)

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3, ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
  If[Head[expn]===RootSum,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
  9]]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, CsCh,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,

```

```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
  fi;

  leaf_count_optimal := leafcount(optimal);
  ExpnType_result := ExpnType(result);
  ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex

```



```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```

```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```
#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#      Albert Rich to use with Sagemath. This is used to
#      grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#      'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#      issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #instance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```



```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```